Thermal and Solutal Buoyancy Effects on Viscous Dissipative and Chemically Reactive Fluid Flow past a Uniformly Moving Plate with Variable Suction

Ananda Reddy.N, Vidya Sagar, Raju. M. C, Varma. S. V. K.

Abstract— An attempt is made to study the chemical reaction effect on an unsteady free convection flow past a semi- infinite vertical plate with viscous dissipation. The governing equations of motion, energy, and species concentration are reduced into a set of ordinary differential equations by applying regular perturbation technique and then solved analytically. The effects of various parameters on the velocity, temperature and concentration are presented and discussed through graphs.

Index Terms— chemical reaction, free convection, viscous dissipation.

I. INTRODUCTION

Combined heat and mass transfer problems with chemical reactions are of importance in recent years in many processes such as drying, evaporating at the surface of a water body, transferring energy in a wet cooling tower, and following in a desert cooler. Possible applications of this type of flows can be found in many industries. For example, in the power industry, electrical energy is extracted directly from a moving conducting fluid. Gokhale and Samman [1] studied the effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet was considered by Cortell [2]. Singh [3] examined the effects of variable fluid properties and viscous dissipation on mixed convection fluid flow past a vertical plate in porous medium. Vyas et al. [4] considered dissipative heat and mass transfer in porous medium due to continuously moving plate. Ravi Kumar et al. [5] considered, heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. Kandasamy et al. [6] studied the effect of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in presence of suction or injection. Muthucumaraswamy [7] considered the effect of chemical reaction effects on vertical oscillating plate with variable temperature. Makinde and Sibanda [8] examined the effects of chemical reaction on boundary layer flow past a vertical stretching surface in the presence of internal heat generation. Rao and Shivaiah [9] considered chemical reaction effects on unsteady MHD flow past semi-infinite vertical porous plate with viscous dissipation. Buoyancy and chemical reaction effects on MHD mixed convection heat and mass transfer in a porous medium with thermal radiation and Ohmic heating was examined by Pal and Talukdar [10]. Ibrahim et al. [11] studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source suction. Ibrahim and Makinde [12] studied the effect of chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Balamurugan et al. [13] considered the effect of chemical reaction and thermal diffusion effects on MHD three dimensional free convection coquette flow with heat absorption. Ananda Reddy et al. [14] investigated the effect of thermo diffusion and chemical effects with simultaneous thermal and mass diffusion in MHD mixed convection flow with Ohmic heating. Hence an attempt is made to study the chemical reaction effect on an unsteady free convection flow past a semi- infinite vertical plate with viscous dissipation.

II. FORMULATION OF THE PROBLEM

An unsteady convective heat and mass transfer flow of a viscous, incompressible, chemically reactive and dissipative Newtonian fluid flow through a porous medium past a semi-infinite moving vertical plate is considered. The x^*-axis is taken along the vertical plate in the fluid flow direction and y^*-axis is taken normal to the plate. Let u^* *and* v^* be the components of velocity in x^* and y^* directions respectively. It is assumed that

- Flow is unsteady and laminar
- The plate is long enough in the x^*-direction, so all the physical variables are functions of y^* and t^* only.
- First order chemical reaction and viscous dissipation effects are taken into account.

Revised Version Manuscript Received on December 08, 2015.

Dr. Ananda Reddy N, Department of Mathematics, A.I.T.S., Rajampet, A.P., India.
Vidya Sagar, Research Scholar, Department of Mathematics, S.V.U., Tirupati, A.P., India
Dr. Raju M. C., Department of Mathematics, A.I.T.S., Rajampet, A.P., India
Prof. Varma. S. V. K., Department of Mathematics, S.V.U., Tirupati, A.P., India.

- The plate temperature and concentration are varying with time.

Under the above assumptions the governing equations for this flow are given below

Continuity equation:

$$\frac{\partial v^*}{\partial y^*} = 0$$

(2.1)

Momentum equation:

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \vartheta \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T_\infty\right) + g\beta^*\left(C^* - C_\infty\right) \quad (2.2)$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu}{\rho C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2$$

(2.3)

Diffusion equation:

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_r^*\left(C^* - C_\infty\right)$$

(2.4)

The boundary conditions for the velocity, temperature and concentration fields are

$$u^* = U_0,\; T^* = T_w^* + \varepsilon\left(T_w^* - T_\infty\right)e^{n^* t^*},\; C^* = C_w^* + \varepsilon\left(C_w^* - C_\infty\right)e^{n^* t^*} \quad at \quad y^* = 0 \quad (2.5)$$

$$u^* \to 0, \quad T^* \to T_\infty, \quad C^* \to C_\infty \quad as \quad y^* \to \infty \quad (2.6)$$

From the continuity equation (2.1),

$$v^* = -V_0\left(1 + \varepsilon A e^{n^* t^*}\right)$$

(2.7)

The negative sign indicates that the suction is normal and directed towards to the plates.

In order to write the governing equations and boundary conditions in dimensional less form, the following non-dimensional quantities are introduced.

$$u = \frac{u^*}{U_0},\quad y = \frac{V_0 y^*}{\vartheta},\quad t = \frac{t^* V_0^2}{\vartheta},\quad \Pr = \frac{\rho \vartheta C_p}{\kappa},$$

$$\theta = \frac{T^* - T_\infty}{T_w^* - T_\infty},\quad \phi = \frac{C^* - C_\infty}{C_w^* - C_\infty},\quad n = \frac{n^* \vartheta}{V_0^2}$$

$$Gr = \frac{\vartheta g \beta\left(T_w^* - T_\infty\right)}{U_0 V_0^2},\quad Gm = \frac{\vartheta g \beta^*\left(C_w^* - C_\infty\right)}{U_0 V_0^2},$$

$$E = \frac{U_0^2}{C_p\left(T_w^* - T_\infty\right)},\quad Kr = \frac{\vartheta K r^1}{V_0^2}, Sc = \frac{\vartheta}{D}$$

(2.8)

In view of equations (2.7) and (2.8), equations (2.2) - (2.4) reduced to the following dimensionless form.

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi$$

(2.9)

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + E\left(\frac{\partial u}{\partial y}\right)^2$$

(2.10)

$$\frac{\partial \phi}{\partial t} - \left(1 + \varepsilon A e^{nt}\right)\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - K_r \phi \quad (2.11)$$

The corresponding dimensionless boundary conditions are

$$u = 1,\quad \theta = 1 + \varepsilon e^{nt},\quad \phi = 1 + \varepsilon e^{nt} \quad at \quad y = 0$$

(2.12)

$$u \to 0,\quad \theta \to 0,\quad \phi \to 0 \quad at \quad y \to \infty \quad (2.13)$$

III. SOLUTION OF THE PROBLEM

Equations (2.9)-(2.11) are coupled non-linear partial differential equations and these cannot be solved in the closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the plate in the following form;

$$u(y,t) = u_0(y) + \varepsilon e^{nt} u_1(y) + O\left(\varepsilon^2\right)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O\left(\varepsilon^2\right) \quad (2.14)$$

$$\phi(y,t) = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2)$$

Substituting equation (2.14) in set of equations (2.9)-(2.11) and equating non-harmonic and harmonic terms and neglecting the higher order terms of O (ε^2) , the following set of equations are obtained.

$$u_0^{11} + u_0^1 = -Gr\theta_0 - Gm\phi_0 \tag{2.15}$$

$$u_1^{11} + u_1^1 - nu_1 = -Au_0^1 - Gr\theta_1 - Gm\phi_1 \tag{2.16}$$

$$\theta_0^{11} + \Pr\theta_0^1 = -E\Pr u_0^{1^2} \tag{2.17}$$

$$\theta_1^{11} + \Pr\theta_1^1 - n\Pr\theta_1 = -A\Pr\theta_0^1 - 2\Pr Eu_0^1 u_1^1 \tag{2.18}$$

$$\phi_0^{11} + Sc\phi_0^1 - KrSc\phi_0 = 0 \tag{2.19}$$

$$\phi_1^{11} + Sc\phi_1^1 - Sc(n + K_r)\phi_1 = -A\phi_0^1 \tag{2.20}$$

The corresponding boundary conditions can be written as

$$u_0 = 1, u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 1,\ \phi_0 = 1,\ \phi_1 = 1\ at\ y = 0 \tag{2.21}$$

$$\begin{aligned} &u_0 \to 0,\ u_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0, \\ &\qquad \phi_0 \to 0,\quad \phi_1 \to 0\quad at\ y \to \infty \end{aligned} \tag{2.22}$$

The equations (2.15)-(2.18) are still coupled non-linear, whose exact solutions are not possible. So we expand u_0, u_1, θ_0 and θ_1 in the following form, taking the Eckert number E<<1 as perturbation parameter.

$$\begin{aligned} u_0 &= u_{01}(y) + Eu_{02}(y) \\ u_1 &= u_{11}(y) + Eu_{12}(y) \\ \theta_0 &= \theta_{01}(y) + E\theta_{02}(y) \\ \theta_1 &= \theta_{11}(y) + E\theta_{12}(y) \end{aligned} \tag{2.23}$$

By substituting (2.23) in the set of equations (2.15)-(2.18), the following equations are obtained

$$u_{01}^{11}(y) + u_{01}^1(y) = -Gr\theta_{01}(y) - Gm\phi_0(y) \tag{2.24}$$

$$u_{02}^{11}(y) + u_{02}^1(y) = -Gr\theta_{02}(y) \tag{2.25}$$

$$u_{11}^{11}(y) + u_{11}^1(y) - nu_{11}(y) = -Au_{01}^1(y) - Gr\theta_{11} - Gm\phi_1 \tag{2.26}$$

$$u_{12}^{11}(y) + u_{12}^1(y) - nu_{12}(y) = -Au_{02}^1(y) - Gr\theta_{12}(y) \tag{2.27}$$

$$\theta_{01}^{11}(y) + \Pr\theta_{01}^1(y) = 0 \tag{2.28}$$

$$\theta_{02}^{11}(y) + \Pr\theta_{02}^1(y) = -\Pr u_{01}^{1^2} \tag{2.29}$$

$$\theta_{11}^{11}(y) + \Pr\theta_{11}^1(y) - n\Pr\theta_{11}(y) = -A\Pr\theta_{01}^1(y) \tag{2.30}$$

$$\begin{aligned} &\theta_{12}^{11}(y) + \Pr\theta_{12}^1(y) - n\Pr\theta_{12}(y) \\ &\qquad = -A\Pr\theta_{02}^1(y) - 2\Pr u_{01}^1(y)u_{11}^1(y) \end{aligned} \tag{2.31}$$

With the corresponding boundary conditions

$$\begin{aligned} &u_{01} = 1,\ u_{02} = 0,\ u_{11} = 0,\ u_{12} = 0 \\ &\theta_{01} = 1,\ \theta_{02} = 0,\ \theta_{11} = 1,\ \theta_{12} = 0\ \ at\ \ y = 0 \end{aligned} \tag{2.32}$$

$$\begin{aligned} &u_{01} \to 0,\ u_{02} \to 0,\ u_{11} \to 0,\ u_{12} \to 0 \\ &\theta_{01} \to 0,\ \theta_{02} \to 0,\ \theta_{11} \to 0,\ \theta_{12} \to 0\ at\ y \to \infty \end{aligned} \tag{2.33}$$

Solutions of the equations (2.24)-(2.31) subject to the corresponding boundary conditions are obtained as follows.

$$\begin{aligned} u(y,t) = &\left\{ \begin{matrix} R_3e^{-y} - R_1e^{-p_ry} - R_2e^{-m_1y} + E\Big(R_4e^{-2y} + R_5e^{-2p_ry} + \\ R_6e^{-2m_1y} + R_7e^{-a_1y} + R_8e^{-a_2y} + R_9e^{-a_3y} + R_{10}e^{-p_ry} + R_{11}e^{-y}\Big) \end{matrix} \right\} \\ &+ \varepsilon e^{nt}\Big\{ l_1e^{-y} + l_2e^{-p_ry} + l_3e^{-m_1y} + l_4e^{-m_2y} + l_5e^{-m_3y} + l_6e^{-m_4y} \\ &+ E\Big(A_1e^{-m_4y} + A_2e^{-2y} + A_3e^{-2p_ry} + A_4e^{-2m_1y} + A_5e^{-a_1y} + A_6e^{-a_2y} + A_7e^{-a_3y} \\ &+ A_8e^{-p_ry} + A_9e^{-y} + A_{10}e^{-a_4y} + A_{11}e^{-a_5y} + A_{12}e^{-a_6y} + A_{13}e^{-a_7y} + \\ &A_{14}e^{-a_8y} + A_{15}e^{-a_9y} + A_{16}e^{-a_{10}y} + A_{17}e^{-a_{11}y} + A_{18}e^{-a_{12}y}\Big)\Big\} \end{aligned}$$

$$\begin{aligned} \theta(y,t) = &\left\{ e^{-p_ry} + E\left(\begin{matrix} P_3e^{-2y} + P_4e^{-2p_ry} + P_5e^{-2m_1y} \\ + P_6e^{-a_1y} + P_7e^{-a_2y} + P_8e^{-a_3y} + P_9e^{-p_ry} \end{matrix} \right) \right\} \\ &+ \varepsilon e^{nt} \left\{ \begin{matrix} L_1e^{-m_3y} - \dfrac{A\Pr}{n}e^{-p_ry} + E\Big(P_{10}e^{-2y} + P_{11}e^{-2p_ry} + \\ P_{12}e^{-2m_1y} + P_{13}e^{-a_1y} + P_{14}e^{-a_2y} + P_{15}e^{-a_3y} + P_{16}e^{-p_ry} \\ + P_{17}e^{-a_4y} + P_{18}e^{-a_5y} + P_{19}e^{-a_6y} + P_{20}e^{-a_7y} + P_{21}e^{-a_8y} \\ + P_{22}e^{-a_9y} + P_{23}e^{-a_{10}y} + P_{24}e^{-a_{11}y} + P_{25}e^{-a_{12}y} + P_{26}e^{-m_3y} \end{matrix} \right. \end{aligned}$$

$$\phi(y,t) = e^{-m_1y} + + \varepsilon e^{nt} P_1e^{-m_1y} + P_2e^{-m_2y}$$

IV. RESULTS AND DISCUSSION

In order to look into the physical insight of the problem, the effects of various physical parameters viz., Schmid number (Sc), chemical reaction parameter (Kr), thermal Grashof number (Gr), modified Grashof number (Gm), and Prandtl number (Pr) are studied numerically with the help of graphs.

Fig.1. depicts the variations in concentration profiles for different values of Schmidt number, from this figure, it is noticed that concentration distribution decreases as Schmidt number increases. Physically it is true because Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity (viscosity) and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection process as in the present problem. Near the vicinity of the plate, concentration appears to be very high, whereas it reaches to the stationary position far away from the plate. In Fig.2 effect of chemical reaction parameter (Kr) on concentration is presented. As the values of chemical reaction parameter Kr increase, concentration appears to be decreased. Kandasamy et al. [6] also showed that chemical reaction process condenses the concentration boundary layer. Therefore our result is in best agreement with that result. Fig.3 depicts the temperature profiles for different values of Prandtl number. From this figure it is noticed that temperature decreases when Prandtl number is increasing. Physically this is true, because as the values of Prandtl number increase thermal conductivity of the fluid decreases, hence the thickness of the thermal boundary layer is reduced. In fig.4 effect of chemical reaction parameter (Kr) on temperature is presented. As Kr increases temperature decreases, thus chemical reaction process condenses the temperature boundary layer. In fig.5. effect of Schmidt number on temperature is presented. This shows that temperature decreases as the Schmidt number increases. Physically it is true because of the increase in Schmidt number the viscosity of the fluid increases and hence temperature decreases.

In figures 6 and 7, effects of Grashof number and modified Grashof number on velocity are shown. From these figures it is evident that the velocity increases with the increasing values of these two parameters. In figure 8, effect of Schmidt number on velocity is presented. This shows that velocity decreases as the Schmidt number increases. Physically it is true because of the increase in Schmidt number the viscosity of the fluid increases and hence velocity decreases. A similar effect is noticed in the presence of chemical reaction parameter from figure 9. Figure 10 depicts the velocity profiles for different values of Prandtl number. Prandtl number is the ratio of momentum diffusivity to thermal conductivity. It is known that lower thermal conductivity material has high velocity and higher thermal conductivity material has lower velocity. Figure 2.10.witness that velocity decreases when Prandtl number increases.

V. CONCLUSION

A theoretical analysis is presented for the problem of thermal and solutial buoyancy effects on viscous dissipative and chemically reactive fluid flow past a uniformly moving plate with variable suction, the following conclusions are

(i) Velocity decreases with an increase in Schmidt number, Chemical reaction and Prandtl number whereas velocity increases with an increase in modified Grashof number and Grashof number.

(ii) Temperature decreases with an increase in Schmidt number, Chemical reaction and Prandtl number.

(iii) Concentration decreases with an increase in Schmidt number and Chemical reaction.

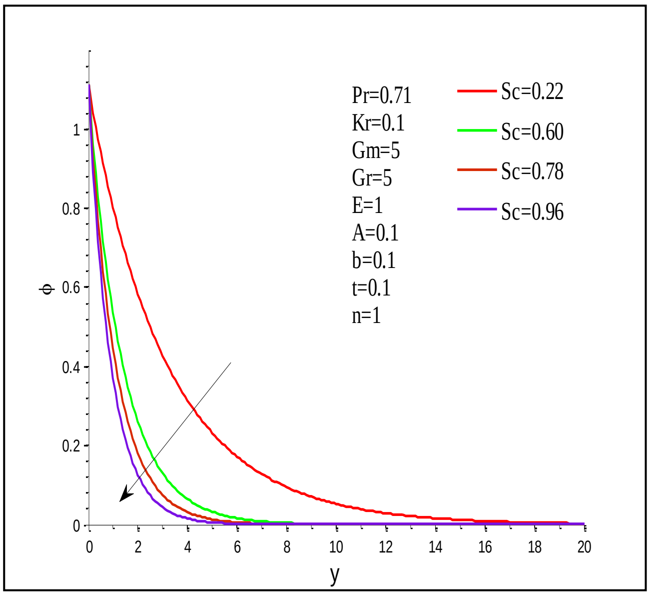

Fig.1. Effect of Schmidt number on concentration

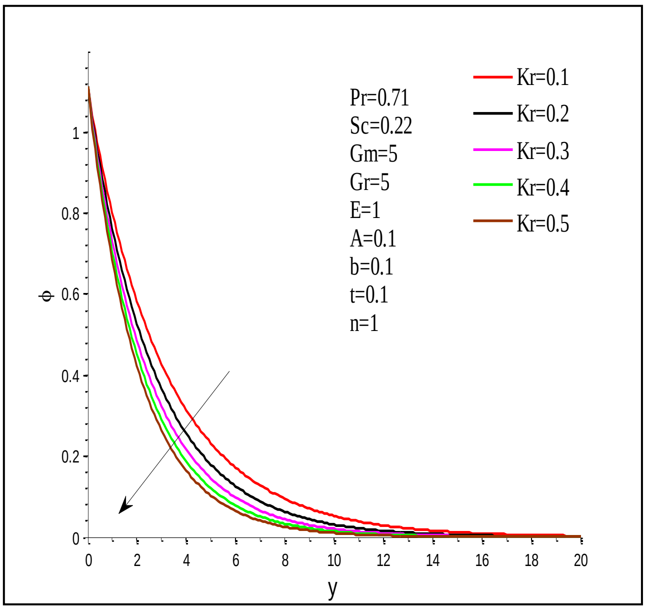

Fig.2. Effect of Chemical reaction on Concentration

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

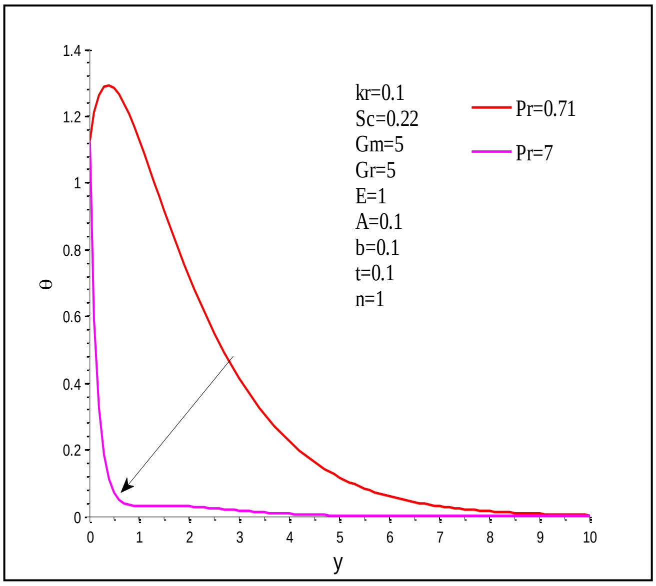

Fig.3. Effect of Prandtl number on Temperature

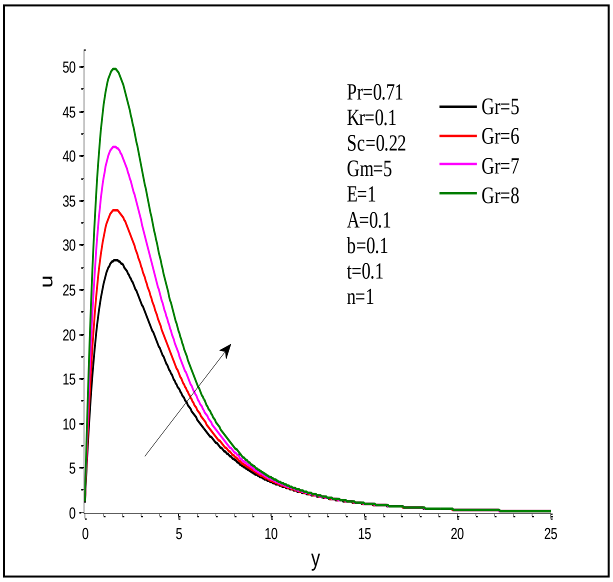

Fig.6. Effect of Grashof Number on velocity

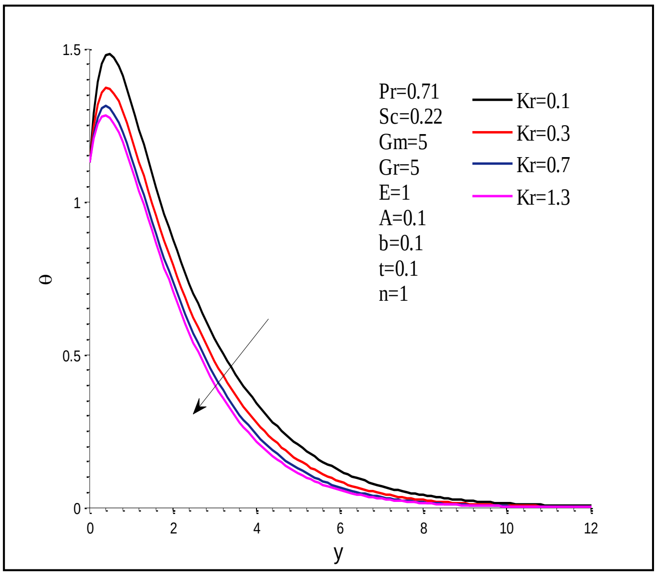

Fig.4. Effect of Chemical reaction on Temperature

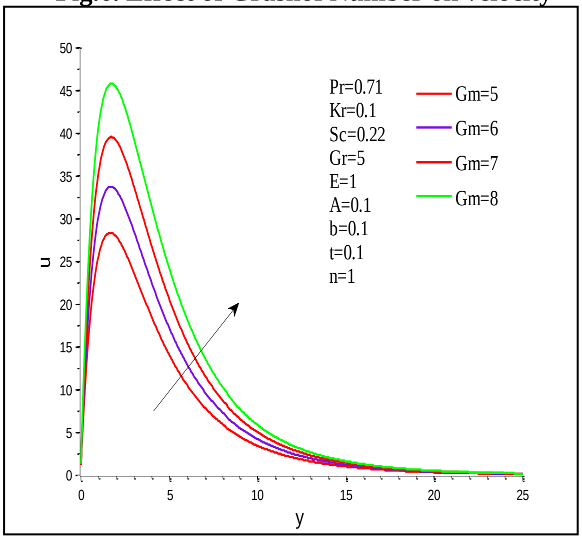

Fig.7. Effect of Grashof Modified number on velocity

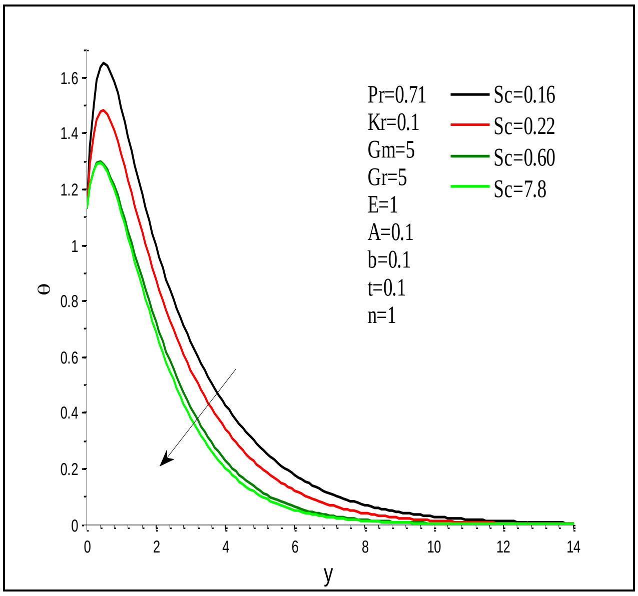

Fig.5. Effect of Schmidt number on Temperature

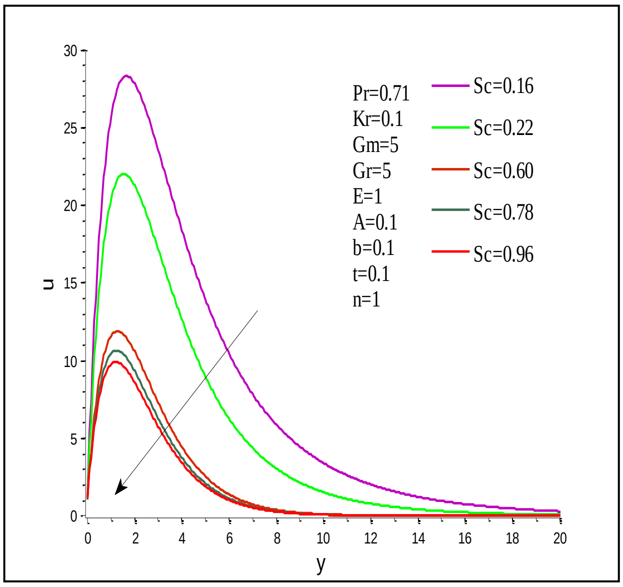

Fig.8. Effect of Schmidt number on velocity

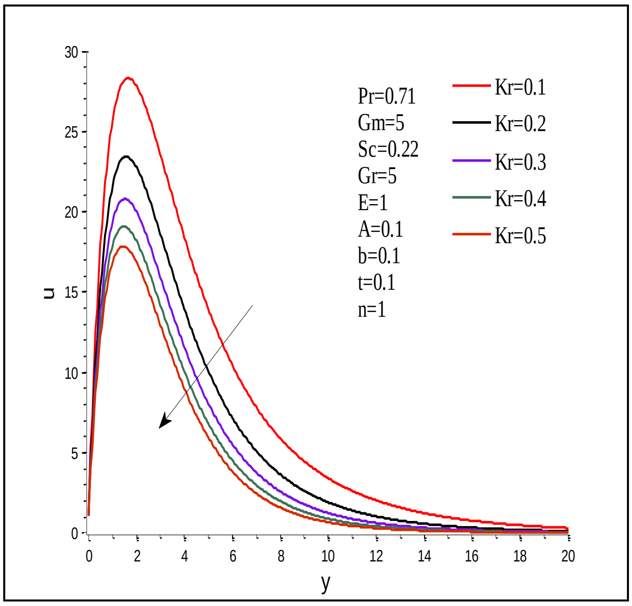

Fig.9. Effect of chemical reaction on velocity

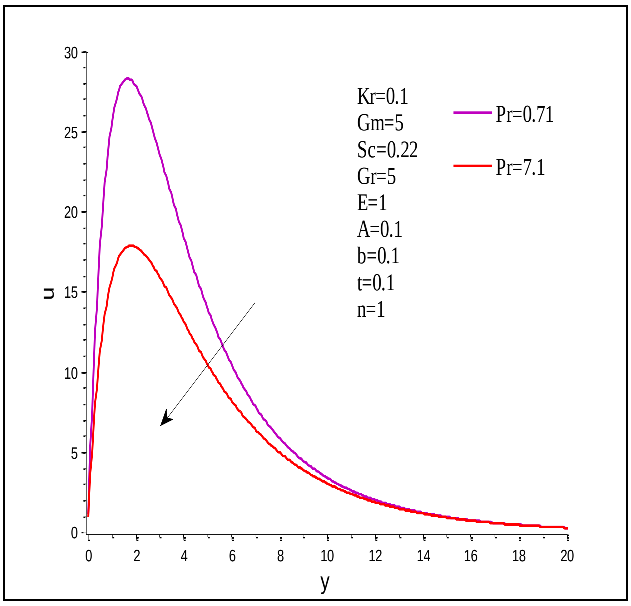

Fig.10. Effect of Prandtl number on velocity

REFEREMCES

1. M. Y. Gokhale and F. M. AL. Samman, "Effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux", Int. J. Heat and Mass Transfer, 46, (2003), pp. 999-1011.
2. R. Cortell, "Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet," Physics Letters A: General, Atomic and Solid State Physics, vol.372 (5), (2008), pp. 631–636.
3. P. K. Singh, "Effects of variable fluid proper ties and viscous dissipation on mixed convection fluid flow past a vertical plate in porous medium", International Journal of Scientific & Engineering Research, Vol.3 (7), (2012), pp. 1-10.
4. P. Vyas, A. Rai, K.S. Shekhawat, "Dissipative heat and mass transfer in porous medium due to continuously moving plate", Applied Mathematical Sciences, Vol.6 (87), (2012), pp. 4319-4330.
5. V. ravi kumar, m. c. raju and g. s. s. raju, "heat and mass transfer effects on mhd flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source", international journal of contemporary mathematical sciences, Vol.7 (32), (2012), pp.1597-1604.
6. R. Kandaswamy, B. Wahib, Md. Raj and B. Azme Khamis, "Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in presence of suction or injection", Theatrical Applied Mechanics, 33(2), (2006), pp.123-148.
7. R. Muthucumaraswamy, "Chemical reaction effects on vertical oscillating plate with variable temperature". Chemical Industry and Chemical Engineering Quarterly, 16 (2), (2010), pp.167-173.
8. O. D. Makinde, P. Sibanda, "Effects of chemical reaction on boundary layer flow past a vertical stretching surface in the presence of internal heat generation", International Journal of Numerical Methods for Heat & Fluid Flow, Vol.21 (6), (2011), pp.779 – 792.
9. J. Rao, S. Shivaiah, "Chemical reaction effects on unsteady MHD flow past semi-infinite vertical porous plate with viscous dissipation", Appl. Math. Mech. –Engl. Ed., 32 (8), (2011), 1065-1078.
10. D. Pal, B. Talukdar, "Buoyancy and chemical reaction effects MHD mixed convection heat and mass transfer in a porous medium with thermal radiation and Ohmic heating", Communications in Nonlinear Science and Numerical Simulation, vol.15 (10), (2010), pp. 2878-2893.
11. F. S. Ibrahim, A. M. Elaiw, and A. A. Bakier, "Effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source suction", Communications. Nonlinear. Science. Numerical simulation, 13, (2008), pp. 1056-1066.
12. F. S. Ibrahim, and O. D. Makinde, "Chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction", Scientific Research and Essays, Vol.5 (19), (2010), pp. 2875-2885.
13. K. S. Balamurugan, S. V. K. Varma, K. Ramakrishna Prasad and N. Ch. S. N. Iyengar, "Chemical reaction and Thermal diffusion effects on MHD three dimensional free convection couette flow with heat absorption", International Journal of Advances in Sciences and Technology, Vol.3 (1), (2011), pp.58-92.
14. N. Ananda Reddy, S. V. K. Varma and M. C. Raju, "Thermo diffusion and chemical effects with simultaneous thermal and mass diffusion in MHD mixed convection flow with Ohmic heating". Journal of Naval Architecture and Marine Engineering vol.6 (2009), pp. 84-93.