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A Viewpoint on Multi-Dimensional Centrality

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Abstract—Complex networks are a becoming a more and more used tool in order to represent real-life systems. Subject to inherent interdisciplinary constraints, the creation of a realist model and its interpretation are more often than not challenging. One of the most used factors in network analysis and interpretation is centrality, which measures the influence of a node in the network or the diffusion power of that node. While its computation is a relative simple problem in basic, uni-dimensional networks, this measure proves more difficult to define in multi-dimensional networks. As practice shows that multi-dimensional models are more accurate, the pressure to create a valid and easily computable influence measure increases. The aim of this paper is to present an overview of the methods used to achieve a multi-dimensional centrality. In the end, a novel method for computing centrality in multi-dimensional networks is proposed.

Index Terms — centrality, complex networks, multi-dimensional, vector

I. INTRODUCTION

During the past 10 years a mixture of graph theory, networks and data analysis coalesced into a blooming study field, commonly known as Network Science [1] that specializes in investigation of large data interaction. The upsurge in large data collections represented a catalyst for using complex networks based systems as models for understanding and discovering interactions in such large data sets [2]. In itself, the distinction between Network Science (based on complex networks) and Graph Theory (based on graphs) is highly empirical and coalesces from the type of data modeled. Although the shape of any complex network is essentially a graph, the abstract mathematical notions found in Graph Theory are not enough in order to describe the complex network properties as each node and edge in the network model a characteristic and/or interaction in a real system, therefore having a clearly defined meaning. A key characteristic of Network Science, its intrinsic interdisciplinary nature, marks the fact that more than mathematical knowledge is required in order to understand a complex network. Moreover, the chosen model for emulating the real-world system influences deeply the results [4] and the limitation of current models [3] require a quest for new metrics [5] that allow higher precision models of real systems [6]. In this regard, complex networks can be characterized through various parameters like dimensions (number of nodes or edges as well as network diameter), qualitative overall measurement (density, clustering coefficient, clusters) and quantitative overall measurements (e.g.: average degree (weighted with the edge's magnitude, or not), average path length) [3].

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However, one of the most interesting ways that can be used to characterize a complex network is centrality, which aims at presenting the influence a node has on the network. This paper aims to explore centrality and its models for real-life systems.

II. LITERATURE REVIEW

There are several ways to evaluate centrality, resulting in many specific named centrality measures: the most common ones are betweeness centrality, which reflects the number of shortest paths from a node to all of the other nodes, and closeness centrality, which shows the medium distance towards all the other nodes. Depending on the real system providing the data, one might choose one or other measure. An often misunderstood notion, centrality has clear similitudes to power (defined as "the ability to control resource flows" [7]) however, the nuances that support centrality refer more to "the structural arrangement of resource flows that facilitate the diffusion". [7] Indeed, a structural arrangement is a prerequisite in the control, but it is not the only factor to be considered. This leads to a two-dimensional view by considering concentration of resources as well as their diffusion. In one of the first papers[8] that explored the concept of multidimensional centrality by using hyper-graphs, while others like [9] refer to more approachable and common two-dimensional graphs. The centrality concept proposed in [8] measures the number of paths of any length between two nodes, considering an attenuation factor along a direct edge and a different attenuation factor along an indirect edge. However, most work revolves around hyper-graphs and speculates an algorithm like k-shell, which is a "maximal subset of vertices such that each is reachable from each of the others by at least k vertex independent paths" [10]. These types of techniques use a k-shell decomposition of the hyper-graphs and then assign the subset indicators to nodes. In a very technical paper [11], k-shell value, degree centrality and betweeness centrality were studied and the results postulated that k-shell predicts better the influence on a network, as opposed to any type of centrality. This situation lead too many quests of defining centrality or a centrality-like measure based on k-sets like the ones in [10], [12], and [13]. The discussion can be further extended in taking into account a special class of hyper-graphs, which allows more than one node type, the multipartite graphs, or networks [14]. In this case the betweenness centrality is either heterogeneous or homogenous and can be presented under the form of a sum of all the rapports between the number of shortest paths of two nodes that are in the same part, but are different from the part of the node under scrutiny and the number of such shortest paths that pass through the node under scrutiny [14].

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The picture can become even more complex by adding another dimension – time, forming what is commonly known as dynamic networks, a more realistic model of real-life systems. These types of networks present, as expected, dynamic centrality [15]. The dynamic centrality metric creates a network of all the possible paths in chronological order in the network in order to determine if there exists "a path that connects the source and destination nodes through intermediaries at different times" [15]. Therefore, this measurement is characterized by two parameters: time and interactions length. Referring to length is impossible not to mention spatial networks and the centrality problems imposed by them. Spatial networks refer to a geographical place, with Euclidean space properties [16] and such networks have specific centrality problems, which require taking into account, not only the resource flow but also the inherent resource limitation specific to spatial networks. In these cases, a balance needs to be struck between the "structural properties of the system and the relevant dynamics on the system" [17]. In a short conclusion for this section, centrality is a complex measure, as it depends on the network and its underlying properties.

III. Composite Vectorial Centrality

As presented in the previous section, centrality depends on the network type, on the nodes and edges types as well as on various constraints (in either space or time). Either way, centrality, in its essence, is a way to characterize network flows. However, in fluid mechanics, flow is a vectorial measurement and these lead me to approach centrality as a vectorial measure in itself. Time is included in the fluid dynamics approach as just another dimension. Moreover, constrains, like flow velocity, pressure, density are all accounted for. This notion can easily translate into network science, by taking into account that the constraints present in fluid dynamics, are similar to spatial, time, interactions length, and so forth from network science. In order to illustrate this, a simple network will be considered, eight nodes, belonging to two different types (Nodes 2, 5 and 8 are of Type 1, while the others are Type 2), thus being a multi-partite (bi-partite) graph. These nodes interact with each other through three edge types (represented using different colors, unscaled by weight). For the simplicity of the argument, this network is static, although, as it will be shown, it can easily be dynamic. The described sample network is presented in Figure 1. In order to compute the proposed composite vectorial centrality of this network (or of any network) a four-step algorithm is offered:

Step 1: Determine the dimensions pertaining to the network and its interpretation.

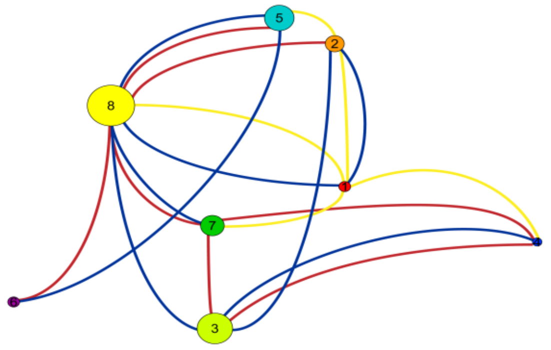

Figure 1. Sample multi-dimensional network

For the sample network, a dimension will be considered for each edge type and the nodes can be split into two dimensions. This creates five dimensions: three for edges (designated E1-E3), two for nodes (labeled N1 and N2).

Step 2: For each edge dimension, for each node dimension compute betweeness centrality (BC) in the limited network for each node. Continue until no edge dimensions are left.

In the sample network, BC is computed for E1N1, E1N2; E2N1, E2N2; E3N1, E3N2.

Step 3: For each edge dimension compute betweeness centrality in the limited network for each node, node type notwithstanding. Continue until no edge dimensions are left.

Compute BC for E1, E2, E3. This allows the quantification of edges that cross node types.

Step 4: For each node, create a vector that includes every BC computed for that node by using a unitary vector to describe origins.

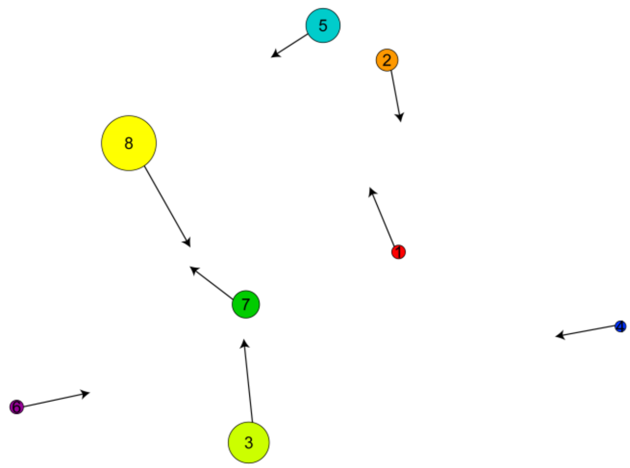

Figure 2. Sample betweeness centrality vector field flow

For a random node *i*, this creates a vectorial flow direction, described in a vectorial manner in Equation (1)

$$BC(Ni) = \sum_{j=0}^{D} \vec{u}_j \times BC_j^{Ni}. \quad (1)$$

where *Ni* is node *I*, u_j is the unity vector for space *j*, *D* is the total number of dimensions computed (in the sample case *D=10*) and is multiplied with the computed BC for that space and node. This describes actually a vector, its size, and its direction. Its module is easily computed. The sample network's centrality field now looks like Figure 2. It is a vector field that should describe the information / resource flow. Looking at the Type 2 nodes, the flow direction of the ensemble is evident, towards the upper-left side (Figure 3)

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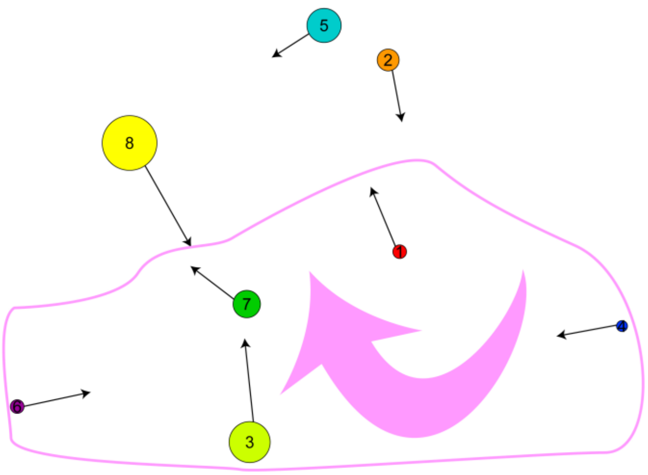

Figure 3. Flow direction for Type 2 nodes

In real-life system with larger networks, as for example opinion networks, similar flow patterns should appear if in *Step 1*, the dimensions are well chosen based on the properties desired to be analyzed. For example, let us consider the blog-sphere complex network from [18] that shows republican-conservatory polarization (Figure4).

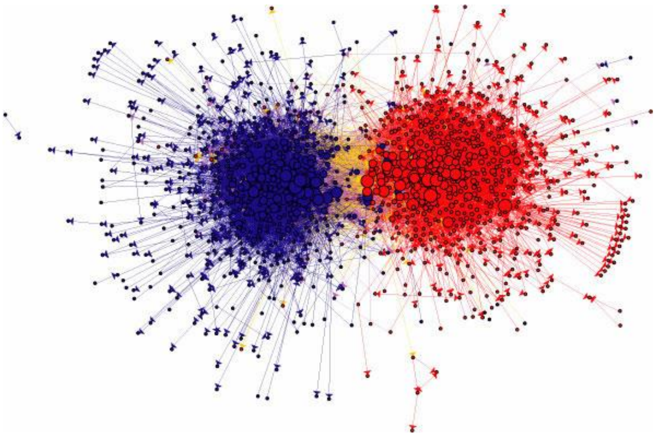

Figure 4. Blog polarizations [18]

If this polarization is compared with vector field lines from, for example, a magnetic field (Figure 5) the similarities are striking.

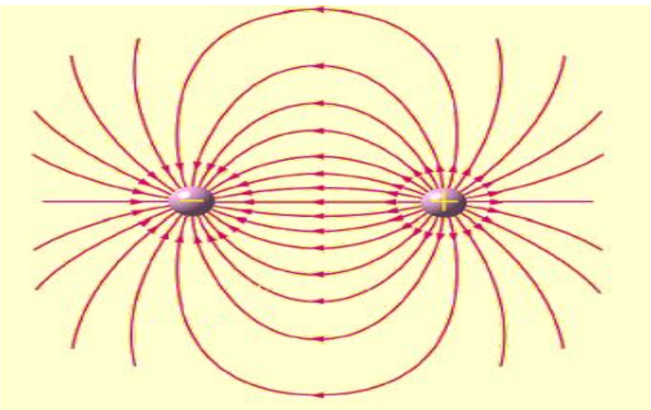

Figure 5. Magnetic vector field (Image credits <u>Chanchocan</u>)

It is clear that further investigation is warranted in order to describe this process; however, the preliminary results are encouraging.

IV. Conclusion

This paper, after conducting a literature review of centrality models offers a solution of modeling a vectorial centrality measure, which can be extended to as many dimensions as needed. Through similitudes with fluid dynamics, characteristics for 2- or 3- dimensions vectorial fields could be extrapolated in a further study.

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