

Peristaltically Induced Electroosmotic Flow of Jeffrey Fluid with Zeta Potential and Navier-Slip at the Wall

Laxmi Devindrappa, N. B. Naduvinamani, G.C Shit

Abstract: This paper concern with the electro-osmotically modulated peristaltic of Jeffrey fluid with zeta potential and Navier-slip boundary condition at the channel wall. The Poisson-Boltzmann equation for electrical potential distribution is assumed to accommodate the electrical double layer. Poisson-Boltzmann equations are simplified by using Debye-Huckel linearization approximation. The closed form analytical solutions are calculated by using low Reynolds number and long wavelength assumptions. Influence of various parameters like electro-osmotic, Jeffrey fluid parameter, Slip parameter and Zeta potential on the flow are discussed through the nature of graphs.

Index Terms: Jeffrey fluid model, Peristaltic transport, Slip Velocity and Zeta Potential.

I. INTRODUCTION

The phenomena of peristaltic are of great importance in many engineering and biological systems. Latham [1] has coined the idea of fluid transport in peristalsis. Devakar et al. [2] studied the effect of porous medium with non-Newtonian fluids. Kothandapani and Srinivas [3] have studied peristaltic transport of a Jeffrey fluid. Qayyum et al. [4] have discussed the unsteady squeezing flow of Jeffrey fluid. Akbar and Nadeem [5] have analyzed the blood flow with Jeffrey fluid model. Xiaoyi and Haitao [6] studied the analytical solution of electro-osmotic on peristalsis. Bhatti et al. [7] have analyzed the peristaltically induced motion of Jeffrey nano fluid. Goswami et al. [8] studied the electro-kinetically modulated peristaltic transport. Yadav et al. [9] studied the peristaltic pumping through porous medium. Combined effects of peristalsis and electro-osmosis analyzed by Tripathi et al. [10]. Ranjit and Shit [11] investigated the micro-channel with different zeta potential. Ranjit et al. [12] studied the two layered electro-osmotic flow. Mondal and Shit [13] have discussed the electro-osmotic flow. Ranjit and Shit [14] have analyzed the flow through a micro-channel. In this study peristaltic transport of Jeffrey fluid model with navier-slip boundary condition with zeta potential at the wall were investigated. By assuming of long wavelength and low Reynolds number, the governing flow problem is solved. The

reduced equations are solved analytically. The collision of all the parameters of interest is taken into reflection with the help of nature of graphs.

II. MATHEMATICAL MODEL

The geometry of the wall surface is

$$H(x,t) = a + b \cos \frac{2\pi}{\lambda} (X - ct). \quad (1)$$

where b is amplitude and λ is the wave length, c is the velocity of wave propagation and X is the direction of wave propagation.

The advective equations for Jeffrey fluid are given by

$$\bar{T} = -P\bar{I} + \bar{S}. \quad (2)$$

$$\bar{S} = \frac{\mu}{1 + \lambda_1} (\nabla \otimes \nabla + \lambda_2 \nabla \otimes \nabla). \quad (3)$$

The transformation between laboratory frame (\bar{X}, \bar{Y}) and wave frame (\bar{x}, \bar{y}) frames is given by

$$x = \bar{X} - ct, y = \bar{Y}, w = \bar{W} \text{ and } v = \bar{V} \quad (4)$$

Where W and V are velocity constituents within in the laboratory frame and w and v are the velocity components within the wave frame.

The equations governing the electroosmotic flow are taken as

$$\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (5)$$

$$\rho \left[w \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right] = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho_e E_x. \quad (6)$$

$$\rho \left[w \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}. \quad (7)$$

where E_x denote electro kinetic force. The Poisson's equation is

$$\Delta^2 \Phi = -\frac{\rho_e}{\epsilon}.$$

(8) Where ρ_e is the density of the total ionic charge and ϵ is the permittivity. The Boltzmann equation

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$$n^{\pm} = n_0 \text{Exp} \left[\pm \frac{e z \bar{\Phi}}{K_B T} \right].$$

(9) where n_0 represents concentration of ions at the bulk, e is the electric charge, K_B is the Boltzmann immutable, and T is the average temperature of the electrolytic solution. Introducing the dimensionless quantities

$$\begin{aligned} \bar{x} &= \frac{x}{\lambda}, \bar{y} = \frac{y}{a}, \bar{w} = \frac{w}{c}, \bar{v} = \frac{v}{c\delta}, \\ \delta &= \frac{a}{\lambda}, p = \frac{a^2 \bar{p}}{\mu_0 c \lambda}, \bar{t} = \frac{ct}{\lambda}, \bar{\tau} = \frac{a\tau}{\mu_0 c}, \\ \phi &= \frac{b}{a}, \bar{\Phi} = \frac{\Phi}{a^2}, R_e = \frac{\rho c a}{\mu} \end{aligned} \quad (10)$$

The equations governing the flow become

$$\frac{\partial w}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (11)$$

$$R_e \delta \left[w \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right] = -\frac{\partial p}{\partial x} + \delta \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + m^2 U_{hs} \Phi. \quad (12)$$

$$\delta^3 R_e \left[w \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial x} + \delta^2 \frac{\partial \tau_{xy}}{\partial x} + \delta \frac{\partial \tau_{yy}}{\partial y}. \quad (13)$$

where

$$\begin{aligned} \tau_{xx} &= \frac{2\delta}{1+\lambda_1} \left[1 + \frac{\delta \lambda_2 c}{a} \left(w \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right] \frac{\partial w}{\partial x}, \\ \tau_{yy} &= \frac{2\delta}{(1+\lambda_1)} \left[1 + \frac{\delta \lambda_2 c}{a} \left(w \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right] \frac{\partial w}{\partial y}, \\ \tau_{xy} &= \frac{1}{1+\lambda_1} \left[1 + \frac{\delta \lambda_2 c}{a} \left(w \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right] \left(\frac{\partial w}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right), \end{aligned}$$

Utilizing long wavelength approximation and dropping terms of order δ and higher, Eqs. (11) - (13) reduces to

$$\frac{\partial}{\partial y} \left(\frac{1}{1+\lambda_1} \frac{\partial w}{\partial y} \right) + m^2 U_{hs} \Phi = \frac{\partial p}{\partial x}. \quad (14)$$

$$\frac{\partial p}{\partial y} = 0. \quad (15)$$

The non dimensional conditions are

$$\frac{\partial w}{\partial y} = 0 \text{ at } y=0 \text{ and } w = -\beta \frac{\partial w}{\partial y} \text{ at } y=h. \quad (16)$$

where $\beta = \frac{s_i}{a}$ is the velocity slip parameter, s_i is the slip

length for hydrophobic channel $m = a e z \sqrt{\frac{2n_0}{\epsilon K_B T}}$ is

electroosmotic parameter and $U_{hs} = -\frac{E_x \epsilon \zeta}{\mu c}$ is the

maximum electroosmotic velocity. Applying Debye-Hückel linearization approximation, Poisson-Boltzmann equation decreases to

$$\frac{\partial^2 \Phi}{\partial y^2} = m^2 \Phi. \quad (17)$$

The boundary conditions for electrical potential are

$$\frac{\partial \Phi}{\partial y} = 0 \text{ at } y=0, \Phi = \zeta \text{ at } y=h. \quad (18)$$

where ζ is the zeta potential imposed at the channel wall.

Solution of the Poisson-Boltzmann equation (17) subjected to boundary conditions (18) give rise to

$$\Phi = \frac{\zeta \text{Cosh}[m y]}{\text{Cosh}[m h]}. \quad (19)$$

III. ANALYTICAL SOLUTION

Solving the Eq. (14) with the boundary conditions (16), we get

$$w = \frac{1}{2} (1 + \lambda_1) \left(\frac{dp}{dx} (h^2 - y^2 + 2h\beta) + 2U_{hs} \zeta \right) \frac{\text{Cosh}[m y]}{\text{Cosh}[m h]} - \frac{2U_{hs} \zeta m \beta \text{Tanh}[mh]}{2U_{hs} \zeta m \beta \text{Tanh}[mh]}. \quad (20)$$

The volume flux q through each cross section of the micro-channel in the wave frame is given by

$$\begin{aligned} q &= \int_0^h w dy \\ &= -\frac{dp}{dx} \frac{(h^3 m + 3h^2 m \beta)(1 + \lambda_1)}{3m} - \frac{3U_{hs} h m \zeta - 3U_{hs}}{(1 + \lambda_1) \left((-1 + h m^2 \beta) \zeta \text{Tan}[hm] \right)} \frac{3m}{3m}. \end{aligned} \quad (21)$$

The expression for pressure gradient from Eq. (21) has the form

$$\frac{dp}{dx} = -\frac{3m}{(h^3m + 3h^2 m \beta)(1 + \lambda_1)} \left((1 + \lambda_1) \left(\frac{3U_{hs} h m \zeta - 3 U_{hs}}{(-1 + h m^2 \beta) \zeta \text{Tan}[h m]} \right) \right) \quad (22)$$

Volume flow rate is

$$\bar{Q}(x, t) = \int_0^h (w + 1) dy = q + h. \quad (23)$$

$$Q = \frac{1}{T} \int_0^T \bar{Q} dt = q + 1. \quad (24)$$

The pressure rise per wave length is

$$\Delta p = \int_0^1 \left(\frac{dp}{dx} \right) dx. \quad (25)$$

IV. RESULTS AND DISCUSSION

In order to estimate the numerical computation of the analytical expressions (20) – (25), the parametric values are chosen as per the data available in [11].

Figures 1(a)-(d) show the variations of pressure gradient. From Figure 1(a) it is seen that, pressure gradient decreases with escalate in Jeffrey fluid parameter λ_1 . The effect of electro-osmotic parameter m is depicted in Figure 1 (b). noted that axial pressure gradient escalate with diminishing Electro-osmotic. From Figure 1(c) observed that, with an escalate in the velocity slip the axial pressure gradient decreases. Effect of zeta potential ζ is shown in Figure 1 (d). noted that axial pressure gradient escalate with escalating zeta potential.

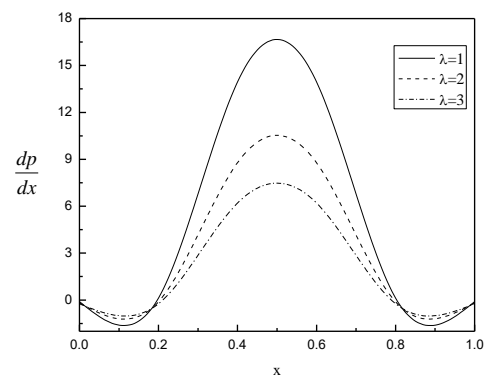
It can be noticed from Figure 2(a) that for an escalate in Jeffrey fluid λ_1 causes decrease in pressure rise. From Figure 2 (b). It is noted that Electro-osmotic parameter elevates pressure differences with increasing averaged volumetric flow rate in $\Delta p > 0$, the free pumping region $\Delta p = 0$ and $\Delta p < 0$. The effect of slip parameter β is depicted in Figure 2 (c). It is noted that pressure rise decreases with increasing slip parameter in $\Delta p > 0$, $\Delta p = 0$ and $\Delta p < 0$. From Figure 2(d) it is revealed that with an increase in the zeta potential there is enhancement $\Delta p > 0$, $\Delta p = 0$ and $\Delta p < 0$.

The development of trappings is another attention grabbing topic in peristaltic transport. The effects of electro-osmotic m , β and zeta potential ζ on the trapping are illustrated in Figs. 3, 4 and 5. Figure 3. indicate that for ascending values of electro-osmotic m , the size of trapped bolus becomes larger. Figure 4. Indicate that for large value of β the size of trapped bolus decreases. Figure 5. Indicate that for increasing values of zeta potential ζ , there is no changes.

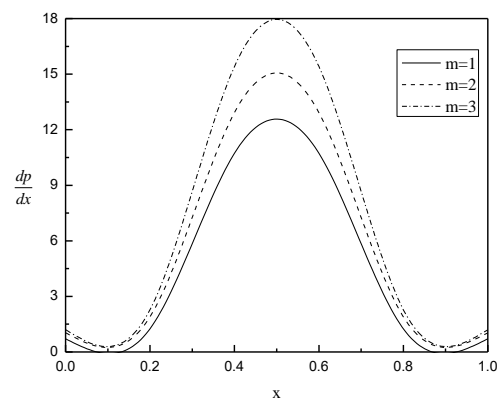
V. CONCLUSION

In the present work, we have analyzed peristaltic transport of Jeffrey fluid with nerver-slip boundary condition with zeta potential at the wall. Closed form solutions are derived for the pressure gradient and pressure rise. The main observations of the present analysis are as follows.

- It is concluded that pressure gradient decreases with the increase of Jeffrey fluid parameter λ_1 and slip parameter β . while it increases by increasing electro-osmotic parameter m and zeta potential ζ .
- It is concluded that pressure rise decreases with the increase in Jeffrey fluid parameter λ_1 and slip parameter β . However it increases with an increase in m and ζ .
- Trapped bolus size enhances with increase in electro-osmotic m and for large value of slip parameter β the size of trapped bolus decreases.

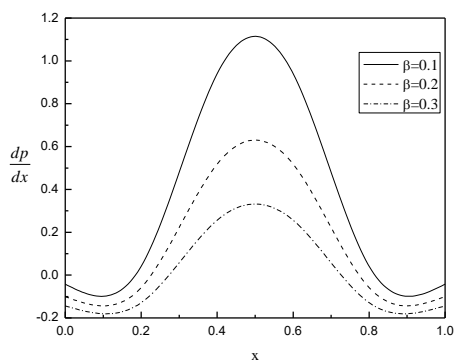


(a)

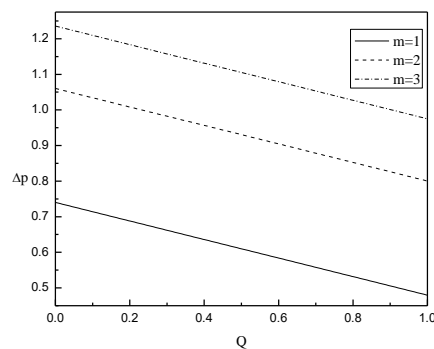


(b)

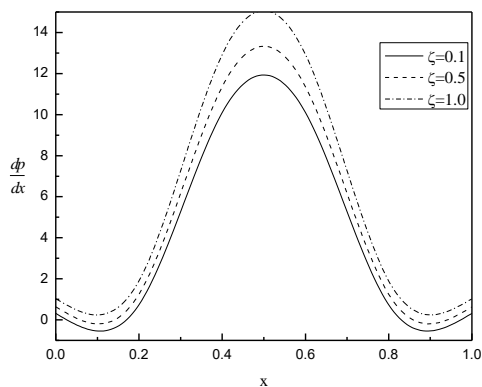
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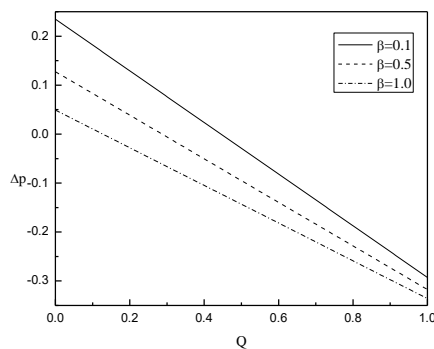
(c)



(b)

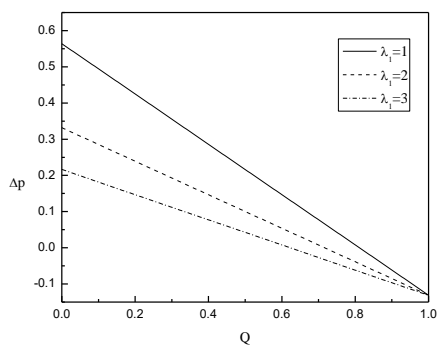


(d)

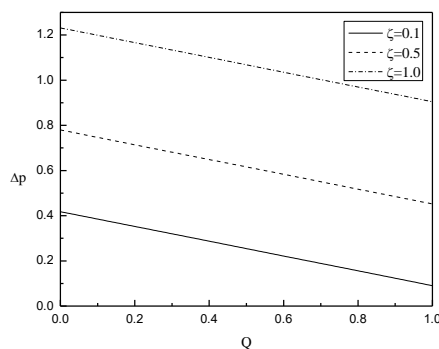


(c)

- Fig. 1. Axial pressure gradient for
- (a) $\phi = 0.7, m = 2, \zeta = -0.5, \beta = 0.01$ and $U_{hs} = 1$.
 - (b) $\phi = 0.7, \lambda = 1, \zeta = 0.1, \beta = 0.1$ and $U_{hs} = 1$.
 - (c) $\phi = 0.4, \lambda = 1, \zeta = -0.1, m = 3$ and $U_{hs} = 1$.
 - (d) $\phi = 0.7, \lambda = 1, \beta = 0.1, m = 2$ and $U_{hs} = 1$.

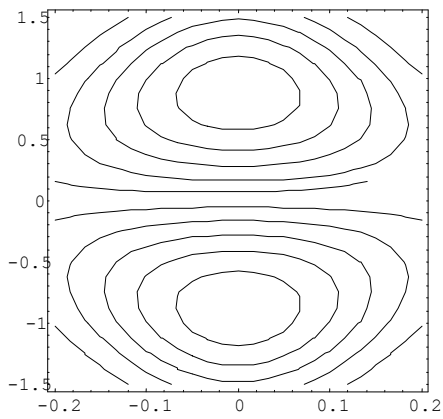


(a)

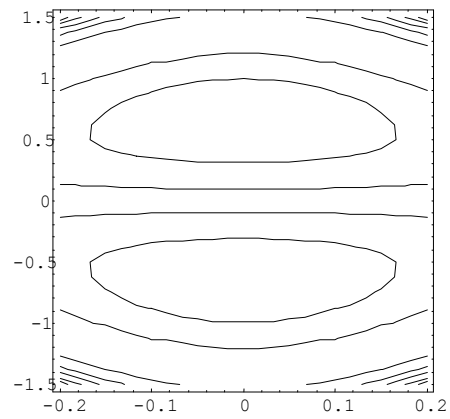


(d)

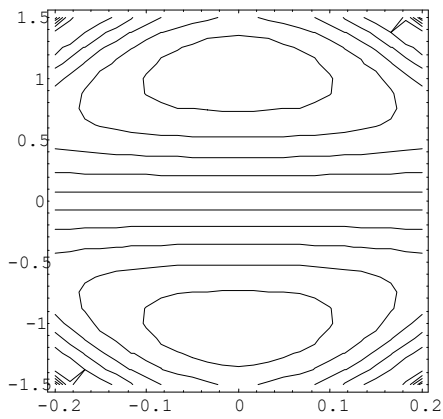
- Fig. 2. Pressure rise with time-averaged flux for
- (a) $\phi = 0.7, m = 2, \zeta = -0.5, \beta = 0.01$ and $U_{hs} = 1$.
 - (b) $\phi = 0.7, \lambda = 1, \zeta = 1, \beta = 0.1$ and $U_{hs} = 1$.
 - (c) $\phi = 0.4, \lambda = 1, \zeta = -0.1, m = 3$ and $U_{hs} = 1$.
 - (d) $\phi = 0.7, \lambda = 1, \beta = 0.1, m = 2$ and $U_{hs} = 1$.



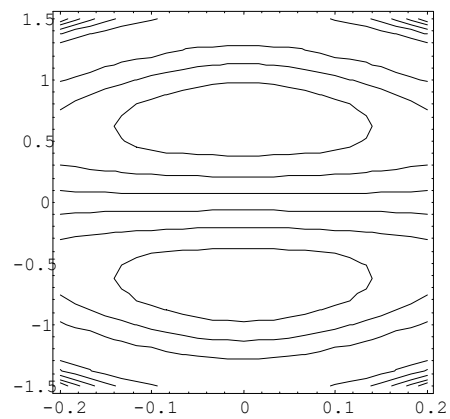
(a) $m = 1$



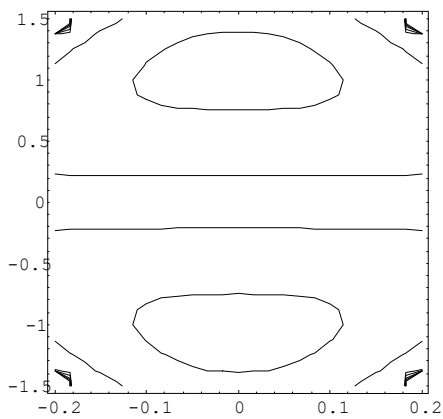
(a) $\beta = 0.0$



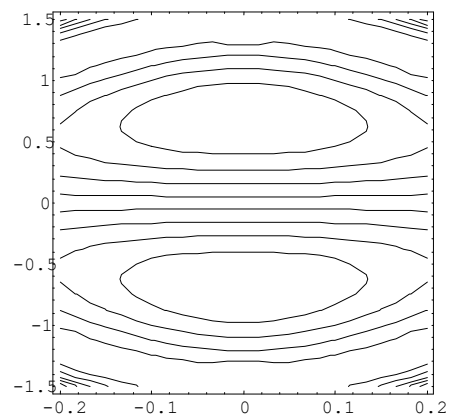
(b) $m = 10$



(b) $\beta = 0.05$



(c) $m = 40$

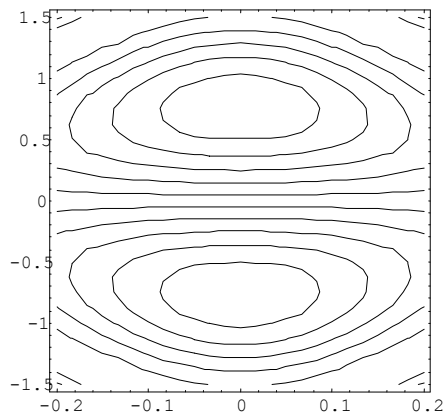


(c) $\beta = 0.1$

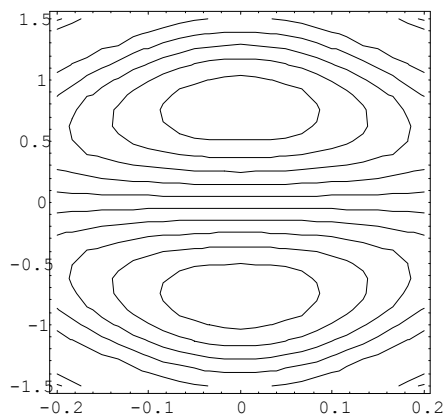
Fig. 3. Streamlines for different values of m (a) $m = 1$, (b) $m = 10$, (b) $m = 40$. The other parameters chosen are $\phi = 0.9, \lambda = 0.3, \zeta = -1, \beta = 0.05$ and $U_{hs} = 1$.

Fig. 4. Streamlines for different values of β (a) $\beta = 0.0$, (b) $\beta = 0.05$, (b) $\beta = 0.1$.The other parameters chosen are $\phi = 0.4, \lambda = 0.4, \zeta = -0.1, m = 3$ and $U_{hs} = 1$.

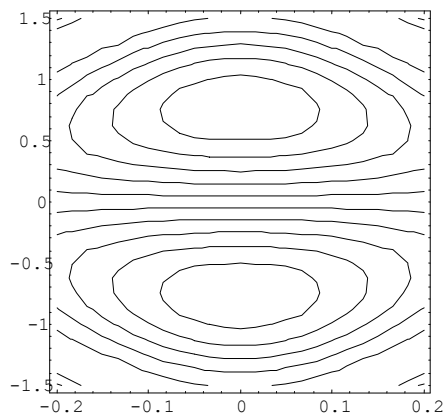
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(a) $\zeta = -0.1$



(b) $\zeta = -0.5$



(c) $\zeta = -1.0$

Fig. 5. Streamlines for different values of ζ (a) $\zeta = -0.1$, (b) $\zeta = -0.5$, (c) $\zeta = -1.0$. The other parameters chosen are $\phi = 0.6$, $\lambda = 0.3$, $\beta = 0.1$, $m = 2$ and $U_{hs} = 1$.

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