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Numerical Integration of Arbitrary Function over Multidimensional Cubes using Haar Wavelet Method

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***Abstract:** In this paper, we investigate numerical integration of arbitrary function over multidimensional cubes by using quadrature method, Haar wavelet method has been used to describe method for multiple integral problems, the high accuracy and wide applicability of the Haar wavelet approach will be illustrated several numerical examples.*

***Index Terms:** Numerical integration, Haar wavelet, multiple integral, n-dimensional cube*

I. INTRODUCTION

The problem is considered in this paper is the numerical integration of arbitrary function over n- dimensional cube which appeared in Biomodeling, chemical engineering, mechanical engineering, computer graphics, sophisticated market drives liquidity in financial sector etc. several approximate formulas were developed for quadrature rule over a polygonal, rectangular, cubic domain such as cubature formula, Gauss Legendre quadrature rule, Generalized Gaussian quadrature rule etc. numerical integration is expensive in computational time and require higher order integration to obtain an accurate solution, many authors have been work out in the field of double and triple integrals over polygonal, triangle, curved arc, polyhedral, tetrahedral, sphere, cone, cylinder, cuboid region [Rathod and Nagaraja, 2004, Shivaram, 2013, Islam and Hossain, 2009, Haq and Aziz, 2010, Aziz and Khan, 2011, Mamatha & Venkatesh, 2015, Fengying Zhou and Xiaoyong Xu, 2017], automatic numerical integration of arbitrary function multi dimensional cube and rectangular domain by adaptive algorithm method are discussed in [Paulvan, Ridder, 1976 and Genz, A.C, A.A. Malik, 1980]. Recently, Multiple integrals over n-dimensional cubes and ball are evaluated numerically by using generalized Gaussian quadrature rule [Sarada and Nagaraja, 2014]. In this paper, new approach is presented to evaluating multiple integrals over multi dimensional cube by haar wavelet method, numerical solution are obtained by the present approach are compared in terms of convergence, accuracy and computational efficiency

II. HAAR WAVELET METHOD

The discrete Haar function are constructed by using scaling function $H_1(x)$ and the mother function $H_2(x)$ using dilation and translation, $H_1(x)$ and $H_2(x)$ are expressed as follows

$$H_1(x)=\begin{cases}1, & if \quad x\in[0,1)\\ 0, & otherwise\end{cases} \tag{1}$$

and

$$H_2(x)=\begin{cases}1, & if \quad x\in[0,\frac{1}{2})\\ -1, & if \quad x\in[\frac{1}{2},0)\\ 0, & otherwise\end{cases} \tag{2}$$

The other Haar wavelets discrete function $H_{jk}(x)$ are constructed using translation parameter k and dilation parameter j as

$$H_{jk}(x)=(\sqrt{2})^j H_2(2^j x-k)$$

The explicit form of the function $H_{jk}(x)$ is defined as

$$H_{jk}(x)=\begin{cases}1, & if \quad x\in[a_{jk},\frac{a_{jk}+b_{jk}}{2})\\ -1, & if \quad x\in[\frac{a_{jk}+b_{jk}}{2},0)\\ 0, & otherwise\end{cases} \tag{3}$$

Where

$$j\geq 0, k=0,1,2,----,2^j-1$$

$$a_{jk}=\frac{k}{2^j} \quad and \quad b_{jk}=\frac{k+1}{2^j}$$

Using the orthogonal basis of L^2 ([0, 1)) the Haar wavelet function H_{jk} (x) can be expressed by Haar series function f(x) of infinite terms as

$$f(x)=\sum_{j=0}^{\infty}\sum_{k=0}^{2^j-1}C_{jk}H_{jk}(x) \tag{4}$$

By considering into finite term approximation, we get

$$f(x)=\sum_{j=0}^{\infty}\sum_{k=0}^{2^j-1}C_{jk}H_{jk}(x) \tag{5}$$

$$\int_0^1 f(x)*dx=\sum_{j=0}^{\infty}\sum_{k=0}^{2^j-1}C_{jk}*\int_0^1 H_{jk}(x)*dx \tag{6}$$

Eqn..(6) can be reduced to single term, since

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$$\int_0^1 H_{jk}(x)*dx = 0 \qquad \{(j,k) \neq (0,0)\}$$

Thus the approximate integral of Eqn. (6) as

$$\int_0^1 f(x)*dx = C_{00}$$

Where C_{00} are Haar coefficients and grid points

$$x_i = \frac{2i-1}{2^{n+2}}, \qquad i = 1,2,3,---2^{n+1}$$

Eqn. (5) reduces to

$$\int_0^1 f(x)dx = C_{00} = \frac{1}{2^{n+1}} \sum_{i=1}^{2^{n+1}} f(x_i) \tag{7}$$

In order to changing the variable u = a + (b- a)x are applied to Eqn. (7) then

$$\int_a^b f(u)du = \frac{(b-a)}{2^{n+1}} \sum_{i=1}^{2^{n+1}} f(a + \frac{(b-a)(2i-1)}{2^{n+1}}) \tag{8}$$

Eqn. (8) rewritten as

$$\int_a^b f(u)du = \frac{(b-a)}{2M} \sum_{i=1}^{2M} f(a + \frac{(b-a)(2i-1)}{2M}) \tag{9}$$

Where $M = 2^n$

2.1. For single integral

$$\int_{a_1}^{a_2} f(x_1)dx_1 = \frac{(a_2-a_1)}{2M} \sum_{i_1=1}^{2M} f(A)$$

2.2. For double integral

$$\int_{a_1}^{a_2}\int_{a_3}^{a_4} f(x_1,x_2)dx_1dx_2 = \frac{(a_2-a_1)(a_4-a_3)}{4M^2} \sum_{i_1=1}^{2M}\sum_{i_2=1}^{2M} f(A,B)$$

2.3. For triple integral

$$\int_{a_1}^{a_2}\int_{a_3}^{a_4}\int_{a_5}^{a_6} f(x_1,x_2,x_3)dx_1dx_2dx_3 = \frac{(a_2-a_1)(a_4-a_3)(a_6-a_5)}{8M^3} \sum_{i_1=1}^{2M}\sum_{i_2=1}^{2M}\sum_{i_3=1}^{2M} f(A,B,C)$$

2.4. For multiple integral

$$\int_{a_1}^{a_2}\int_{a_3}^{a_4}\int_{a_5}^{a_6}\int_{a_7}^{a_8} f(x_1,x_2,x_3,x_4)dx_1dx_2dx_3dx_4$$
$$= \frac{(a_4-a_3)(a_6-a_5)(a_2-a_1)(a_8-a_7)}{16M^4} \sum_{i_1=1}^{2M}\sum_{i_2=1}^{2M}\sum_{i_3=1}^{2M}\sum_{i_4=1}^{2M} f(A,B,C,D)$$

$$\int_{a_1}^{a_2}\int_{a_3}^{a_4}\int_{a_5}^{a_6}\int_{a_7}^{a_8}\int_{a_9}^{a_{10}} f(x_1,x_2,x_3,x_4,x_5)dx_1dx_2dx_3dx_4dx_5$$
$$= \frac{(a_4-a_3)(a_6-a_5)(a_8-a_7)(a_2-a_1)(a_{10}-a_9)}{32M^5} \sum_{i_1=1}^{2M}\sum_{i_2=1}^{2M}\sum_{i_3=1}^{2M}\sum_{i_4=1}^{2M}\sum_{i_5=1}^{2M} f(A,B,C,D,E)$$

In generally

$$\int_{a_1}^{a_2}\int_{a_3}^{a_4} \ldots\ldots \int_{a_{n-1}}^{a_n} f(x_1,x_2,x_3,\ldots\ldots,x_n)dx_1dx_2\ldots\ldots dx_n =$$
$$\frac{(a_2-a_1)(a_4-a_3)\ldots\ldots(a_n-a_{n-1})}{2^n M^n} (\sum_{i_1=1}^{2M}\sum_{i_2=1}^{2M}\ldots\ldots\sum_{i_n=1}^{2M} f(A,B,C,\ldots\ldots))$$

Where

$$A = a_1 + \frac{(a_2-a_1)(i_1-0.5)}{2M}, \qquad B = a_3 + \frac{(a_4-a_3)(i_2-0.5)}{2M}$$
$$C = a_5 + \frac{(a_6-a_5)(i_3-0.5)}{2M}, \qquad D = a_7 + \frac{(a_8-a_7)(i_4-0.5)}{2M}$$
$$E = a_9 + \frac{(a_{10}-a_9)(i_5-0.5)}{2M}, \qquad F = a_{11} + \frac{(a_{12}-a_{11})(i_6-0.5)}{2M}$$

III. NUMERICAL RESULT

We have computed the multiple integral of arbitrary functions with limits of constant terms are approximated numerically by proposed method, results are tabulated in table.1 and 2 and compare with Sarada and Nagaraja, 2014 results are more accurate and easy to compute the mathematical equations of various order, we consider some examples to show that the present formulation may be applied to integrate the arbitrary function over n – dimensional cubes and some of which cannot be evaluated analytically

$$f_1 := 8\,(1 + 2(x_1 + x_2 + x_3))^{-1}$$

$$f_2 := x_3^2\, x_4\, e^{x_3 x_4} (x_1 + x_2 + 1)^{-2}$$

$$f_3 := x_1\, x_2^2 \sin(x_3)\,(4 + x_4 + x_5 + x_6)^{-1}$$

$$f_4 := \cos(x + y)$$

$$f_5 := \sin(10x_1)$$

$$f_6 := \cos(x_1 + x_2 + x_3 + x_4 + x_5)$$

$$f_7 := \left((x_1^2 + 0.0001)((x_2 + 0.25)^2 + 0.0001)\right)^{-1}$$

$$f_8 := (x_1 + x_2 + x_3)^{-2}$$

$$f_9 := \frac{1}{2^n}$$

Table: 1

Exact value	Order N	Obtained numerical results	Error	Error (Sarada & Nagaraja. 2014)
$\int_0^1\int_0^1\int_0^1 f_1\ dx_1 d$ = 2.152142832	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	2.150086352 2.151626716 2.152122162 2.152137665 2.152141540 2.152142835	0.00205648 0.00051611 6 2.067E-05 5.167E-06 1.292E-06 3E-09	0.49E -8
$\int_0^1\int_0^1\int_0^1\int_0^1 f_2$ $dx_4\,dx_3\,dx_2\,dx$ = 0.5753641449 035616	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	0.568995706 0.573763754 0.575300021 0.575348108 0.575360121 0.575364142	0.00636843 9 0.00160039 6.41239E-0 5 1.60368E-0 5 4.02E-06 2.50356E-0	0.32E -4

			9	
$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{\frac{\pi}{2}}$ $dx_6\,dx_5\,dx_4\,dx_3\,d$ = 1.4347618883 97263	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	1.431278445 1.433889811 1.434726989 1.434753166 1.434759709 1.434761850	0.00348344 3 0.00087207 7 3.48994E-0 5 8.7224E-06 2.1794E-06 3.83973E-0 8	0.49E -8
$\int_{0}^{2\pi}\int_{0}^{2\pi} f_4\ dx_1\,d$ = -4	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	-4.30971672 -4.07485119 -4.00296222 -4.00074077 -4.00018515 -4.00008231	0.30971672 7 0.07485119 3 0.00296222 2 0.00074077 2 0.00018515 4 8.231E-05	0.41E -7
$\int_{0}^{1}\int_{0}^{1}\int_{0}^{1}\int_{0}^{1} f_5$ dx_2 = 0.1839071529 076452	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	0.191799495 6 0.185836913 6 0.183983803 3 0.183926311 2 0.183911942 4 0.183909281 2	0.00789234 3 0.00192976 1 7.66504E-0 5 1.91583E-0 5 4.78949E-0 6 2.12829E-0 6	0.83E -7
$\int_{0}^{\frac{\pi}{2}}\int_{0}^{\pi}\int_{0}^{\pi}\int_{0}^{\pi}\int_{0}$ $dx_1\,dx_2\,dx_3\,dx_4\,d$ = 16	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	16.28232082 16.07007653 16.00279736 16.00069928 16.00000752 16.00000003	0.28232082 0.07007653 0.00279736 0.00069928 7.52E-06 3E-08	0.19E -11
$\int_{0}^{1}\int_{0}^{1} f_7\ dx_1\,dx_2$ = 499.1249442	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	147.2763481 275.4184600 497.1703940 499.1008728 499.1198018 499.1226584	351.848596 1 223.706484 2 1.95455022 4 0.02407142 4 0.00514242 4 0.0022858	0.16E -7
$\int_{0}^{1}\int_{0}^{1}\int_{0}^{1} f_8\ dx_1\,d$ = 0.8630462173 553432	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	0.803648175 0 0.832776326 1 0.856900630 8 0.859967695 1 0.861505523 9 0.863045883 2	0.05939804 2 0.03026989 1 0.00614558 7 0.00307852 2 0.00154069 3 3.34155E-0 7	0.39E -4

Table: 2

Exact value	Order N	Obtained numerical results	Error	Error (Sarada & Nagaraja. 2014)
For n = 2 $\int_0^1\int_0^1 \cdots \int_0^1 \frac{1}{2^n}$ $dx_1\,dx_2 \ldots\ldots dx_n$ = 1	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000	0 0 0 0 0 0	0
For n=3 $\int_0^1\int_0^1 \cdots \int_0^1 \frac{1}{2^n}$ $dx_1\,dx_2 \ldots\ldots dx_n$ = 1	N=5 N=10 N=50 N=100 N=200 N=300	1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000	0 0 0 0 0 0	0.99E -16
For n=4 $\int_0^1\int_0^1 \cdots \int_0^1 \frac{1}{2^n}$ $dx_1\,dx_2 \ldots\ldots dx_n$ = 1	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000	0 0 0 0 0 0	0.99E -16
For n=5 $\int_0^1\int_0^1 \cdots \int_0^1 \frac{1}{2^n}$ $dx_1\,dx_2 \ldots\ldots dx_n$ = 1	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000	0 0 0 0 0 0	0.99E -16
For n=6 $\int_0^1\int_0^1 \cdots \int_0^1 \frac{1}{2^n}$ $dx_1\,dx_2 \ldots\ldots dx_n$ = 1	N:=5 N:=10 N:=50 N:=100 N:=200 N:=300	1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000 1.00000000000 00000000	0 0 0 0 0 0	0.55E -14

IV. CONCLUSIONS

In this study, we implemented quadrature approach with Haar wavelet technique has been applied to solve the multiple integrals problems over an n-dimensional cube, the proposed Haar wavelet method has been performed using MAPLE - 13 Software, the validity of our results is good accuracy with the previous authors

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