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Numerical Solution of Generalised Burgers-Huxley Equation Using Nodal Integral Method

Kushal Dinkar Badgujar, Suneet Singh

Abstract- In this article, a modified nodal integral method (MNIM) is developed to solve generalised Burgers-Huxley equation which is governing equation for various nonlinear wave phenomena. Nodal integral methods have been earlier used for solving differential and partial differential equations in various areas of physics. These methods have been known to be significantly more accurate than traditional finite volume/difference approaches. The developed scheme is verified by comparing it to analytical solutions of the equation for different values of parameters present in this equation. It is observed that MNIM yields quite accurate results even with coarse grids.

Keywords: modified nodal integral method, solve generalised Burgers-Huxley equation, nonlinear wave phenomena. Nodal integral methods.

I. INTRODUCTION

Burgers-Huxley equation has been used in many fields of chemistry, biology, combustion, metallurgy, mathematics, and engineering. There are various schemes to solve Burgers-Huxley equation analytically using different transformations. Few methods are available to solve Burgers-Huxley equation numerically. Wang et. al. have obtained solution of Burgers-Huxley analytically using relevant transformation [1]. Yefimova and Kudryashov have got exact solution of Burgers-Huxley equation through Cole-Hope transformation [2]. J. Diaz, J.Ruiz-Ramirez, J. Villa have developed a conditionally bounded and symmetry-preserving scheme to solve Burgers-Huxley numerically [3].Uri Ascher, Steven Ruuth, Raymond Spiteri have designed a Implicit-Explicit Runge-Kuttamethods for numerical solution of Burgers equation[4]. Ismail et al. [5] and Hashim et al. [6,7] have solved Burger's–Huxley equation using Adomian decomposition method. Rathish Kumar et al. have proposed three-step Taylor–Galerkin Finite Element Method (3TGFEM) to analyze the behavior of the Singularly Perturbed Generalized Burgers–Huxley (SPGBH) equation [8]. Kaushik and Sharma [9] have constructed a uniformly convergent numerical method on non-uniform mesh for singularly perturbed unsteady Burger–Huxley equation. Sari et al. have established the numerical solutions of the corresponding equation, by combining the high-order schemes in space and a fourth-order Runge-Kutta scheme in time [10]. Mohammadi R. found the cubic B-spline collocation scheme to find numerical solution of the generalized Burger's–Huxley equation which is based on the finite-difference formulation for time integration and cubic B-spline functions for space integration [11]. Nodal methods are classified under the class of coarse-mesh methods. The governing differential equations are approximately satisfied the on finite size brick-like elements which are obtained by discretizing the space of independent variables of the problem. The schemes were called nodal because in the early development of nodal schemes, these brick-like elements were referred to as nodes. Nodes in nodal methods are analogous to the elements of the finite element approach. As in the space–time finite element method (FEM), time in the nodal approach is treated in the similar manner as any spatial direction. Nuclear industry has utilized the full advantage of developments in coarse mesh methods. Nuclear industry's complex equations such as neutron diffusion and neutron transport [12–17] are based on coarse mesh methods. Lawrence [18] has given a review of nodal methods which are developed and used by the nuclear industry. Coarse-mesh schemes are also extensively used for fluid flow and heat transfer problems [19-22]. Some other branches of science and engineering are also taking benefits of similar approaches to develop efficient schemes. Hennert has presented Nodal methods as a general class of computational schemes [23]. Nodal integral method is applied to the steady-state Boussinesq equations for natural convection, as well as to several steady-state incompressible flow problems [24,25,26]. Esser and Witt [27] came up with a nodal scheme for the two-dimensional, vorticity-stream function formulation of the well known Navier–Stokes equations. A second and a third-order nodal integral method has been developed by Michael et al. [28] for Convection–diffusion equation, and further its results are compared with that of using the LECUSSO scheme [29]. They proved that the nodal integral method is as accurate as LECUSSO scheme, as well as it significantly takes less CPU time than the very efficient LECUSSO scheme. Nodal integral method is also formulated and used for the time-dependent heat conduction problem. Rizwan-uddin has developed modified nodal integral method to solve a burgers equation numerically [30,31]. In this paper, modified nodal integral method is extended to solve Burgers-Huxley equation numerically. The significance of the nodal integral method is transverse integration process in which PDE is converted into ODE, thus m set of ODEs are generated for each of PDE where m is number of independent variables. The set of ODEs can be solved easily to obtain solution of original PDE. Since closed form exact solution of set of ODEs are not possible, only linear part of ODEs is solved while nonlinear part is treated as inhomogeneous or pseudo-source termwith known particular solution.

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As linear part of ODEs is solved exactly, fundamental solution relevant to the problem is guaranteed. The fundamental solution can be linear, quadratic, trigonometric, exponential (hyperbolic) depending on linear part of ODEs. In following sections Burgers-Huxley equation is stated and solved numerically using modified nodal integral method.

II. BURGERS-HUXLEY EQUATION

The generalised form of Burgers-Huxley equation is written as follows [1,2].

$$\frac{\partial u}{\partial t} + \alpha u^{\delta}\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \beta f(u) \qquad (1)$$

where $f(u) = \left(1-u^{\delta}\right)(u^{\delta}-\gamma)$. Here α,β are non negative numbers and γ has value in the range of (0,1). For $\alpha = 0$ and $\delta = 1$ above equation becomes Huxley equation which represent pulse propagation in nerve fiber or wall motion in liquid crystal. Moreover, for $\beta = 0,\ \alpha = 1$ it is Burger's equation. It is also pointed out that it is well known generalized Burgers equation for $\delta > 1$ and $\beta = 0, \alpha = 0$ which is prototype model for describing the interaction between reaction mechanisms, convection effects and diffusion transport.

The initial condition and boundary conditions for Burgers-Huxley equation are given as follows.

$$u(x,0) = \left\{\frac{\gamma}{2} + \frac{\gamma}{2}tanh(C_1 x)\right\}^{\frac{1}{\delta}} \qquad \text{for every } x \in [x_l, x_r],$$

$$u(x_l,t) = \left\{\frac{\gamma}{2} + \frac{\gamma}{2}tanh[C_1(x_l - C_2.t)]\right\}^{\frac{1}{\delta}} \qquad \text{for every } t \geq 0\,,$$

$$u(x_r,t) = \left\{\frac{\gamma}{2} + \frac{\gamma}{2}tanh[C_1(x_r - C_2.t)]\right\}^{\frac{1}{\delta}} \qquad \text{for every } t \geq 0\,,$$

where the constants C_1 and C_2 are given by

$$C_1 = \frac{-\alpha\delta + \delta\sqrt{\alpha^2 + 4\beta(1+\delta)}}{4(1+\delta)}.\gamma,$$

$$C_2 = \frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)(-\alpha + \sqrt{\alpha^2+4\beta(1+\delta)})}{2(1+\delta)}.\gamma,$$

The particular solution of the Burgers-Huxley equation is given by the expression

$$u(x,t) = \left(\frac{\gamma}{2} + \frac{\gamma}{2}tanh(C_1(x - C_2.t))\right)^{\frac{1}{\delta}}$$

III. METHODOLOGY

The spatial domain, which has lower and upper bound as x_l and x_r is divided into m number of cells of size $2a$ as shown in Fig. 1, The temporal domain is divided into equal time steps of size 2τ. The space is indexed with subscript i while time is indexed with subscript j. Fig. 1 shows the discretisation of temporal as well as spatial domain. The nodes are identified by its coordinates (*i*,*j*). A node centred local coordinate system is defined such that a cell is given by $-a \leq x \leq a$ and $-\tau \leq x \leq \tau$ as shown in Fig. 2. Averaging equation (1) over node (*i*,*j*) in independent variable x and t (i.e. operating with $\frac{1}{2a}\int_{-a}^{a} dx$ and with $\frac{1}{2\tau}\int_{-\tau}^{\tau} d\tau$) respectively, following set of ordinary differential equations is obtained,

$$\frac{d\bar{u}^x(t)}{dt} = \frac{1}{2a}\int_{-a}^{a}\left(\frac{\partial^2 u(x,t)}{\partial^2 x} - \alpha u(x,t)^{\delta}\frac{\partial u(x,t)}{\partial x} + \beta f(u)\right)dx \qquad (2)$$

$$\frac{d^2\bar{u}^t(x)}{d^2 x} - \alpha\overline{u^{\delta}}^t(x)\frac{d\bar{u}^t(x)}{dx} = \frac{1}{2\tau}\int_{-\tau}^{\tau}\left(\frac{\partial u(x,t)}{\partial t} + \beta f(u)\right)dt \qquad (3)$$

The time averaged and space averaged velocities are given as follows,

$$\bar{u}_{i,j}^x(t) = \frac{1}{2a}\int_{-a}^{a} u(x,t)dx \qquad (4)$$

$$\bar{u}_{i,j}^t(x) = \frac{1}{2\tau}\int_{-\tau}^{\tau} u(x,t)d\tau \qquad (5)$$

where pseudo source terms are defined as

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$$\bar{s}_1^x(t) = \frac{d\bar{u}^x(t)}{dt} \tag{6}$$

$$\bar{s}_2^t(x) = \frac{d^2\bar{u}^t(x)}{d^2x} - \alpha\overline{u^{\delta}}^t(x)\frac{d\bar{u}^t(x)}{dx} \tag{7}$$

It should also be noted that following approximation is used to get equations (2) and (3) referred from [31,32].

$$\frac{1}{2\tau}\int_{-\tau}^{\tau}\left[\alpha u^{\delta}(x)\frac{d\bar{u}^t(x,t)}{dx}\right] \approx \left[\frac{1}{2\tau}\int_{-\tau}^{\tau}\alpha u^{\delta}(x)\right]\left[\frac{1}{2\tau}\int_{-\tau}^{\tau}\frac{d\bar{u}^t(x,t)}{dx}\right]$$

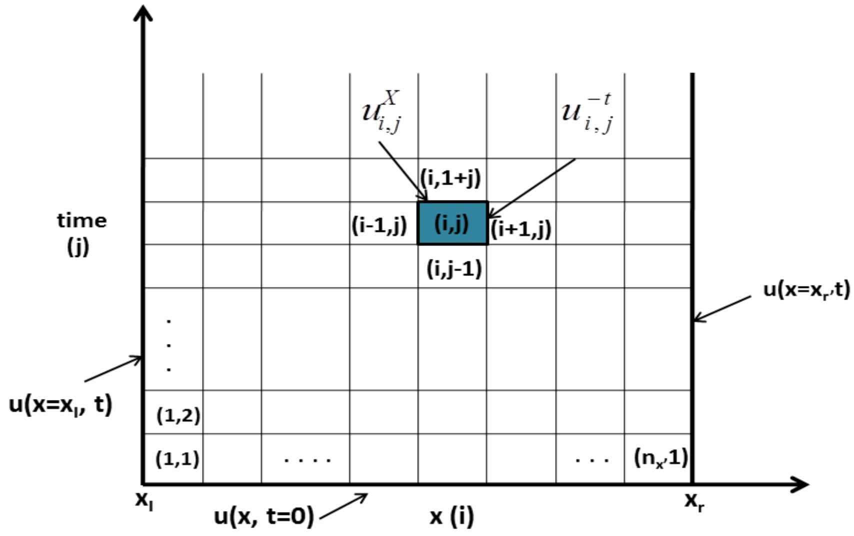

Figure 1: Discretisation in time and space domain. A node (i,j) and its surrounding nodes.

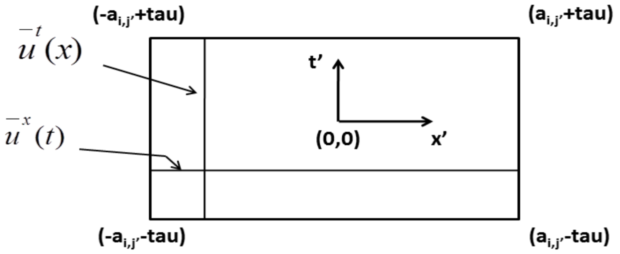

Figure 2: Dimensions of each node in local coordinate system andaveragevelocities $\bar{u}_{i,j}^x(t), \bar{u}_{i,j}^t(t)$.

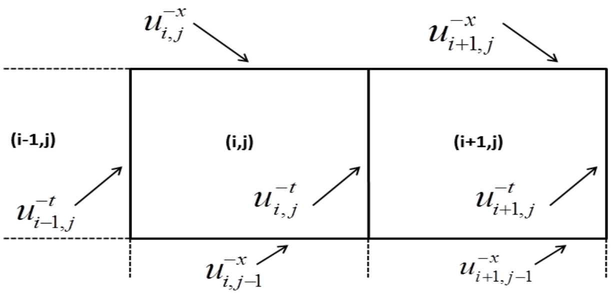

Figure 3: Surface averaged velocities of the nodes

The pseudo-source term Eq. (6) is expanded in Legendre polynomials andtruncated at zeroth order. is the equation is integrated from $t = -\tau$ to t yielding,

$$\bar{u}^x(t) = \bar{u}^x(-\tau) + \bar{s}_1^{x,t}(\tau + t) \tag{8}$$

Substituting $t = +\tau$ gives rise to following result

$$\bar{u}^x(\tau) = \bar{u}^x(-\tau) + 2\tau\bar{s}_1^{x,t} \tag{9}$$

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Using the definition of surface averaged variables as defined in Fig. 3, the previous equation can be written as

$$\bar{u}_{i,j}^{x} = \bar{u}_{i,j-1}^{x} + 2\tau\bar{s}_{1,(i,j)}^{x,t} \quad (9)$$

Now, expanding the pseudo source term in equation (7)and truncating it at zeroth order,

$$\frac{d^2\bar{u}^t(x)}{d^2x} - \alpha\overline{u^{\delta}}^t(x)\frac{d\bar{u}^t(x)}{dx} = \bar{s}_2^{t,x}(x) \quad (10)$$

The term $\alpha\overline{u^{\delta}}^t(x)$is space-dependent time averaged velocity in nonlinear convection term $\alpha\overline{u^{\delta}}^t(x)\frac{d\bar{u}^t(x)}{dx}$. It is calculated as average of time averaged velocity $\overline{u^{\delta}}^t$ at $x = +a$ and$x = -a$. It is denoted by $u_{i,j}^0$

$$u_{i,j}^0 = \alpha\frac{\left[\overline{u^{\delta}}^t{}_{i,j} + \overline{u^{\delta}}^t{}_{i-1,j}\right]}{2}$$

Solving Eq. (10) in terms of derivative $\frac{d\bar{u}^t}{dx}$ at $x = -a$, yields

$$\bar{u}^t(t) = \bar{u}^t(-a) + \frac{d\bar{u}^t(-a)}{dx}\left(\frac{e^{\left[u_{i,j}^0(x+a)\right]} - 1}{u_{i,j}^0}\right) + \frac{\bar{s}_2^{t,x}}{u_{i,j}^0}\left(\frac{e^{[\,u_{i,j}^0(x+a)]} - 1}{u_{i,j}^0} - (x+a)\right) \quad (11)$$

In terms of derivative $\frac{d\bar{u}^t}{dx}$at $x = a$, the solution of the same equation is,

$$\bar{u}^t(x) = \bar{u}^t(a) - \frac{d\bar{u}^t(a)}{dx}\left(\frac{1 - e^{-\left[u_{i,j}^0(a-x)\right]}}{u_{i,j}^0}\right) - \frac{\bar{s}_2^{t,x}}{u_{i,j}^0}\left(\frac{1 - e^{-\left[u_{i,j}^0(a-x)\right]}}{u_{i,j}^0} - (a-x)\right) \quad (12)$$

By changing subscript i to $i+1$ in Eq. (11) $\frac{d\bar{u}_{i+1,j}^t(-a)}{dx}$ can be written as,

$$\frac{d\bar{u}_{i+1,j}^t(-a)}{dx} = \frac{u_{i+1,j}^0}{\left[e^{\left(2\,u_{i+1,j}^0\right)} - 1\right]}\left[\bar{u}^t(a) - \bar{u}^t(-a)\right] - \bar{s}_{2,(i+1,j)}^{t,x}\left[\frac{1}{u_{i+1,j}^0} - \frac{2a}{\left[e^{2u_{i+1,j}^0 a} - 1\right]}\right] \quad (13)$$

In above equation, the term $\frac{d\bar{u}_{i+1,j}^t(-a)}{dx}$ indicates the derivative of the velocity at the left boundary of node (i+1,j). Similarly, Eq.(12) can be evaluated at $x = +a$ and solved for $\frac{d\bar{u}_{i,j}^t(a)}{dx}$

$$\frac{d\bar{u}_{i,j}^t(a)}{dx} = \frac{u_{i,j}^0}{\left[1 - e^{\left(2\,u_{i,j}^0\right)}\right]}\left[\bar{u}^t(a) - \bar{u}^t(-a)\right] - \bar{s}_{2,(i,j)}^{t,x}\left[\frac{1}{u_{i,j}^0} - \frac{2a}{\left[1 - e^{-2u_{i,j}^0}\right]}\right] \quad (14)$$

In above equation, the term $\frac{d\bar{u}_{i,j}^t(a)}{dx}$ indicates the derivative of the velocity at the right boundary of node (i,j).Since, at the interface between i^{th} and $i+1^{th}$ spatial node the derivatives $\frac{d\bar{u}_{i+1,j}^t(-a)}{dx}$ and $\frac{d\bar{u}_{i,j}^t(a)}{dx}$ must be same, leading to a relation among $u_{i+1,j}^t$, $u_{i,j}^t$ and $u_{i-1,j}^t$

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$$\left\{\frac{u_{i,j}^0}{\left[1-e^{-2\,u_{i,j}^0 a}\right]}\right\}u_{i-1,j}^t-\left\{\frac{u_{i,j}^0}{\left[1-e^{-2\,u_{i,j}^0 a}\right]}+\frac{u_{i+1,j}^0}{\left[e^{2\,u_{i+1,j}^0 a}-1\right]}\right\}u_{i,j}^t+\left\{\frac{u_{i+1,j}^0}{\left[e^{2\,u_{i+1,j}^0 a}-1\right]}\right\}u_{i+1,j}^t$$
$$=\left\{\frac{2a}{\left[1-e^{-2\,u_{i,j}^0 a}\right]}-\frac{1}{u_{i,j}^0}\right\}\bar{s}_{2,(i,j)}^{t,x}+\left\{\frac{1}{u_{i+1,j}^0}-\frac{2a}{\left[1-e^{-2\,u_{i+1,j}^0 a}\right]}\right\}\bar{s}_{2,(i+1,j)}^{t,x} \qquad (15)$$

The above equation can be written in following compact form

$$K_{i,j}\bar{u}_{i-1,j}^t-L_{i,j}u_{i,j}^t+M_{i+1,j}u_{i+1,j}^t=N_{i,j}\bar{s}_{2,(i,j)}^{t,x}+O_{i+1,j}\bar{s}_{2,(i+1,j)}^{t,x} \qquad (16)$$

Here, coefficients in the curly brackets have been replaced by symbols for sake of clarity and brevity. It is also pointed out that surface averaged velocities such as $\bar{u}_{i-1,j}^t$ are defined in Fig. 3. We have four unknowns i.e.$\bar{u}_{i,j}^t, \bar{u}_{i,j}^x, \bar{s}_{1,(i,j)}^{x,t}, \bar{s}_{2,(i,j)}^{t,x}$ and till now we have two Eqs. (9and13) relating them so we need two more constrain to solve them. We obtain the first constrain by double integrating each term of Eq. (1) with$\frac{1}{4a\tau}\int_{-\tau}^{\tau}\int_{-a}^{a}dtdx$,

$$\frac{1}{2\tau}\int_{-\tau}^{\tau}\frac{d\bar{u}^x(t)}{dt}dt=\frac{1}{2a}\int_{-a}^{a}\left(\frac{d^2\bar{u}^t(x)}{dx^2}-\bar{u}^t(x)\frac{d\bar{u}^t(x)}{dx}+\frac{1}{2\tau}\int_{-\tau}^{\tau}\beta f(u)\right)dx \qquad (17)$$

Considering the definition of $\bar{s}_{1,(i,j)}^{x}$and $\bar{s}_{2,(i,j)}^{t}$ above equation simplifies as

$$\bar{s}_{1,(i,j)}^{x,t}=\bar{s}_{2,(i,j)}^{t,x}+\frac{1}{4a\tau}\int_{-\tau}^{\tau}\int_{-a}^{a}\beta f(u)dtdx \qquad (18)$$

The second constrain is obtained by knowing the fact that

$$\bar{u}_{i,j}^{tx}=\bar{u}_{i,j}^{xt} \qquad (19)$$

First we calculate $\bar{u}_{i,j}^{tx}$ by operating Eq. (11) with $\frac{1}{2a}\int_{-a}^{a}dx$ and then $\bar{u}_{i,j}^{xt}$ by operating Eq. (8) with $\frac{1}{2\tau}\int_{-\tau}^{\tau}dt$.So we get,

$$\bar{u}_{i,j}^{tx}=\left\{\frac{1}{2u_{i,j}^0 a}-\frac{1}{\left[e^{2\,u_{i,j}^0 a}-1\right]}\right\}\bar{u}_{i,j}^t+\left\{1-\frac{1}{2u_{i,j}^0 a}+\frac{1}{\left[e^{2\,u_{i,j}^0 a}-1\right]}\right\}\bar{u}_{i-1,j}^t$$
$$+\left\{\frac{1}{\left(u_{i,j}^0 a\right)^2}-\frac{a}{u_{i,j}^0}+\frac{2a}{u_{i,j}^0\left[e^{2\,u_{i,j}^0 a}-1\right]}\right\}\bar{s}_{2,(i,j)}^{x,t}$$

The coefficients in curly brackets are assigned constants P, Q, T respectively, reducing above equation as follows

$$\bar{u}_{i,j}^{tx}=P_{i,j}\bar{u}_{i,j}^t+Q_{i,j}\bar{u}_{i-1,j}^t+T_{i,j} \qquad (20)$$

Moreover, Eq. (8) is operated with $\frac{1}{2\tau}\int_{-\tau}^{\tau}dt$ to yield following equation.

$$\bar{u}_{i,j}^{xt}=\bar{u}_{i,j-1}^x+\tau\bar{s}_{1,(i,j)}^{x,\tau} \qquad (21)$$

Now , there are four equations namely Eqs. (9), (16),(18) and (19) with four unknowns $\bar{u}_{i,j}^t, \bar{u}_{i,j}^x, \bar{s}_{2,(i,j)}^{t,x}, \bar{s}_{2,(i,j)}^{x,t}$.These equations are simplified to two equations with two unknowns with following procedure.

1. Obtain $\bar{s}_{1,(i,j)}^{x,t}$ from Eq. (9) and substitute it in Eq. (21) and Eq. (18) to get following expression respectively,

$$\bar{u}_{i,j}^{xt}=\bar{u}_{i,j-1}^x+\frac{\bar{u}_{i,j}^x-\bar{u}_{i,j-1}^x}{2} \qquad (22)$$

$$\frac{\bar{u}_{i,j}^x-\bar{u}_{i,j-1}^x}{2\tau}=\bar{s}_{2,(i,j)}^{x,t}+F_{i,j}(u) \qquad (23)$$

where $F_{i,j}(u)=\frac{1}{4a\tau}\int_{-\tau}^{\tau}\int_{-a}^{a}\beta f(u)dtdx$

$F_{i,j}(u)$ can approximated as follows,

$$F_{i,j}(u)=\frac{\bar{u}_{i,j}^x+\bar{u}_{i,j-1}^x+\bar{u}_{i,j}^t+\bar{u}_{i-1,j}^t}{4}$$

2. Equate RHS of Eq. (20) and Eq. (22), get the value of $\bar{s}_{2,(i,j)}^{t,x}$ as follows,

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$$\bar{s}^{t,x}_{2,(i,j)} = \frac{\left\{\left[\frac{\bar{u}^x_{i,j} - \bar{u}^x_{i,j-1}}{2}\right] - P_{i,j}\bar{u}^t_{i,j} - Q_{i,j}\bar{u}^t_{i-1,j}\right\}}{T_{i,j}}$$

Putting $\bar{s}^{t,x}_{2,(i,j)}$in Eq. (23) and simplifying further, one gets first of the two final algebraic equations in the numerical scheme relating only $\bar{u}^t_{i,j}$and $\bar{u}^x_{i,j}$ as follows

$$\left[\frac{T_{i,j} - \tau}{\tau}\right]\bar{u}^x_{i,j} - \left[\frac{T_{i,j} + \tau}{\tau}\right]\bar{u}^x_{i,j-1} + 2P_{i,j}\bar{u}^t_{i,j} + 2Q_{i,j}\bar{u}^t_{i-1,j} - 2T_{i,j}F_{i,j}(u) = 0 \qquad (24)$$

3. Substitute $\bar{s}^{t,x}_{2,(i,j)}$ in Eq. (16) one gets second expression relating only $\bar{u}^t_{i,j}$and $\bar{u}^x_{i,j}$ as given below

$$\left[\frac{P_{i,j}N_{i,j}}{T_{i,j}} + \frac{O_{i+1,j}Q_{i+1,j}}{T_{i+1,j}} - L_{i,j}\right]\bar{u}^t_{i,j} + \left[K_{i,j} + \frac{Q_{i,j}N_{i,j}}{T_{i,j}}\right]\bar{u}^t_{i-1,j} + \left[M_{i+1,j} + \frac{P_{i+1,j}O_{i+1,j}}{T_{i+1,j}}\right]\bar{u}^t_{i+1,j}$$
$$- \frac{N_{i,j}}{T_{i,j}}\left[\frac{\bar{u}^x_{i,j} + \bar{u}^x_{i,j-1}}{2}\right] - \frac{O_{i+1,j}}{T_{i+1,j}}\left[\frac{\bar{u}^x_{i+1,j} + \bar{u}^x_{i+1,j-1}}{2}\right] = 0 \qquad (25)$$

Thus, we have two algebraic equations, namely Eq. (24) and Eq. (25) per node which need to be solved to find out $\bar{u}^t_{i,j}$and$\bar{u}^x_{i,j}$

IV. RESULTS AND DISCUSSION

The results are shown for various values of the parameters. In all simulations, the wavafront is observed for space with lower and upper bounds $x_l = -10$ $x_r = 10$. Since the meshing is uniform, the relationship of ΔX=2×a and and Δt= 2× τ hold good throughout the meshing. The time is discretised with Δt = 0.01.

Example 1. When $\beta = 0.5, \alpha = 1, \delta = 2, \gamma = 0.85$, the wavefront moves forward and attains space averaged velocity 0.4 at x= 0 after 5 seconds as indicated in Fig. (4).

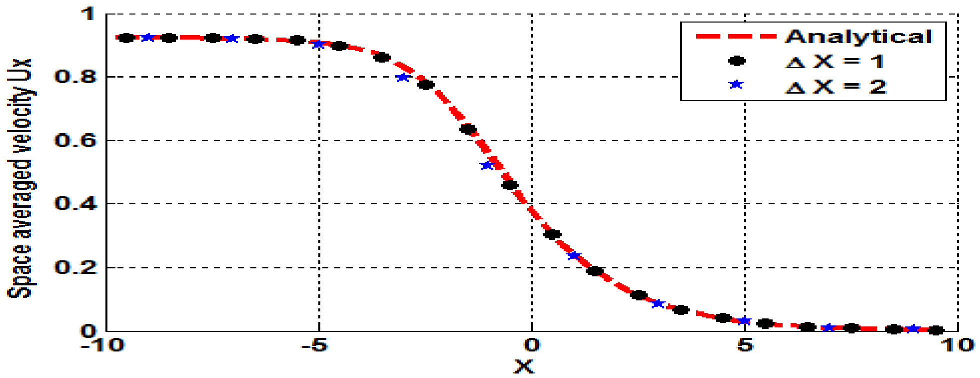

Fig. 4. Numerical solution of generalised Burgers-Huxley equationwith$\beta = 0.5, \alpha = 1, \delta = 2, \gamma = 0.85$

There is match between analytical and numerical results. The numerical results are shown when number of points m = 10 and m = 20 i.e. ΔX = 2 and ΔX = 1 reaspectively.When number of points is 10 there is slight deviation in the nonlinear region as number of points required to capture the curve are insufficient.

Example 2. With parameters $\beta = 0.1, \alpha = 1, \delta = 2, \gamma = 0.85$, the wavefront acquires the shape as shown in Fig. (5)

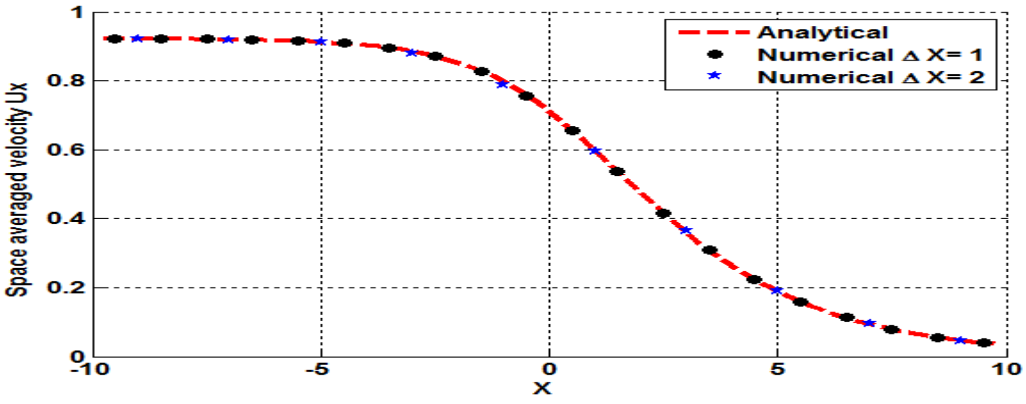

Fig. 5. Numerical solution of generalised Burgers-Huxley equation with $\beta = 0.1, \alpha = 1, \delta = 2, \gamma = 0.85$

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The wavefront in Fig.(5) reaches space averaged velocity of 0.7 at x= 0 at the end of 5 seconds.

In example 2, all the parameters are same as that of in example 1 except β. In example 2, β is reduced from 0.5 to 0.1 i.e. weightage of nonlinear term to the right hand side of equation 1 is reduced. Here, with 10 numbers of points the nonlinearity in the wavefront is captured.

Example 3. By setting the parameters$\beta = 0.5, \alpha = 1, \delta = 1, \gamma = 0.85$,the wavefront reaches velocity of 0.47 at x=0

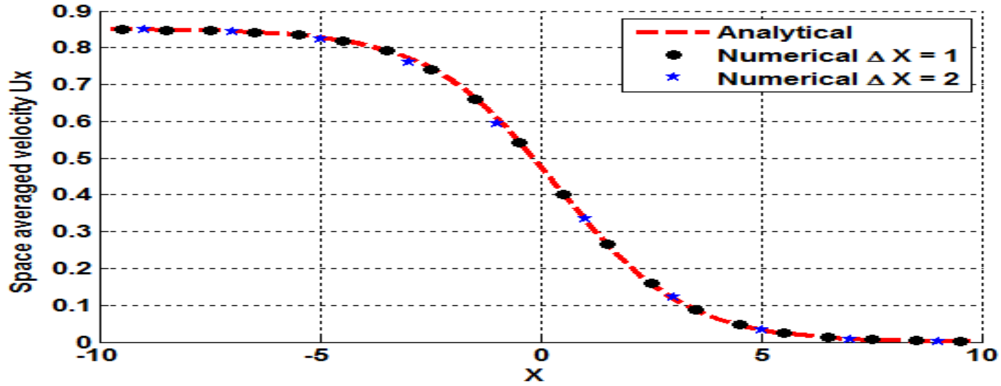

Fig. 6. Numerical solution of generalised Burgers-Huxley equationwith$\boldsymbol{\beta = 0.5, \alpha = 1, \delta = 1, \gamma = 0.85}$

From Fig.(6), it is clear that even with number of points m = 10 there is exact match between analytical and numerical results.

Example 4. After setting the parameters $\beta = 0.1, \alpha = 1, \delta = 1, \gamma = 0.85$, the wavefront reaches to space averaged velocity of 0.58 at x=0 as shown. In example 3 and 4, δ = 1 i.e. the order of nonlinearity is lesser than that of in example 1and 2. All the features of the wave front are captured accurately with even 10 number of points

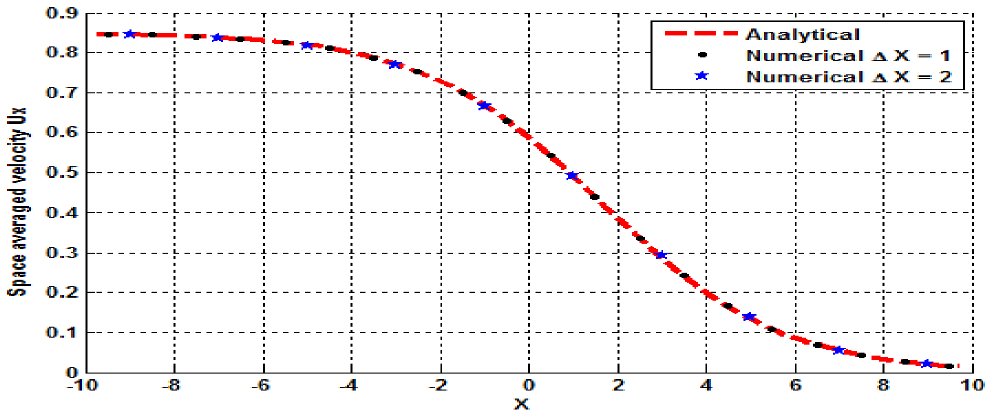

Fig. 7. Numerical solution of generalised Burgers-Huxley equationwith$\boldsymbol{\beta = 0.1, \alpha = 1, \delta = 1, \gamma = 0.85}$

Example 5. For parameters $\beta = 0.5, \alpha = 2, \delta = 1, \gamma = 0.85$ the wavefront forwards with the space averaged velocity of 0.81 at x=0 at the end of t=5 seconds.

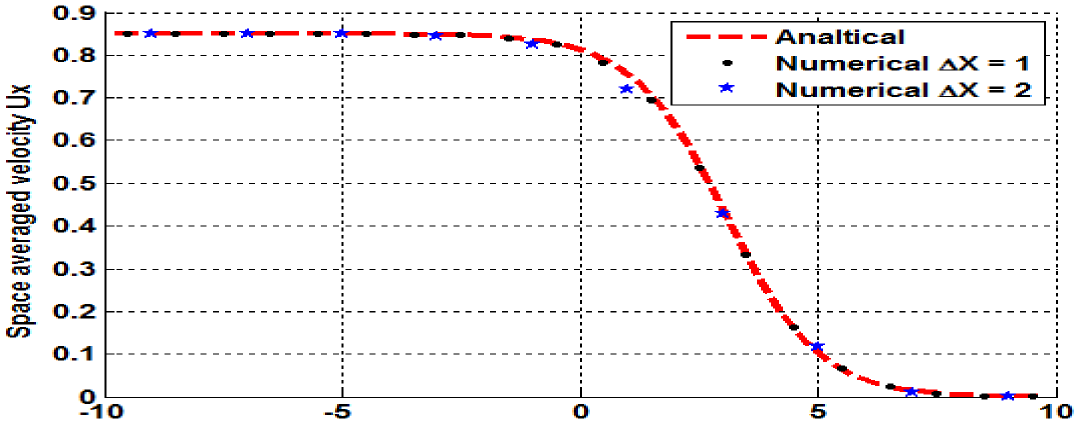

Fig. 8. Numerical solution of generalised Burgers-Huxley equation with$\boldsymbol{\beta = 0.5, \alpha = 2, \delta = 1, \gamma = 0.85}$

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As shown in Fig.(8), the numerical result for the case of $\Delta x = 2$ is in accordance with analytical result. In this example the convection term is scaled high by changing the parameter α from1 to 2.In the results it can seen that only one point out of 10 points is slightly deviated.

Example 6. Fig.(9) shows the nuerical result for parameters $\beta = 0.1, \alpha = 2, \delta = 1, \gamma = 0.85$

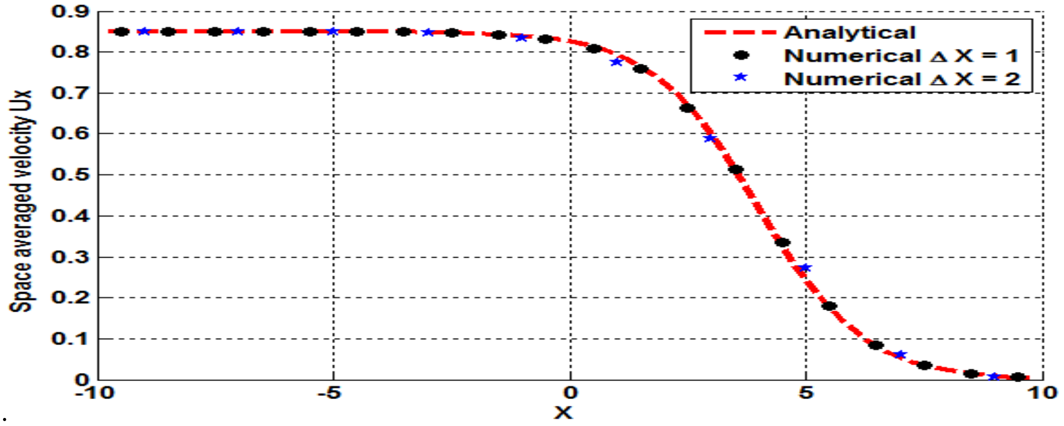

Fig. 9. Numerical solution of generalised Burgers-Huxley equation with$\boldsymbol{\beta = 0.1, \alpha = 2, \delta = 1, \gamma = 0.85}$

The waveform reaches value of 0.825 at x=0 as shown in the figure.

Example 7. When parameters are $\beta = 0, \alpha = 2, \delta = 2$; nonlinear term on the right hand side of the equation 1 is set to zero. The wave front gets shape as shown in Fig. 10.

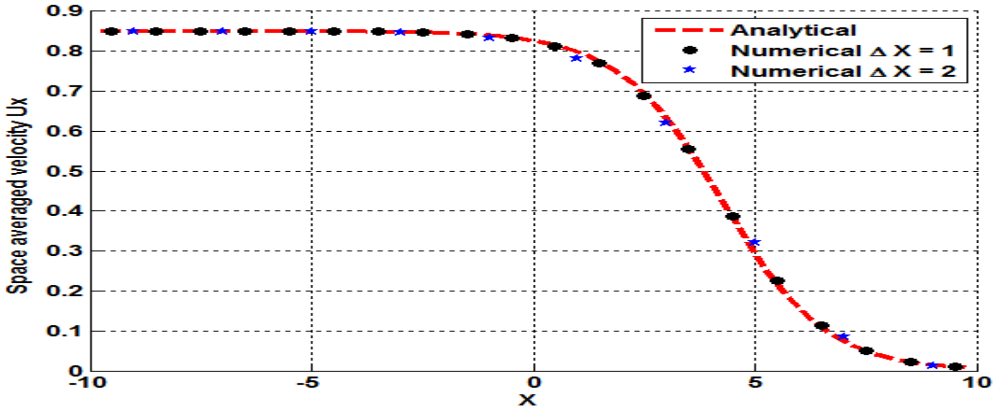

Fig. 10. Numerical solution of generalised Burgers-Huxley equation with $\boldsymbol{\beta = 0, \alpha = 2, \delta = 2}$

The wavefront has space averaged velocity of 0.82 at x = 0 at the end of 5 seconds.

Example 8. Whenparameters are$\beta = 0, \alpha = 1, \delta = 1$;the Eq. (1)leads to burgers equation. Since it is spcial case of diffusion dominanet burgers equation (Reynolds number$Re = 1$), the wavefront tries to diffuse over entire space domain.The solution is illustrated in the Fig.(11).

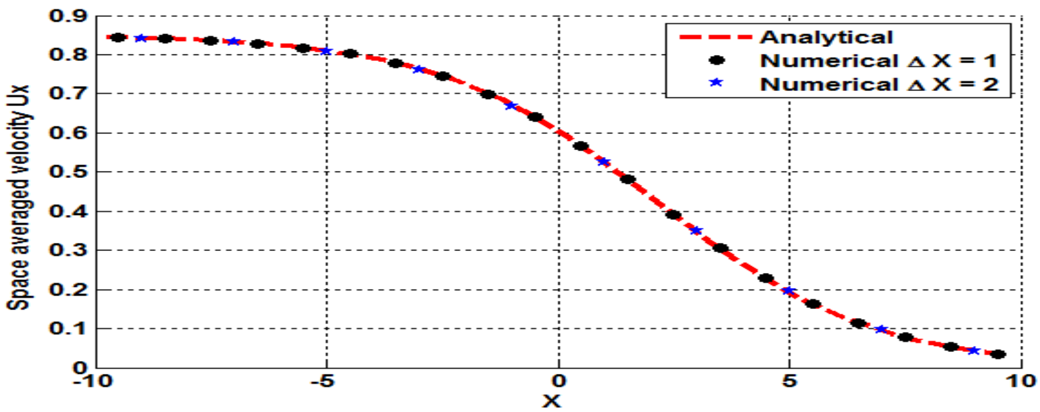

Fig. 11. Numerical solution of generalised Burgers-Huxley equation with $\boldsymbol{\beta = 0, \alpha = 1, \delta = 1}$

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The waveform obtains space averaged velocity of 0.6 at x=0 at t =5 seconds.

V. ERROR ANALYSIS

Simulations are carried out with following parameters to have deep insight of error statistics. Results are evaluated for the parameters $\beta = .1, \alpha = 1, \delta = 2, \gamma = 0.5, \ x_l = -10\ x_r = 10$. The root mean square error between analytical and numerical is evaluated results with following formula.

$$E_{rms} = \sqrt{\frac{\sum_{i=1}^{m}\left(Xi_{analytical} - Xi_{numerical}\right)^2}{m}}$$

The RMS error for various cases of different meshing size in time and space domain for above mentioned parameters are plotted on following surface (Fig.12).

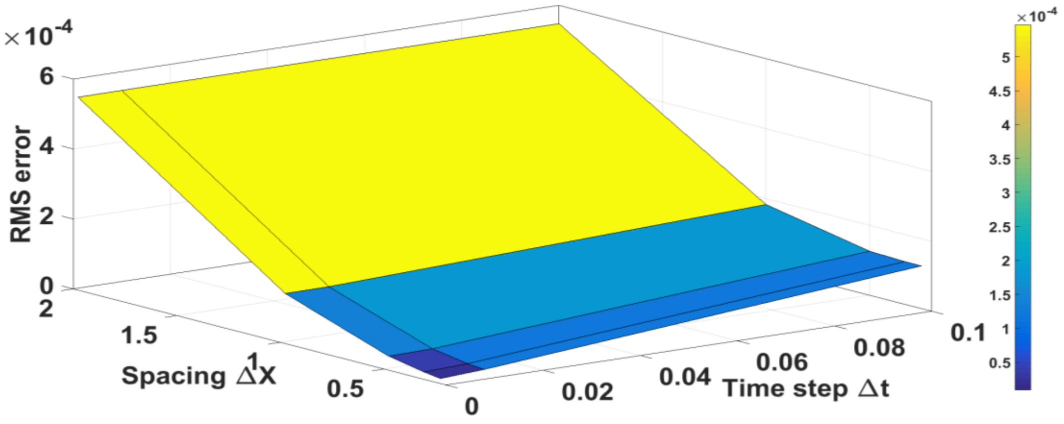

Fig. 12. RMS error for different meshing size in time and space domain.

With reduction of the time step Δt and spacing Δx the RMS error is reducing further. Thus modified nodal integral method is applicable to solve Burgers-Huxley equation efficiently.

VI. CONCLUSIONS

The different methods to solve Burgers-Huxley equation are studied. Numerical solution is formulated with diligence. The Results are tested with coarse grid for various parameters using modified nodal integral method. The solution of Burgers-Huxley equation using modified nodal integral method is efficient and works reasonable well for coarse grid. The root mean square error for the case of $\Delta x = 2\ \&\ \Delta t = 0.1$ is 5.4747×10^{-4}. Thus results with coarse grid are acceptable. Hence modified nodal integral method which is used to solve burgers equation can be extended further to solve Burgers-Huxley.

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