

# Stationary Time Series in Pricing



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**Abstract:** *The methods for analyzing the dynamics of time series are compared in this study, the mechanisms for assessing the accuracy of value forecasting are examined, and a brief description of the models and examples of their use are provided. The problem of choosing the optimal model according to the criterion of the minimum forecasting error is stated and solved. The methods of mathematical modeling, mathematical statistics and econometrics, such as autoregression, moving average, exponential smoothing, and neural network modeling were used to solve this problem. The result of the study is the algorithm for finding the optimal model based on minimizing the forecasting error, as well as the program that implements this algorithm.*

**Index Terms:** ARIMA, autoregression, dynamics analysis, exponential smoothing, forecast estimation, moving average, neural networks, pricing, time series.

## I. INTRODUCTION

The quality of managerial decisions in the modern conditions, especially in the field of pricing, directly depends on a thorough analysis of the environment, where most of economic indicators are expressed as time series. There are many mathematical models for analyzing time series and predicting their future values.

Forecasting is one of the ways to predict the internal and external conditions of operation. Forecasting as a method of reducing the risks caused by uncertainty allows finding out the most likely state of the environment in the future (political, scientific, technical, financial, environmental, and social). Forecasting allows assessing the immediate and remote consequences of the decisions made.

The use of various models in combination provides a more accurate idea of the predicted parameters. The purpose of the study is to review the existing methods for analyzing time series, create an algorithm for finding the optimal pricing model by the criterion of minimal error, and its software implementation. Statistics on online air tickets sales are selected as the source data.

## II. LITERATURE REVIEW

The most important theoretical aspects of applying the

forecasting methods and models are reflected in the papers of Russian and foreign authors (A.A. Burdina, A.G. Granberg, Yu.N. Lapygin, V.E. Krylov, A.P. Chernyavsky, Jingfei Yang M., J.S. Armstrong, Ferrer R.C. Kinnunen, J. Valuing, Mabert, and others) [1]–[4]. The analysis of various studies revealed the development of methodological and methodical issues regarding the application of forecasting methods and models for solving pricing problems. However, the cited studies do not sufficiently address the most important issues of neural network modeling in solving pricing problems, which justifies the need for research in this field.

## III. METHODS

General description. The forecasting method is a sequence of actions that must be performed to obtain a forecasting model. The forecasting model is a functional representation that adequately describes the desired process and is the basis for obtaining its future values. Methods are divided into two groups: intuitive and formalized [5], [6]. Intuitive forecasting methods deal with expert judgments and evaluations. They are often used in marketing, economics, and politics today, because a system where the behavior needs to be forecasted is either very complex and not amenable to mathematical description or very simple and does not need such a description [3]. Formalized methods are the forecasting methods described in the literature, which result in forecasting models, i.e., they determine a mathematical relationship that allows calculating the future value of the process or make a prediction. Algorithm. The models should be divided into two groups: domain models (mechanics, thermodynamics, and fundamental analysis) and time series models that are seeking for dependencies within a process. The domain models are mathematical forecasting models that use domain laws to build them. For example, the model for the weather forecast contains the equations of fluid dynamics and thermodynamics. The forecast of population development is made using a model built on a differential equation. The forecast of the blood sugar level in a patient with diabetes is made based on a system of differential equations. In other words, dependencies that are inherent in a particular field are used in such models. Such kind of models have an individual approach to the development [7], [8]. The time series models are mathematical forecasting models that seek to find the dependence of the future value on the past value within the process and calculate the forecast based on this dependence. These models are universal for various fields, i.e., their general appearance does not change depending on the nature of the time series. We can use neural networks to predict air temperature, and then use a similar model on neural networks to predict stock market indices.

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The time series models can be divided into two groups: statistical and structural [4], [9].

In statistical models, the dependence of the future value on the past value is presented as an equation. These include:

1. regression models (linear regression, nonlinear regression);
2. autoregressive models (ARIMAX, GARCH, ARDLM);
3. exponential smoothing model; and
4. maximum uniformity sampling model.

In structural models, the dependence of the future value on the past is presented as a certain structure and rules for transition along it. These include: neural network models, Markov's chain models, models based on classification regression trees, etc. [10].

Let us set the task of finding the optimal forecasting model with the criterion of the minimum error.

Suppose some stationary time series  $\{Y_t\}$  is given (Fig.1). The forecasted time series  $\{\hat{Y}_t\}$  can be obtained, using the mathematical model  $\xi \in M$ , where  $M$  is the set of mathematical models.



Fig. 1. Specified time series

The accuracy of the model can be described in different ways, based on the calculation of various errors, including the mean absolute error of forecasting (MAE), root mean squared error (RMSE), mean absolute percentage error (MAPE), etc. [8]:

$$v = \sqrt{\frac{\sum_{t=1}^n (Y_t - \hat{Y}_t)^2}{\sum_{t=1}^n Y_t^2 + \sum_{t=1}^n \hat{Y}_t^2}} \quad (1)$$

This ratio shows the degree of similarity of the time series series  $\{Y_t\}$  and  $\{\hat{Y}_t\}$ . The closer it is to zero, the closer are the compared rows.

The following minimization problem is obtained:

$$\sqrt{\frac{\sum_{t=1}^n (Y_t - \hat{Y}_t(\xi))^2}{\sum_{t=1}^n Y_t^2 + \sum_{t=1}^n \hat{Y}_t^2(\xi)}} \rightarrow \min_{\xi \in M} \quad (2)$$

In this case, the optimal model can be expressed by the following formula:

$$\xi^* = \operatorname{argmin}_{\xi \in M} \left( \sqrt{\frac{\sum_{t=1}^n (Y_t - \hat{Y}_t(\xi))^2}{\sum_{t=1}^n Y_t^2 + \sum_{t=1}^n \hat{Y}_t^2(\xi)}} \right) \quad (3)$$

### Autoregression models AR(p)

Forecasting using the autoregression (AR) model is based on the previous values.

The AR model of the  $p$  order is generally described by the following equation:

$$\hat{Y}_t = \sum_{i=1}^p \alpha_{t-i} Y_{t-i} + \beta + \varepsilon \quad (4)$$

where [4], [9]  $\hat{Y}_t$  is the forecasting value at time  $t$ ;

$\alpha_i$  is the parameters of the AR model;

$Y_{t-i}$  is the time series values;

$\beta \in R$  is the free member of the model; and

$\varepsilon \sim N(0, \sigma^2)$  is the random exposure – white noise.

The parameters of the AR model can be chosen using the

values of the time series and the ordinary least squares method. Having built the charts for two models of orders 1 and 5, the following charts are obtained (Fig. 2a, Fig. 2b).

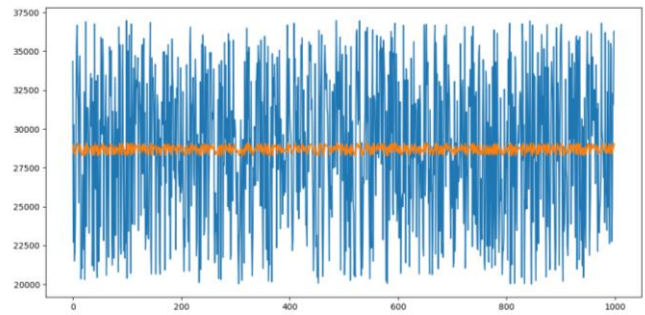


Fig. 2a. AR model (1)

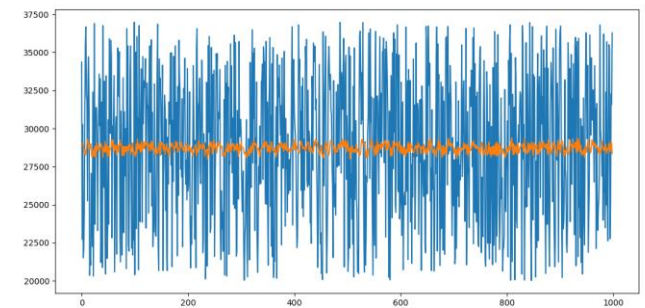


Fig. 2b. AR model (5)

### Moving average models MA(q)

The simple moving average (MA) method is based on the assumption that the future value of a variable depends on the average  $q$  of its previous values.

The MA model of order  $q$  is described as follows [1], [10]:

$$\hat{Y}_t = \mu + \varepsilon_t + \sum_{i=1}^q \omega_i \varepsilon_{t-i} \quad (5)$$

where  $\hat{Y}_t$  is the dependent variable at time  $t$ ;

$\mu$  is the constant average of the process;

$\varepsilon_t$  is the error at time  $t$ ; and

$\omega_1, \omega_2, \dots, \omega_q$  are the estimated ratios.

$$\hat{Y}_{t+1} = \frac{1}{q} \sum_{i=1}^{q-1} Y_{t-i} \quad (6)$$

The application of this model to the original time series results in the following values (Fig. 3):

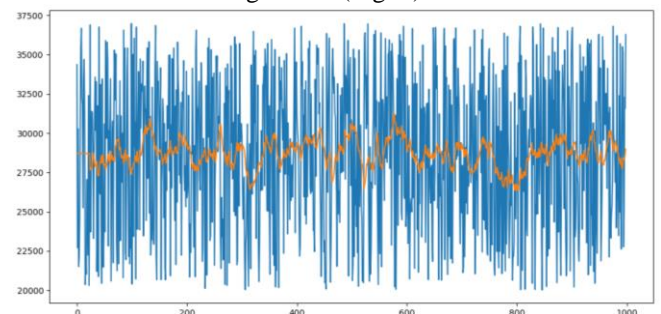


Fig. 3. MA model (1)

There is a modification of this method – a weighted moving average model, WMA(q) [2, 3, 8].

In the general case, a weighted moving average (WMA) is any average that sets different weights for the observed values of a random variable. The idea of its calculation is to assign more weight to new observations and less weight to older observations.

The equation of WMA is as follows:

$$\hat{Y}_t = \frac{\sum_{i=1}^q Y_{t-i} \cdot w_i}{\sum_{i=1}^q w_i} \quad (7)$$

where  $w_i$  is the weights.

There is a method of the exponential WMA, the specifics of which is that the weights decrease exponentially rather than linearly, which ensures better accuracy in forecasting. The following chart is obtained for a given time series in the calculations (Fig. 4):

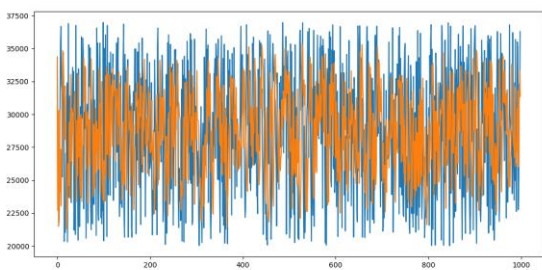


Fig. 4. WMA model (6)

**Integrated autoregression – moving average model ARIMA**

The ARIMA model (AutoRegressive Integrated Moving Average) belongs to the class of linear methods, as it describes the stationary and nonstationary time series equally well. Besides, this model does not use independent variables, which means that only the information contained in the data is used for forecasting.

The model is described by the following equation [9], [10]:

$$\hat{Y}_t = \sum_{i=1}^p \alpha_{t-i} Y_{t-i} + \beta + \sum_{i=1}^q \omega_i \varepsilon_{t-i} + \mu \quad (8)$$

Some nonstationary time series can be reduced to stationary using the sequential difference operator. Let us assume that there is a time series  $y_t$ , to which this operator had been applied to  $d$  times, and after which the series became stationary  $\Delta^d y_t$  and satisfying the conditions of the ARIMA model (p, q). The model of autoregression and moving average will look as follows:

$$\Phi(L)y_t = \delta + \Theta(L)\varepsilon_t, \varepsilon_t \sim iid(0, \sigma^2), \quad (9)$$

where  $\Phi(L) = 1 - \varphi_1 L - \dots - \varphi_p L^p$  and  $\Theta(L) = 1 - \theta_1 L - \dots - \theta_q L^q$  are the polynomials of the shift operator.

In this case,  $y_t$  will be called an integrated process of autoregression and a moving average, or ARIMA (p, d, q).

This model allows building highly accurate forecasts with a small forecasting ranging. It is also quite flexible and may be suitable for describing various time series. Besides, ARIMA models are easily checked for their adequacy. However, the disadvantages of this method include the need for a large amount of source data and the lack of a simple way to adjust the model parameters (Fig. 5).

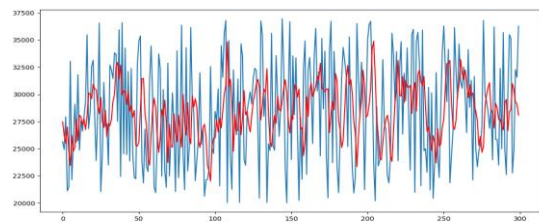


Fig. 5. ARIMA model (2,0,1)

**Exponential smoothing models ES**

Simple exponential smoothing (SES) is described by the following equation:

$$\hat{Y}_{t+1} = \begin{cases} Y_1, & t = 1 \\ \hat{Y}_t + \alpha(Y_t - \hat{Y}_t), & t > 1 \end{cases} \quad (10)$$

where  $\alpha \in (0; 1)$  is the smoothing constant, which describes the rate of decrease in weights. The smaller is its value, the stronger is the influence of the previous values on the current value  $\hat{Y}_t$ . This parameter is selected a priori.

There are the following charts for various  $\alpha$  (Fig. 6):

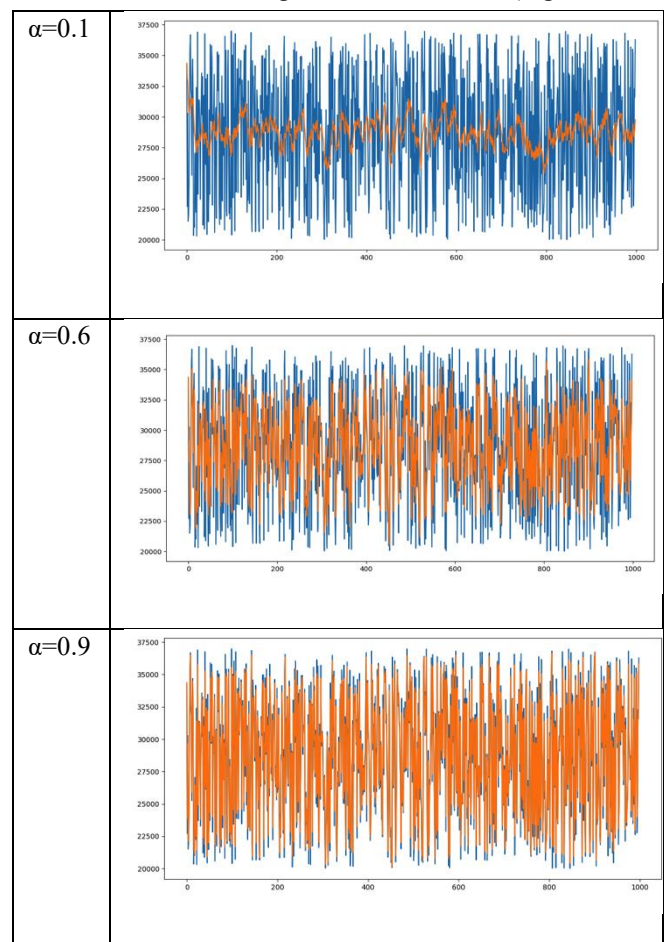


Fig. 6. ES model

Double exponential smoothing (DES) is a modification of the SES method. The idea of this method is as follows: an additional term is introduced in the SES formula, which changes the contribution of the previous value depending on the trend.



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The initial time series is divided into two components: level (intercept)  $l$  and trend  $b$  (slope). The level, or the expected value of the series, is found using the previous methods. Then the exponential smoothing is applied to the trend, assuming that the future direction of change of the series depends on the weighted previous changes. This method can be used to obtain not one, but two forecasting values.

The generalized formulas of the model are as follows [5], [8]:

$$\begin{cases} l_t = \alpha Y_t + (1 - \alpha)(l_{t-1} + b_{t-1}) \\ b_t = \beta(l_t - l_{t-1}) + (1 - \beta)b_{t-1} \\ Y_{t+1} = l_t + b_t \end{cases} \quad (11)$$

where  $\alpha \in [0; 1)$ , and  $\beta \in [0; 1]$ .

A set of functions is obtained as a result. The former describes the level – as before, it depends on the current value of the series, and the second term is now divided into the previous value of the level and trend. The latter is responsible for the trend – it depends on the change in the level at the current step and on the previous trend value. Ratio  $\beta$  plays the role of weight in the exponential smoothing here. Finally, the final forecast is the sum of the model level and trend values (Fig. 7).

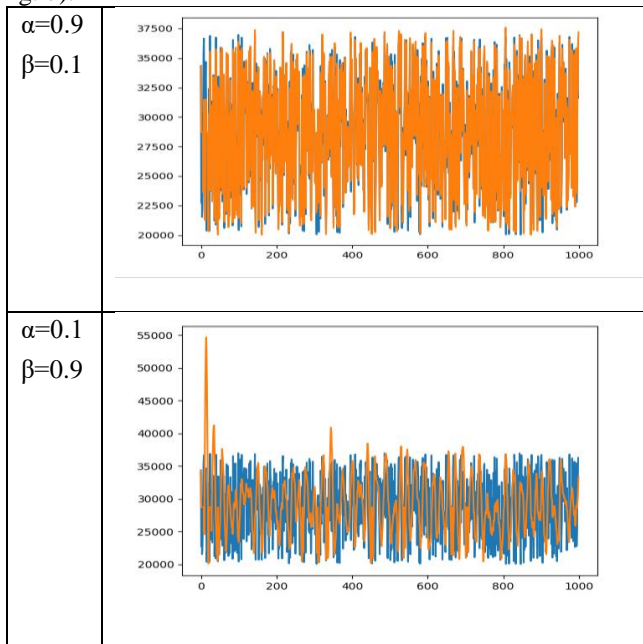


Fig. 7. DES model

In turn, the DES method also has a modification – triple exponential smoothing or the Holt-Winter's (HW) method.

The idea of this method is to add another, third, component – seasonality. Accordingly, the method is applicable only if the series is not deprived of this seasonality, which is true in this case. The seasonal component in the model will explain the repeated fluctuations around the level and trend, and it will be described by the length of the season – the period after which the repetition of the fluctuations begins. The independent component is formed for each observation in the season – for example, if the season length is seven (week-based seasonality), then seven seasonal components are obtained, piece by piece for each of the days of the week.

The system of equations of the model is as follows [2], [5]:

$$\begin{cases} l_t = \alpha(Y_t - s_{t-L}) + (1 - \alpha)(l_{t-1} + b_{t-1}) \\ b_t = \beta(l_t - l_{t-1}) + (1 - \beta)b_{t-1} \\ s_t = \gamma(Y_t - l_t) + (1 - \gamma)s_{t-L} \\ Y_{t+m} = l_t + mb_t + s_{t-L+1+(m-1)modL} \end{cases} \quad (12)$$

The level now depends on the current value of the series net of the corresponding seasonal component, the trend remains unchanged, and the seasonal component depends on the current value of the series minus the level and on the previous value of the component. At the same time, components are smoothed through all available seasons – for example, if this is a component responsible for Monday, then it will be averaged only with other Mondays. Once a seasonal component is available, the forecast can be provided for not just one, or even two, but arbitrary  $m$  steps forward (Fig. 8).

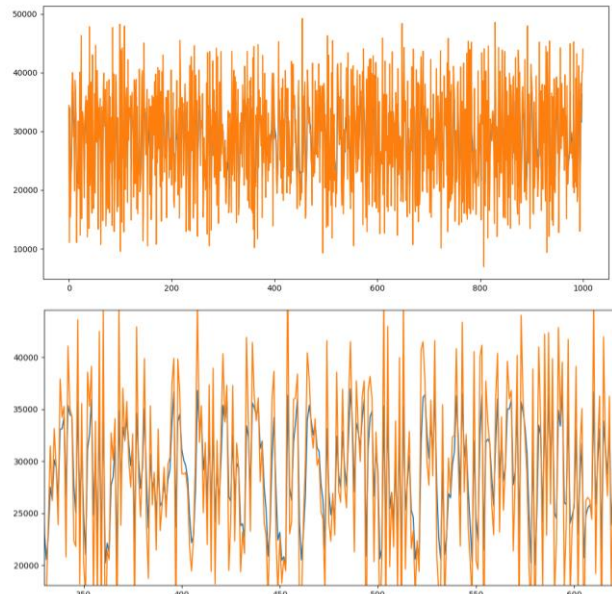


Fig. 8. HW method (at different scales)

The Brutlag's method can also be included in the model to build confidence intervals:

$$\begin{cases} y_{\max\_t} = l_{t-1} + b_{t-1} + s_{t-T} + md_{t-T} \\ y_{\min\_t} = l_{t-1} + b_{t-1} + s_{t-T} - md_{t-T} \\ d_t = \gamma \cdot |Y_t - Y_{t+1}| + (1 - \gamma)d_{t-T} \end{cases} \quad (13)$$

where  $T$  is the season length,  $d$  is the forecasted deviation, and the remaining parameters are taken from triple smoothing (Fig. 9).

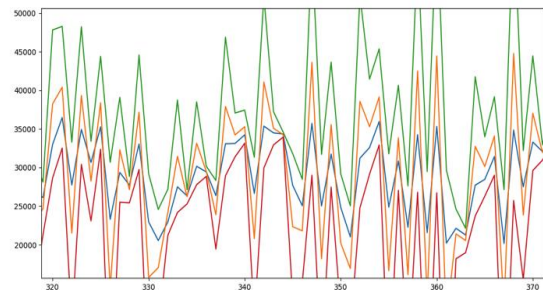


Fig. 9. HW method with confidence intervals

IV. RESULTS

Today there are no algorithms for designing neural networks, thus, the selection of parameters and architecture of neural networks is a creative process in some way. A neural network was used to solve the problem of pricing optimization. The learning rate of the model was chosen as 0.1. The learning was carried out by the gradient descent method implemented by the error back-propagation algorithm [8]. Part of the price statistics data was used to train the neural network. After the program application, the result was obtained as shown in Fig. 10.

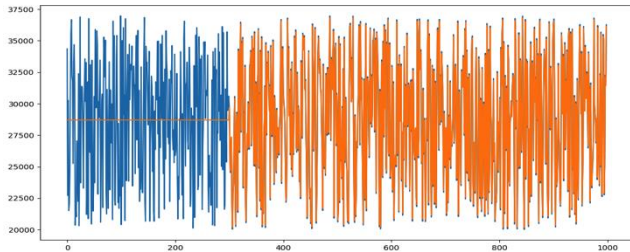


Fig. 10. Results of the program

Finding the Theil's ratios for the described models provides the following values (Table I).

Table I. Theil's ratios

Model	Theil's ratio	Model	Theil's ratio
AR(1)	0.11812	ES(0.6)	0.0553
AR(5)	0.11827	ES(0.9)	0.0155
MA(1)	0.1214	DES(0.1,0.9)	0.1414
WMA(6)	0.0421	DES(0.9,0.1)	0.0306
ARIMA(2,0,1)	0.0918	HW	0.1162
ES(0.1)	0.1098	Nnet	0.0644

It is shown in Table I that the best result for a given time series was achieved using the ES method with the parameter  $\alpha = 0.9$ .

V. DISCUSSION

The methods and models of forecasting are explored in the study, and the feasibility of using methods in the pricing process is justified. The mechanism for forecasting prices based on neural network modeling is proposed. The methods for analyzing the dynamics of time series are compared in this study, the mechanisms for assessing the accuracy of forecasting values are examined, and a brief description of the models and examples of their use is provided. The problem of choosing the optimal model according to the criterion of the minimum forecasting error is stated and solved. The families of mathematical modeling methods, mathematical statistics, and econometrics, such as autoregression, moving average, exponential smoothing, and neural network modeling, were used to solve this problem.

VI. CONCLUSION

As such, the problem of choosing the optimal model for solving the pricing problem in the enterprise has been stated and solved in the study. The main methods of analyzing and

forecasting time series for solving the pricing problem in the enterprise were explored. The algorithm for finding the best time series approximation model has been formed based on the results. The software product has also been developed to solve the task, which automates the work of a forecasting specialist. The result of the work is an algorithm for finding the optimal model based on minimizing the forecasting error and the program that implements this algorithm. Due to the presence of complex patterns in the time series, which cannot be determined by linear methods, the forecasting problems are often solved using neural networks. Therefore, there are plans to improve the neural network model in further studies and to consider the use of methods for analyzing nonstationary time series.

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