Nonlinear Interpolation in Hedge Algebras Associating Genetic Algorithm to Solve the Bell-Shaped Function Approximation Problems

Nguyen Tien Duy, Nghiem Van Tinh, Nguyen Tuan Anh

Abstract: Recently, there have been many works published related to approximation ability of the function using fuzzy logic and hedge algebras. These results showed that the approximation has a large error. In this paper, we propose a new method in improving the approximation accuracy of the function using hedge algebra by executing the normalization and denormalization by nonlinear interpolation. Moreover, we apply genetic algorithm to optimize the algorithms of hedge algebra. The function we choose to be approximate is the bell-shaped function. It is proved in the result that approximation bell surface has a significant decrease compared with the last results. Therefore, the effectiveness of hedge algebra in solving the approximation problems using algorithm can be revealed; as a result, it is advisable that nonlinear interpolation in hedge algebra to these problems such as nonlinear function approximation, fuzzy control be used,…

Keywords: Approximation inference, identification, function approximation, hedge algebras.

I. INTRODUCTION

Nowadays, computer science can be regarded as an important science in the development of information technology. Beside theoretical mathematics, applied mathematics flourishes with the appearance and development of digital computers. In particular, the digital method is the science in the field of applied mathematics researching the approximate solutions of the equations, the problem of function approximation and optimization problems. Solving a problem of function approximation aims at changing a complex function such as an expression form or function as a table with simpler functions.Interpolation is one of the methods to restore the continuous characteristics of a function *y = f (x)* from the discrete data set by measurement or observation. When *f(x)* is a complex function and difficult to compute, it also needs to be approximated by a polynomial. The simplest interpolation is the one by a polynomial.

In many fields of science and technology, as well in cybernetics, it is very difficult to determine the input - output relationship of an element or a system with a function form. Searching the model approximation method based on a template data set is often carried out in order to build an approximation relationship with the acceptable error rate which its relationship is simpler. Therefore, analyzing and synthesizing the system are able to be more efficient 8.

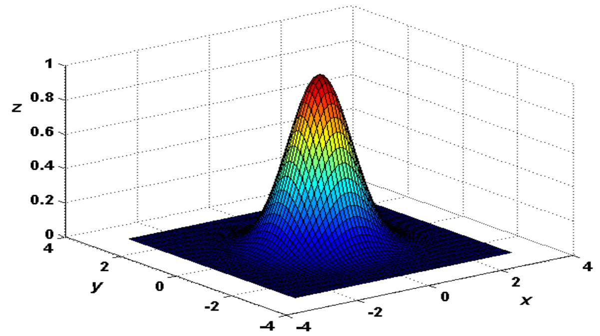

Figure 1. The bell “original” $z(x,y)=e^{-\left(x^2+y^2\right)}$

In the theory of function approximation, the problems of interpolation, even approximation and square approximation (known as the least square method) are often studied.

Many control problems under fuzzy approach have been solved quite efficiently since Prof. Lotfi Zadeh's fuzzy logic theory (California University, Berkeley) first introduced in 1965. Fuzzy logic allows performing linguistic values based on fuzzy set 1, 2, 4.

In addition, a different computing approach on the language terms which are hedge algebras. The theory of hedge algebras has been introduced by N.C. Ho and Wechler on a strictly algebraic structure on linguistic variables 3 since 1990. Whereas each linguistic value of the linguistic variable is quantified by a real value in the range [0, 1], semantically quantifying function formula was constructed based on parameters such as the fuzzy measurement of generating elements and hedges.

In this paper, we study the application of hedge algebras to deal with the problems of approximate inference for a specific fuzzy model based on rule base for solving function approximation. Combining with nonlinear interpolation and GA to optimize the fuzzy parameters, approximate computing values brings the more accurate results.

To be able to compare and evaluate methods, we use the bell - shaped function as an “original” one 6, given by:

$$z(x,y)=e^{-\left(x^2+y^2\right)} \qquad (1.1)$$

Let x, y varies in the range = [-3, 3], *step* = 0.1, we can map the bell surface of the function *z(x, y)* as shown in *Figure 1*.

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II. HEDGE ALGEBRAS

Suppose that there is a linguistic value set that is the language domain of temperature language variable and includes these following words: X = *dom*(TEMPERATURE) = {*Very Very small* < *Very small* < *small* < *Little small* < *Very Little small* < *medium* < *Very Little big* < *Little big* < *big* < *Very big* < *Very Very big*, …}. The linguistic values are used in the approximation inference problems based on knowledge by rules. Obviously, it is necessary to have a structure strong enough which is based on the inherent order of the linguistic values in the domain of linguistic variable. Therefore, we can compute the semantic per linguistic value of linguistic variables in the problems of approximation inference.

Definition of hedge algebras: Each linguistic variable X can be expressed as an algebraic structure AX= (*X*, *G*, *C*, *H*, ≤), called a hedge algebras, in which *X* is a set of terms in X; ≤ denotes the naturally semantic order relationships of the terms in *X*; $G = \{c^-, c^+\}$, $c^- \leq c^+$, called the generating elements (For example, *G = {small, big}); C =* ***{0***, ***W***, ***1}*** is the set of constants, with $\mathbf{0} \leq c^- \leq \mathbf{W} \leq c^+ \leq \mathbf{1}$, to indicate the elements that has the smallest, largest and neutral elements (For example, *W* = *medium*); $H = H^- \cup H^+$, with $H^- = \{h_j: -1 \leq j \leq -q\}$ is the set of negative hedge, $\forall h \in H^-$ then $hc^+ \leq c^+$ and $H^+ = \{h_j: 1 \leq j \leq p\}$ is the positive hedge, $\forall h \in H^+$ then $hc^+ \geq c^+$. For example, H^-= {*Rather*, *Little*}, H^+= {*Very*, *More*}. With $x \in X$, $x = h_n h_{n-1} \ldots h_1 c$, $c \in G$. Moreover, it can be seen that there is a comparable relationship existed among hedges and they can be shown by their signs 1, 4 as follows:

Sign function: *Sgn*: X → {–1, 0, 1} is recursively defined: With *k*, *h*∈H, $c \in \{c^-, c^+\}$

sgn(c+) = +1 and *sgn(c–)* = –1 (2.1)

{h∈H+ | sgn(h) = +1} and

{*h*∈H⁻ | *sgn(h)* = –1} (2.2)

sgn(hc) = +sgn(c) if hc>c and

sgn(hc) = -sgn(c) if hc<c (2.3)

or *sgn(hc) = sgn(h)×sgn(c)* (2.4)

sgn(*khx*) = +*sgn*(*hx*) if *k is* positive to *h* (*sgn*(*k*,*h*) = +1) and *sgn*(*khx*) = -*sgn*(*hx*) if *k* is negative to *h* (*sgn*(*k*,*h*) = +1).

<u>*Generally:*</u>

∀x ∈ H(G), it can be written as follow:

$x = h_m \ldots h_1 c$, with $c \in G$ and $h_1, \ldots, h_m \in H$.

Then:

sgn(x) =

sgn(hm,hm-1)×…×sgn(h2,h1)×sgn(h1)×sgn(c) (2.5)

(sgn(hx) = +1) ⇒ *(hx ≥ x)* and

(sgn(hx) = –1) ⇒ *(hx ≤ x)*

Fuzzy measurement: The concept of "fuzzy" of a fuzzy language information is very important in computing the semantic value of the term 3, 4. The semantics of the language value in AX is built from the sets $H(x)=\{x=h_n h_{n-1} \ldots h_1 c,\ c \in G,\ h_j \in H\} \cup \{x\}$, $x \in X$, it can be regarded as a fuzzy model of *x*. The set *H*(*x*), *x*∈*X* determines the fuzzy measurement *fm* of X and is equal to the "radius" of *H*(*x*). It can be recursively computed from the fuzzy measurement of the generating elements, $fm(c^-)$, $fm(c^+)$ and the fuzzy measurement of hedges μ(*h*), *h* ∈*H*. They are called the fuzzy parameters of X.

fm: X → [0, 1] is called the fuzzy measurement if it meets this following condition:

$$fm(c^-) + fm(c^+) = 1$$

$$\text{and } \sum_{h \in H} fm(hx) = fm(x), \text{ with } \forall x \in X \quad (2.6)$$

with the elements 0, W and 1,

fm(0) = fm(W) = fm(1) = 0 (2.7)

and with ∀*x*, *y*∈ X, ∀*h*∈H,

$$\frac{fm(hx)}{fm(x)} = \frac{fm(hy)}{fm(y)} \quad (2.8)$$

The equation (2.8) does not depend on the elements *x*, *y*, it characterizes as the hedge *h*, called the fuzzy measurement of *h*, denoted as *μ(h)*. The characteristics of *fm(x)* and *μ(h)* are as follows:

We have $x \in X$, $x = h_n h_{n-1} \ldots h_1 c$,

$$fm(hx) = m(h)\, fm(x),\ \forall x \in X \quad (2.9)$$

$$fm(h_n h_{n-1} \ldots h_1 c) = m(h_n) \ldots m(h_1)\, fm(c),$$

$$c \in G \quad (2.10)$$

$$\sum_{i=-1}^{-q} \mu(h_i) = \alpha \text{ and } \sum_{i=1}^{p} \mu(h_i) = \beta,$$

with α, β >0 and α + β = 1 (2.11)

Semantically Quantifying Mapping*: With* the fuzzy parameter set, semantically quantifying value is determined by semantically quantifying mapping function *(SQM - Semantically quantifying Mapping)* υ recursively as follows:

υ(W) = θ = fm(c-) (2.12)

υ(c-) = θ – α.fm(c-) = β.fm(c-) (2.13)

υ(c+) = θ + α.fm(c+) (2.14)

$$v(h_j x) = v(x) + Sgn(h_j x)\left\{\left[\sum_{i=Sgn(j)}^{j} fm(h_i x)\right] - \omega(h_j x)\, fm(h_j x)\right\} \quad (2.15)$$

With:

$$\omega(h_j x) = \frac{1}{2}\left[1 + Sgn(h_j x) Sgn(h_p h_j x)(\beta - \alpha)\right] \in \{\alpha, \beta\}$$

and j, –q ≤ j ≤ p and j ≠ 0 (2.16)

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Semantically quantifying mapping function (*SQMs*) can be directly mapped from linguistic values to semantically quantifying value of its semantics. Therefore, based on SQMs, we can simulate approximation inference methods of human which always ensures the semantics order of the language.

III. NORMALIZATION AND DENORMALIZATION

Figure 2. The diagram of approximation inference system

In the above *Figure 2*, the component *Normalization* performs the conversion from the input real value to semantic value domain of linguistic variables $(x, y) \rightarrow (x_s, y_s)$. The component *Quantified Rule Base Composition & HA-IRMd* has the task of computing the value of semantically quantifying mapping in the input and output linguistic variables for fuzzy model. Based on the rule system of the approximation inference problem, building the input – output relationship surface S_{real}. When the inference approximation set works, with each set of input values (x_s, y_s), this component will find out interpolation value at the output z_s. The component *Denormalization* will convert the value z_s on the semantic value domain to the real value at the output.

The range of any input - output linguistic variables to the approximation inference system is often any domain [a, b]. Semantic value in the hedge algebras is computed in the domain [0, 1]. Many problems using hedge algebras before used to use linear transformations from domain [a, b] to the domain [0, 1] and vice versa 1, 2, 4. The advantage of this linear transformation is simple and fast. In additions, another question is given whether the quality of the approximation inference would be better if we used the nonlinear transformations. For each specific approximation inference system, if we use nonlinear transformations, the optimal value can be easily achieved. The nonlinear transformations can use the polynomial order 2, 3 ... In this paper, we create nonlinearity the interval [a, b] and [0, 1] into three linear segments. Thus, there are four values on each variable domain:

[a, b]	a	y_1	y_2	b
[0, 1]	0	x_1	x_2	1

Using interpolation methods *Lagrange* on value set to perform the **normalization** and **denormalization**.

IV. DESIGN THE APPROXIMATION INFERENCE SYSTEM

As can be seen in the *Figure 2*, in order to make the approximation inference set give good results, designing an approximation inference based on the hedge algebras must consider computations of the components. In this paper, interpolation on the semantic relations input - output surface used is linear interpolation. The approximation inference set was built by 3 hedge algebras for 3 linguistic variables *Lx*, *Ly* and *Lz* corresponds to the variables x, y and z.

A. Select the parameter set and the rule system

First, determine the components of the hedge algebras for input - output variables. Hedge algebras for linguistic variables *Lx*, *Ly*, *Lz* of the variables *x*, *y*, *z* are:

1) The set of generating element G = {*N*, *P*}, with $c^{-}=N$ (*Negative*) and $c^{+} = P$ (*Positive*).

2) The set of hedges chosen: H^{-} = {*L* (*Little*)} and H^{+} = {*V* (*Very*)}.

The sign relationship of the hedges to the others and the generating elements are defined in the following sign table:

Table 1. The sign relationship of the hedges and the generating elements

	V	*L*	*N*	*P*
V	+	+	−	+
L	−	−	+	−

Rule base system is quantatively built on the variability of the bell-shaped function (1.1) 5 in the following *Table 2*.

Table 2. Rule table for approximation inference Lz

Lx \ *Ly*	*0*	*VN*	*LN*	*W*	*LP*	*VP*	*1*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
VN	*0*	*VVN*	*VN*	*LVN*	*VN*	*VVN*	*0*
LN	*0*	*VN*	*P*	*VP*	*P*	*VN*	*0*
W	*0*	*LVN*	*VP*	*1*	*VP*	*LVN*	*0*
LP	*0*	*VN*	*P*	*VP*	*P*	*VN*	*0*
VP	*0*	*VVN*	*VN*	*LVN*	*VN*	*VVN*	*0*
1	*0*	*0*	*0*	*0*	*0*	*0*	*0*

Rule table for approximation inference in *Table 2* can be clarified as follows:

If Lx = VN and Ly = VN then Lz = VVN

If Lx = VN and Ly = LN then Lz = VN

If Lx = LN and Ly = VN then Lz = VN

If Lx = LN and Ly = LN then Lz = P

B. Optimize the parameters

Applying GA 7 aims at optimizing the fuzzy parameters of the hedge algebras and the nonlinear points in the variation domain of the language value and semantic value domain. The objective function used is:

$$g = \max\left(\frac{1}{1+\boldsymbol{err}}\right) \tag{4.1}$$

with:

$$err = \sqrt[2]{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(z_{gt}(i,j) - z_{ha}(i,j)\right)^2} \tag{4.2}$$

The optimal parameters are received as in *Table 3*:

Table 3: The optimal fuzzy parameters

	L_x, L_y	L_z
fm(N)	*0.5*	*0.493402*
α = μ(L)	*0.511290*	*0.491153*

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With the optimal parameter set found by GA, we can compute the semantically quantifying value of linguistic terms based on applying the formula (2.12) to (2.16) in rule system as shown in *Table 4*.

Table 4:Semantically quantifying value ofrule system

Lx_s \ Ly_s	*0*	*0.1194*	*0.3751*	*0.5000*	*0.6249*	*0.8806*	*1*
0	*0*	*0*	*0*	*0*	*0*	*0*	*0*
0.1194	*0*	*0.0650*	*0.1278*	*0.1883*	*0.1278*	*0.0650*	*0*
0.3751	*0*	*0.1278*	*0.7422*	*0.8688*	*0.7422*	*0.1278*	*0*
0.5000	*0*	*0.1883*	*0.8688*	*1.0000*	*0.8688*	*0.1883*	*0*
0.6249	*0*	*0.1278*	*0.7422*	*0.8688*	*0.7422*	*0.1278*	*0*
0.8806	*0*	*0.0650*	*0.1278*	*0.1883*	*0.1278*	*0.0650*	*0*
1	*0*	*0*	*0*	*0*	*0*	*0*	*0*

Nonlinear conversion curve using interpolation method Lagrange to the normalization and denormalization as shown in the Figuge 3.

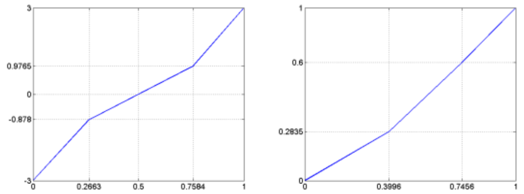

Figure 3. Normalizationcurve and denormalization curve

The input - output relationship surface S_{real} of the approximation inference set corresponds to the values in *Table 4* as shown in *Figure 4*.

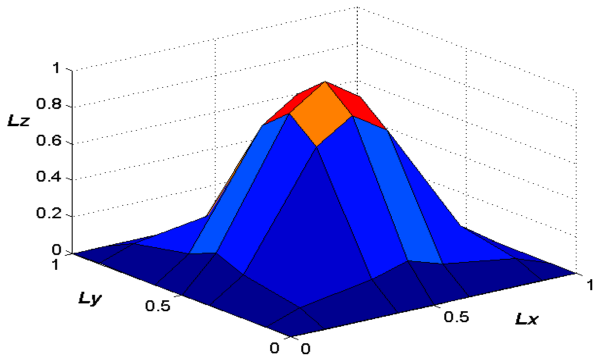

Figure 4. The input - output relationship surface S_{real}

V. COMPUTING RESULT

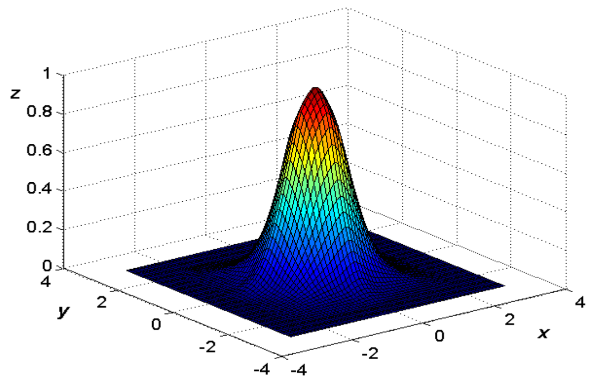

Figure 5. Approximation bell-shaped

Suppose *x*, *y* varies in the range [-3, 3], with *step* = 0.1, the total of models is $n \times n = 61 \times 61 = 3721$. The approximation inference set computed according to the hedge algebras gives the approximation curved surface as shown in the *Figure 5*.

According to 4.2, the computing error is: err = 0.3907.

VI. DISCUSSION

In this paper, we propose a method to improve the accuracy of the approximation function using hedge algebras by executing the normalization and denormalization by nonlinear interpolation. We can draw out the computing result of approximation inference set to the optimal parameters found by GA as follows:

- Hedge algebras has the preeminent because of its simple computing method, the number of computation when inferring is small. However, this method brings a more accurate result than the one in 5, 6.

- After having built the inference set, it is easier to optimize the parameters. The accuracy of inference just depends on the fuzzy parameters of hedge algebras, *normalization*, *denormalization* and the interpolation on the input - output relationship.

- To optimize the fuzzy parameters of the hedge algebras, we can easily apply the optimal and efficient algorithms such as GA, PSO,...

- It is completely possible to widen the application of hedge algebras in order to solve the problems of approximation inference based on the rules. Due to the statement of rule system, the semantic order of the language must be assured.

Apply the efficient and optimal methods to optimize the parameters of hedge algebras in the fuzzy control problems.

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