International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-8 Issue-10, August 2019

# Applications of Linear Volterra Integral Equations of First Kind by Using Tarig Transform

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*Abstract— A Tarig Transform is a transform for solving linear Volterra integral equations of first kind. A few applications are solved in related to understand the efficiency. This paper deals with this transform and their inverse transform, convolution theorems are used and derived the exact result.*

*Keywords: Volterra integral equations, Tarig transform, Inverse Tarig Transform, Convolution Theorem*

## I. INTRODUCTION

An Integral transform is a best tool for solving the linear integral equations, system of differential equations and integro-differential equations, etc. The integral equations arise in many scientific and engineering problems such as mathematical and physics model, quantum mechanics, diffraction problems, conforming mapping and water waves [6,8,11,12]. Haarsa [7] has solving on Volterra integral equations of the first kind using Elzaki transform. Abdallah and Shama [1] were solving the applications of differential transform method to integral equations. Aggarwal et al. [2-5] has expressed a new application of Aboodh, Kamal and Mahgoub transforms for solving linear Volterra integral equations. Senthil Kumar et al. [9,10] has got the exact results of Mohand transforms for solving linear Volterra integro-differential equations and linear Volterra integral equations of first kind.

In mathematics the general Volterra integral equation is defined as

$$\phi(x)y(x)=f(x)+\lambda\int_a^x k(x,t)y(t)dt \qquad (1)$$

when $\phi(x)=0$, we are getting the classifying of first kind is given by,

$$f(x)=-\lambda\int_a^x k(x,t)y(t)dt \qquad (2)$$

where $k(x,t)$ is an any function of kernel along with $x$ and $t$ .

## II.TARIG TRANSFORM

The Tarig M.Elzahi was introduced a new transform as "Tarig Transform", defined as

$$T\left[f(t)\right]=\frac{1}{u}\int_0^\infty f(t)e^{-\left(\frac{t}{u^2}\right)}dt,\ u\neq 0 \qquad (3).$$

*Tarig Transform of Some Standard Results*

The Tarig transform of $f(t)$ is defined as $T\left[f(t)\right]=E(u)$ and some standard functions are listed in Table I.

**Table 1: Standard Functions**

| $f(t)$ | $E(u)=T\left[f(t)\right]$ |
|---|---|
| 1 | $u$ |
| $t$ | $u^3$ |
| $t^2$ | $2u^5$ |
| $t^n$ | $n!u^{(2n+1)}$ |
| $e^{at}$ | $\frac{u}{1-au^2}$ |
| $e^{-at}$ | $\frac{u}{1+au^2}$ |
| $\sin at$ | $\frac{au^3}{1+a^2u^4}$ |
| $\sinh at$ | $\frac{au^3}{1-a^2u^4}$ |
| $\cos at$ | $\frac{u}{1+a^2u^4}$ |
| $\cosh at$ | $\frac{u}{1-a^2u^4}$ |

*Tarig Transform of Some Standard Results*

The Inverse Tarig transform $f(t)$ is defined as $f(t)=T^{-1}\left[E(u)\right]$ and some standard functions are listed in Table II.

**Manuscript published on 30 August 2019.**
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**Table 2: Inverse Functions**

| $E(u)$ | $f(t)=T^{-1}\left[E(u)\right]$ |
|---|---|
| $u$ | $1$ |
| $u^3$ | $t$ |
| $2u^5$ | $t^2$ |
| $n!u^{(2n+1)}$ | $t^n$ |
| $\dfrac{u}{1-au^2}$ | $e^{at}$ |
| $\dfrac{u}{1+au^2}$ | $e^{-at}$ |
| $\dfrac{au^3}{1+a^2u^4}$ | $\sin at$ |
| $\dfrac{au^3}{1-a^2u^4}$ | $\sinh at$ |
| $\dfrac{u}{1+a^2u^4}$ | $\cos at$ |
| $\dfrac{u}{1-a^2u^4}$ | $\cosh at$ |

*Convolution of Tarig Transform*

Let $f(t)$ and $g(t)$ having Tarig transform $M(u)$ and $N(u)$ respectively, then

$$T\left[f(t)*g(t)\right]=u.M(u)N(u) \qquad (4).$$

## III.APPLICATIONS OF TARIG TRANSFORM & RESULTS

In this section, few applications are applied in order to solving the effectiveness of Tarig transform in linear Volterra integral equations of first kind.

*Application 1*

Solve the linear Volterra integral equation of first kind $x=\int_0^x e^{(x-t)}u(t)\,dt$ .

Solution: Apply the Tarig Transform

$$T(x)=T\left[\int_0^x e^{(x-t)}u(t)\,dt\right]$$

Using convolution theorem of above integral equation and simplify

$$T(x)=T\left[e^x*u(x)\right]$$

$$u^3=uT\left[e^x\right].T\left[u(x)\right]$$

$$=u\left(\frac{u}{1-u^2}\right).T\left[u(x)\right]$$

$$u=\left(\frac{1}{1-u^2}\right).T\left[u(x)\right]$$

$$T\left[u(x)\right]=u-u^3$$

Operating inverse Tarig transform, we have

$$u(x)=T^{-1}[u]-T^{-1}\left[u^3\right]=1-x\,.$$

*Application 2*

Solve the linear Volterra integral equation of first kind $\sin x=\int_0^x e^{(x-t)}u(t)\,dt$ .

Solution: Apply the Tarig Transform

$$T(\sin x)=T\left[\int_0^x e^{(x-t)}u(t)\,dt\right]$$

Using convolution theorem of above integral equation and simplify

$$\frac{u^3}{1+u^4}=u.\left[T\left(e^x\right).T\left(u(x)\right)\right]$$

$$=u.\frac{u}{1-u^2}.T\left[u(x)\right]$$

$$T\left[u(x)\right]=\frac{u\left(1-u^2\right)}{1+u^4}=\frac{u}{1+u^4}-\frac{u^3}{1+u^4}$$

Operating inverse Tarig transform, we have

$$u(x)=T^{-1}\left[\frac{u}{1+u^4}\right]-T^{-1}\left[\frac{u^3}{1+u^4}\right]=\cos x-\sin x\,.$$

*Application 3*

Solve the linear Volterra integral equation of first kind $x^2=\frac{1}{2}\int_0^x (x-t)u(t)\,dt$ .

Solution: Apply the Tarig Transform

$$T\left(x^2\right)=\frac{1}{2}T\left[\int_0^x (x-t)u(t)\,dt\right]$$

Using convolution theorem of above integral equation and simplify

$$2!.u^5=\frac{1}{2}u.\left[T(x)*T\left(u(x)\right)\right]$$

$$2u^5=\frac{1}{2}u.u^3.T\left[u(x)\right]$$

$$4u=T\left[u(x)\right]$$

Operating inverse Tarig transform, we have

$$u(x)=4T^{-1}[u]=4\,.$$

*Application 4*

Solve the linear Volterra integral equation of first kind $x=\int_0^x e^{-(x-t)}u(t)\,dt$ .

Solution: Apply the Tarig Transform

$$T(x)=T\left[\int_0^x e^{-(x-t)}u(t)\,dt\right]$$

Using convolution theorem of above integral equation and simplify

*Retrieval Number: J10200881019/19©BEIESP*
*DOI: 10.35940/ijitee.J1020.0881019*
*Journal Website: www.ijitee.org*



*Published By:*
*Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)*

International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-8 Issue-10, August 2019

$$u^3 = u.T\left(e^{-x}\right).T\left[u(x)\right]$$

$$u^2 = \left(\frac{u}{1+u^2}\right)T\left[u(x)\right]$$

$$T\left[u(x)\right] = u\left(1+u^2\right) = u+u^3$$

Operating inverse Tarig transform, we have

$$u(x) = T^{-1}\left[u+u^3\right] = 1+x\,.$$

*Application 5*

Solve the linear Volterra integral equation of first kind $x = \int_0^x u(t)\,dt$.

Solution: Apply the Tarig Transform $T(x) = T\left[\int_0^x u(0).u(t-0)\,dt\right]$

Using convolution theorem of above integral equation and simplify

$$u^3 = u.T\left[u(0)\right].T\left[u(x)\right]$$

$$= u.T[1].T\left[u(x)\right]$$

$$= u^2 T\left[u(x)\right]$$

Operating inverse Tarig transform, we have

$$u(x) = T^{-1}[u] = 1\,.$$

## IV.CONCLUSION

In this paper, few applications are applied in order to find out the solutions of Tarig transform for solving linear Volterra integral equations of first kind. The given applications are showed the exact solutions that have been obtained by using very less computational work and spending a very little time. This methodology can be also applied for other linear Volterra Integral Equations.

## V.ACKNOWLEDGMENT

The authors acknowledge and express the appreciation to the reviewer's useful comments and valuable suggestion towards the upgrading of the paper.

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