

# Effects of Mass Suction on MHD Boundary Layer Flow and Heat Transfer over a Porous Shrinking Sheet with Heat Source/Sink

B. Shankar Goud, Pudhari Srilatha, MN Raja Shekar

**Abstract:** An examination is made to think about the impacts of the mass suction on the steady flow of 2-D magneto-hydrodynamic (MHD) boundary layer flows and heat transfer past on a shrinking sheet with source/sink. In the dynamic framework, an-uniform magnetic field acts perpendicular to the plane of flow. The governing non-dimensional partial differential equations are changed into nonlinear ordinary differential equations (ODE's) using similarity transformations. The so derived ordinary differential equations are solved numerically by using the MATLAB solver *bvp5c*. From the keen examinations it is found that the velocity inside the boundary layer increments with increment of wall mass suction, magnetic field and reportedly the thickness of the momentum layer diminishes. There is a reduction in temperature as increases the Prandtl number. With heat source specifications, Hartmann number, heat sink parameter & the temperature increments are seen. Moreover, for strong heat source heat assimilation at the sheet happens.

**Index: Terms:** Heat Transfer, Heat Source/Sink, MHD Boundary layer, attenuation Sheet and *bvp5c*.

## I. INTRODUCTION

The analysis of assorted convection boundary layer flow of an incompressible viscous fluid over a reduction of sheet has a key part to play in most of the manufacturing and technological areas of applications, such as metal and polymer processing industries, paper production, wire drawing and many others. Several authors have studied the complications in boundary layer flow of an electrically conducting viscous fluid about the stretching sheet problems, such as; Postelnicu [1] examined the effect of chemical reaction. Shankar Goud and Raja Shekar [2] studied heat source and thermal radiation effects, Bhattacharyya and Layek [3] analyzed chemical reaction impact with suction or blowing on MHD free convection over on MHD boundary layer flow over a permeable stretching sheet. Ali and Al-Salem [4] reviewed the impact of suction or addition on the boundary layer flows made by continuous surfaces stretched with given skin friction. Uwanta and Hamza [5] analyzed the influence of suction/injection with thermal diffusion effect on unsteady hydromagnetic convective flow of volatile viscous fluid between perpendicular porous plates. Shankar Goud et.al [6] considered the implicit finite

difference manner for the study with heat transfer of MHD flow of a micro polar fluid past a stretching sheet. Effects suction or injection, chemical reaction of heat and mass transfer along a wedge with heat source and concentration was discussed by Kandasamy et.al [7]. Kandasamy et.al [8] scrutinized the effects of chemical reaction with heat radiation in the existence of suction or injection of heat & mass transfer on boundary layer flow over a porous wedge. Shankar Goud and Dharmendar Reddy [9] studied the chemical reaction, heat source effect on MHD heat in addition to mass transfer fluid stream over a moving vertical plate and convective boundary condition. Effects of heat source/ sink on MHD flow and heat transfer over a shrinking sheet with mass suction was analyzed by Krishnendu Battacharrya [10]. Muhaimin et.al [11] examined in the presence of suction. Nonlinear MHD boundary layer of over flow shrinking sheet effects by heat & mass transfer. G. Bal Reddy et.al [12] studied the Keller box solution of MHD boundary layer flow of nanofluid effects exponentially on stretching porous sheet. Shankar Goud [13] analyzed the radiation effects on oscillating vertical plate in porous medium with MHD flow in the presence of the chemical reaction. Finite element method application of heat source & suction influences on an unsteady MHD convective heat and mass transfer flow in a semi-infinite vertical moving in a porous medium was studied by Shankar Goud and MN Rajashekar [14].

In the current analysis, with the help of heat source or sink, the impact of suction on MHD boundary layer flow and heat transfer over a porous stretching sheet are investigated. The governing equations are changed into a set of non-linear ODE's using the similarity transformation. MATLAB in built solver is used to solved these numerically. Results are plotted in figures for different physical specifications concerned in the equations and are explore in detail.

## II. MATHEMATICAL FORMULATION

Take into account the MHD flow of an incompressible viscous electrically conducting fluid, where heat transfer takes place over a porous shrinking sheet which coincides with the plane  $y=0$ . The flow is restricted to  $y>0$  in the occurrence of heat generation and absorption. The x-axis is taken along sheet and y-axis is perpendicular to sheet respectively and variable magnetic field  $B_0$  is employed to normal to the plate. A geometry of the physical problem is given in the figure 1.

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**B.Shankar Goud**, Department of Mathematics, JNTUH College of Engineering Hyderabad, Kukatpally, Telangana State, India. 500085

**Pudhari Srilatha**, Institute of Aeronautical Engineering, Hyderabad, Telangana State, India.

**MN Raja Shekar**, Department of Mathematics, JNTUH College of Engg. Jagitial Nachupalli, Jagitial Dt.-505 501, Telangana State, India.

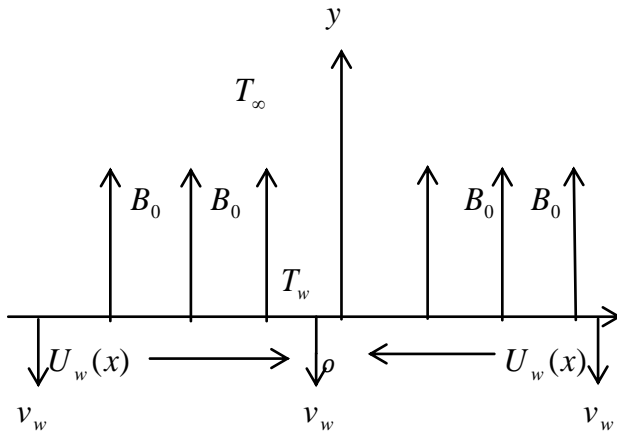


Figure 1: An outline of the physical problem

According to the boundary layer conditions the governing equ's are given by

$$\text{Continuity equation: } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

Energy equations:

$$\rho C_p \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} + Q_0 (T - T_\infty) \quad (3)$$

The appropriate boundary condition are given by

$$u = U_w(x) = -cx, \quad v = v_w, \quad T = T_w \quad \text{at } y \rightarrow 0 \quad (4)$$

$$u \rightarrow 0, \quad T \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

Now bringing in the stream function  $\xi$ , the velocity components  $u$  and  $v$  can be written as

$$u = \frac{\partial \xi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \xi}{\partial x} \quad (5)$$

Equation (1) satisfied using the eqn. (5) and momentum and temperature equations take the following forms:

$$\frac{\partial \xi}{\partial x} \frac{\partial}{\partial x} \left( \frac{\partial \xi}{\partial y} \right) - \frac{\partial \xi}{\partial x} \frac{\partial}{\partial y} \left( \frac{\partial \xi}{\partial x} \right) = \nu \frac{\partial}{\partial y} \left( \frac{\partial^2 \xi}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} \frac{\partial \xi}{\partial y} \quad (6)$$

$$\frac{\partial \xi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \xi}{\partial x} \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad (7)$$

Also the boundary conditions is suited to

$$\frac{\partial \xi}{\partial y} = -cx, \quad \frac{\partial \xi}{\partial x} = v_w \quad \text{at } y \rightarrow 0 \quad (8)$$

$$\frac{\partial \xi}{\partial y} \rightarrow 0, \quad \frac{\partial \xi}{\partial x} \rightarrow 0 \quad \text{as } y \rightarrow \infty$$

Next, we bring in the non-dimensional variables for  $\xi$  and  $T$  as  $\xi = \sqrt{c\nu} x f(\eta)$  and  $(T - T_\infty)/(T_w - T_\infty) = \theta(\eta)$

where  $\eta = y \sqrt{\frac{c}{\nu}}$  is a similarity variable. Using the non-dimensional and similarity variables, eqns. (6) and (7) converted to the following form:

$$f' = (f''' + ff'') / (f' - M^2) \quad (9)$$

$$\theta'' = \text{Pr}(\lambda\theta - f\theta') \quad (10)$$

Here  $\lambda = Q_0/\rho C_p$  is a heat source ( $\lambda < 0$ ) or sink ( $\lambda > 0$ ) parameter.

The boundary conditions (8) and (4) becomes in the following form:

$$\eta \rightarrow 0: f = S, f' + 1 = 0, \theta = 1$$

$$\eta \rightarrow \infty: f' \rightarrow 0, \theta \rightarrow 0,$$

(11)

Where  $S = v_w/\sqrt{c\nu} > 0$  is the mass suction factor.

### III. NUMERICAL PROCEDURE FOR SOLUTION

With the above described numerical procedures, Numerical solutions have been attained for the governing equations (9) and (10) with the assorted boundary condition (11) for some of the non-dimensional specifications, namely Magnetic parameter ( $M$ ), Prandtl number ( $Pr$ ), mass suction parameter ( $S$ ), heat source parameter ( $\lambda$ ). The effects of  $M, Pr, S$  and  $\lambda$  on the fluid temperature and velocity stream over shrinking sheet are explored in detail. The numerical computations are done by using the MATLAB in-built numerical solver bvp5c.

### IV. RESULTS AND FINDINGS

Numerical computations are used to carry out the results for several values of non-dimensional factors. For explaining the results, numerical values are located in figures from B to E. Also, in the table below, the comparison between the current and past results of skin friction factor  $f''(0)$  for dissimilar standards of mass suction parameter ( $S$ ) with  $M^2 = 2$  are shown. In this study a focus is laid on the numerical values of flow parameters namely, the mass suction parameter ( $S$ ), the Hartmann number  $M$ , the heat source or sink parameter ( $\lambda$ ) and the Prandtl number ( $Pr$ ). In order to enforce a steady flow close to the sheet by restricting the initiated vorticity within the boundary layer, large values are assigned to  $S$  and  $M$  indicating a strong magnetic field and wall mass suction.

Figures B to H depict the flow characteristics and temperature field with varying parameter values. The current results can be assured the exactness of applied numerical scheme as the outcomes of Muhaimin et al. in the above table and the computed skin friction coefficients are in perfect agreement. Practically, a lot of importance is given to the influence of Hartmann number  $M$  on temperature & velocity profiles.

This is studied in Figures B and C, as  $M$  goes up, the value of  $f'$  increases indicating the rise in dimensionless velocity. Reasoning to this behaviour is that with a velocity of electrically conducting fluid, there arises a Lorentz force. The thus generated Lorentz force aids to the fluid velocity in the boundary layer area thereby reducing momentum boundary layer width. It can be observed from Figure C that temperature value at a point diminishes with  $M$ . Considering the impact of mass suction specification  $S$  on velocity and temperature profiles, we understand the following.

S	Current study	Muhaimin. <i>et.al</i> [11]	Battacharryya [10]
2	2.414476	2.414214	2.41300
3	3.302816	3.302776	3.30275
4	4.236072	4.236068	4.23609

From figure D, it is clear that for a constant term of  $\eta$ , bigger the value of applied suction, higher is the velocity profile leading to a thinner momentum boundary layer. Figure E illustrates wall mass suction has an effect on not only on velocity field but also on the dimensionless temperature allocation. Clearly it is evident that for fixed  $\eta$ , the temperature  $\theta$  decreases on increasing the value of suction. This reduces the width of thermal boundary layer as a consequence. With raise in Prandtl number  $Pr$ , Figure F depicts the reduce in thermal boundary layer width and the dimensionless distribution of temperature. Also, it's understood that increased value of  $Pr$  means a reduced thermal fluid conductivity. This is the reason for reduced temperature. Also, as momentum equation is the same with or without  $\theta$ , velocity of fluid field is not affected by Prandtl number  $Pr$ . Figure G indicates that on increasing heat sink strength, the dimensionless temperature  $\theta$  decreases while on increasing heat source strength temperature increases. Hence it can be said that increased heat sink parameter reduces the width of thermal boundary layer but alternate behaviour can be observed with heat source parameter. Figure H depicts the varying heat flux at the sheet  $\theta'(0)$  for varying magnitude of  $Pr$  and  $\lambda$ , which is considered important in calculating rate of heat transfer. The sign of  $\theta'(0)$  indicates the heat flux, absorption is positive value and negative is transfer. On increasing Prandtl number & the heat flux increases An observation is made that, for small factor of Prandtl number,  $Pr$  with higher magnitudes of heat source specification ( $\lambda < 0$ ) there is heat retention at the sheet. However, heat transfer increases with increased heat sink parameter ( $\lambda > 0$ ).

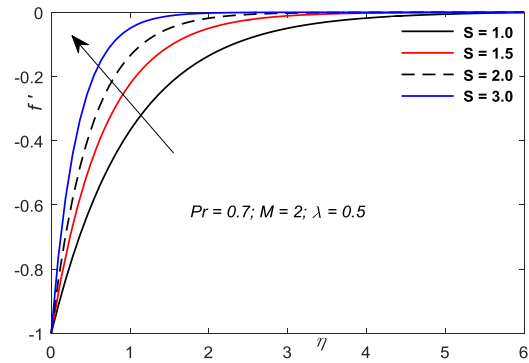


Fig C: Velocity profile for different magnitudes of S

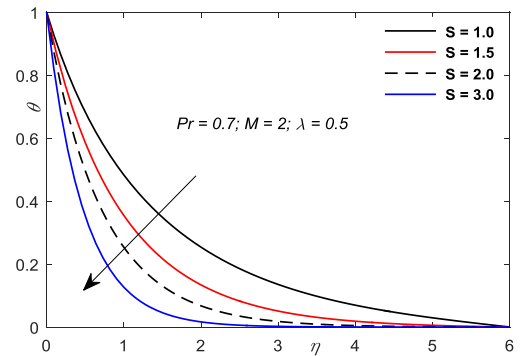


Fig D: Temperature profile for various magnitudes of S

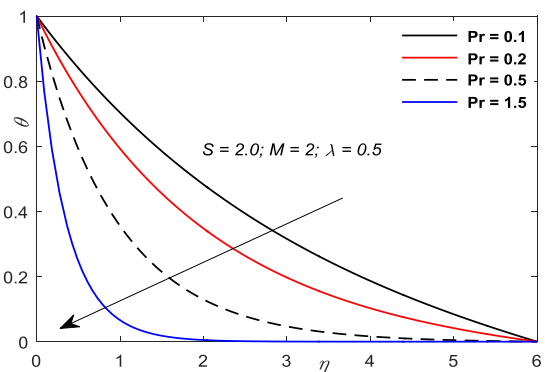


Fig E: Temperature curves considering various magnitudes of Pr

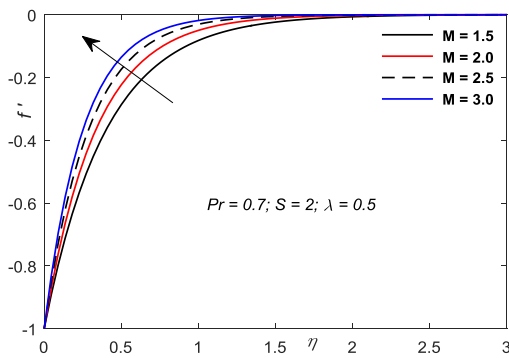


Fig A: Velocity curve for different magnitudes of M

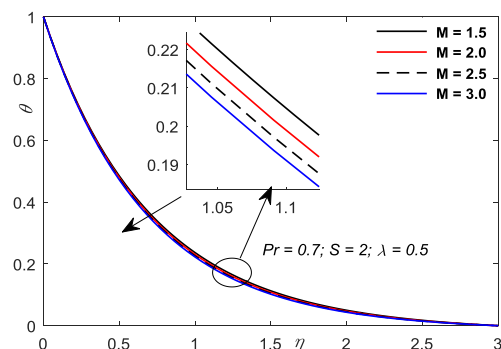


Fig B: Temperature profile for different magnitudes of M

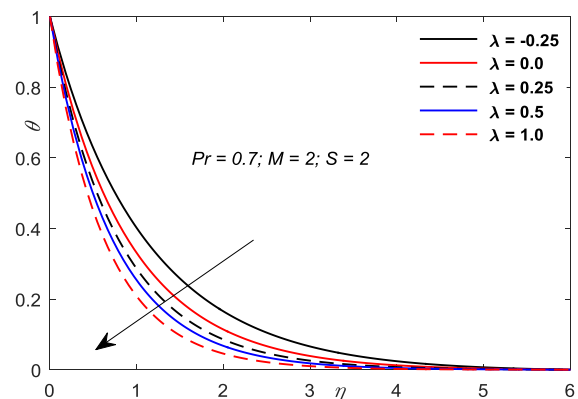


Fig F: Temperature variations for several values



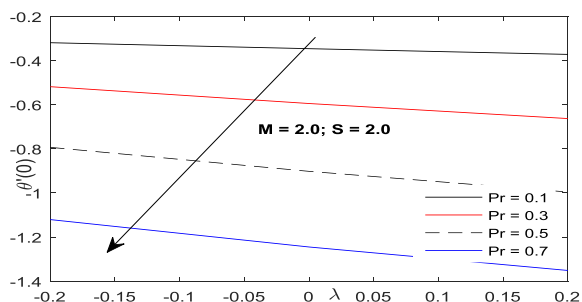


Fig G: Heat flux at the sheet for several magnitudes

## V. CONCLUSION

Subjected to strong suction, a study is made on effect of heat source/sink on MHD boundary layer flows and heat transmits over an attenuation sheet. Similarity transformations helped to obtain the self-similar connections. The required equations are solved by using MATLAB solver bvp5c. It can be said that with an expand in Hartmann number and mass suction specifications, temperature at a point and thickness of momentum boundary layer diminishes. On expanding Prandtl number, temperature and thickness of the boundary layer decreases.

At higher values of heat source constraint, the heat absorption is more on sheet boundary. The transfer of heat is considerably high for higher values of Prandtl constant and heat sink specifications which is a critical aspect of production engineering useful to yield a good quality final product.

## APPENDIX

$u, v$	Velocity components in x and y direction
$\nu = \mu/\rho$	Kinematic fluid viscosity
$\mu$	Coefficient of fluid viscosity
$\rho$	Fluid density
$\sigma$	Electrical conductivity of the fluid
$B_0$	Applied uniform magnetic field
$T, T_\infty \& T_w$	Temperature, Free stream Temperature and Temperature of the sheet
$C_p$	Specific heat
$k$	Fluid thermal conductivity
$c > 0$	Shrinking constant
$v_w > 0$	Wall mass suction parameter
$M = \sqrt{\sigma B_0^2 / c\rho}$	Hartmann constant
$Pr = \mu C_p / k$	Prandtl constant

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## AUTHORS PROFILE



B. Shankar Goud has received Ph.D degree from JNTU Hyderabad and presently working as an Assistant Professor, Department of Mathematics, JNTU University Hyderabad College of Engineering Hyderabad, India. I have 9 years of wide experience in teaching various disciplines like B. Tech, M.Sc, M.Tech etc. He published twenty one research papers. His interested areas are computational fluid dynamics, heat & mass transfer, nanofluids etc.



Ms. P. Srilatha is working as an Assistant professor of mathematics at Institute of Aeronautical Engineering, An autonomous engineering college at Hyderabad. She has 12 years of wide experience in teaching. She has published 8 research papers in her thrust area of research -fluid dynamics.



MN. Raja shekar is a professor of mathematics, JNTUH College of Engineering Jagtial Nachupalli, Jagtial Telangana, His interested areas are computational fluid dynamics, heat & mass transfer, nanofluids etc.