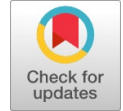


Radiated Force due to Surge Motion by a Pair of Submerged Cylinder in Water



Pankaj Borah, Nijara Konch

Abstract: Evaluation of hydrodynamic coefficients due to surge of submerged structure is great significant to designing a device which can be consider as a device of wave energy. In the present work, a theoretical approach is developed to describe radiation of water wave by fully submerged cylinder placed above a submerged circular plate in water of finite depth which is based on linear water wave theory. The radiation problem due to surge motion by this pair of cylinders have investigated with the suspension of linear water wave theory. To determine the radiated potentials in every area, we utilize the eigenfunction expansion method and variables separation method. Finally, we derived the analytical expressions of Hydrodynamic coefficients i. e. added mass and damping coefficient due to surge and associated unknown coefficients are calculated by utilizing the matching conditions between the physical and virtual boundaries. A set of added mass and damping coefficient have presented graphically for various radius of the submerge cylinder.

Keywords: finite depth, radiation, surge, virtual boundary.

I. INTRODUCTION

The study of radiation problem due to floating structures provide us useful properties such as hydrodynamic property. Water wave radiating problem involving regular structures in water of finite depth are yet to be investigated by many researchers. In our study we have consider the propagating of water wave on a pair of coaxial cylinder with the suspicion of linear water wave theory. In [2] and [6] discussed the interaction of waves with a cylinder in water of uniform depth and determined the analytical expression of velocity potential for both interior and exterior regions. In [3] calculated wave induced force acting on the floating rectangular structure placed at near a wall in water of step type base. In [4-5] discussed water wave forces due to a device consist of two cylinders which placed in water of uniform depth. In [7] gave an analytical expression of diffracted velocity potential for a single vertical cylinder in water of arbitrary depth. In [9] developed non-linear water wave theory and they investigated second order force for a couple of cylinders using Graft's addition theorem. In [10] analysed the scattering and radiation of water wave due to a rectangular oscillating structure considering a bottom still effect. In [11-12] analysed the scattering and radiation of water waves by couple of

cylinders which are placed in water of constant depth. Also in [13] formulated the problem of scattering and radiation by considering a couple of vertical truncated cylinders in water of constant depth.

A rigid submerged structure may undergo six degrees of freedom: three rotational and three translational. We assume a system of coordinate, OXYZ, the rotational motions are in the x, y and z-axes are called roll, pitch and yaw respectively and the translational motions about x, y and z-axes which are called surge, sway and heave respectively. The techniques formulated in this work have many applications including water waves field, acoustics, waves of electromagnetic. The present work is therefore concerned with the hydrodynamics of the interaction of water waves with structures. It includes the equations that govern the fluid motion as well as definitions of integrated quantities which are physically importance of hydrodynamic forces .

II. MATHEMATICAL MODEL

A. Formulation to the problem

Based on linearised water wave propagation in ideal water of finite depth H with oscillating submerged cylinder placed above a circular plat which is fixed at finite height from the sea bottom. Let us consider the radius of submerged cylinder is R that possesses the region $r \leq R, 0 \leq \theta \leq 2\pi, -l_3 \leq z \leq -l_4$ and the radius of lower cylinder is $R_b (\geq R)$ that occupies the region $r \leq R_b, 0 \leq \theta \leq 2\pi, -l_1 \leq z \leq -l_2$. Also we chose Cartesian coordinate system is defined on undisturbed free surface with origin at O , as shown in Fig. 1 in which z - axis is measured vertically upward.

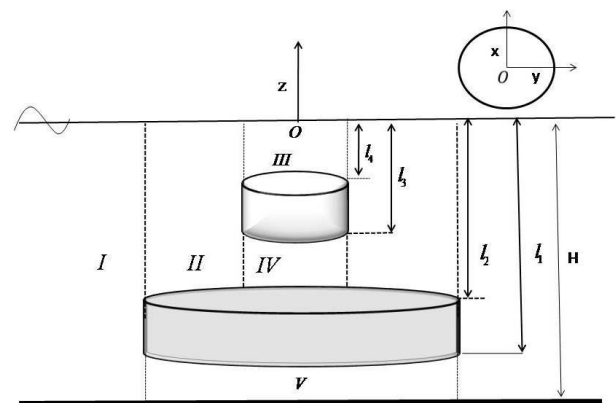


Fig. 1. Design of the device

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*Correspondence Author(s)

Pankaj Borah*, Mathematics department, North Eastern Regional Institute of Science and Technology, Nirjuli, Itanagar, India-791109. Email: pankajborahmajuli@gmail.com

Nijara Konch, Mathematics department, B. Borooh College, Guwahati, India, Email: dh7399@gmail.com

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Clearly, under the assumption of linear water wave theory of time-harmonic motion, we have the velocity potential of the form as

$$\phi(r, \theta, z, t) = \text{Re}[\phi(r, \theta, z)e^{-i\omega t}], \quad (1)$$

where $\text{Re}[\cdot]$ stands for real part of complex variable, $i = \sqrt{-1}$ and $\phi(r, \theta, z)$ is the time-independent velocity potential which satisfies the following Laplace's equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2)$$

B. Governing equation and boundary conditions

In this section we setup the governing equation with boundary conditions which can solve by separation of variables method to determine the radiated potentials. Since we consider the radiated problem only for surge motion due to oscillation of submerged cylinder only. Let us consider the total radiated potential due to surge motion be ϕ_r . Therefore the radiated potential ϕ_r is given by ([10])

$$\phi_r = -i\omega\varphi_{r1}(r, z) \cos \theta \quad (3)$$

From equations (2) and (3) will gives the governing equation and boundary conditions are as follow

$$\frac{1}{r} \frac{\partial \varphi_{r1}}{\partial r} + \frac{\partial^2 \varphi_{r1}}{\partial r^2} + \frac{\partial \varphi_{r1}}{\partial z^2} - \frac{\varphi_{r1}}{r^2} = 0 \quad (4)$$

$$\frac{\partial \varphi_{r1}}{\partial z} - \frac{\omega^2}{g} \varphi_{r1} = 0 \quad (z = 0) \quad (5)$$

$$\frac{\partial \varphi_{r1}}{\partial z} = 0 \quad (z = -H) \quad (6)$$

$$\frac{\partial \varphi_{r1}}{\partial z} = 0 \quad (z = -l_2, r < R_b; z = -l_4, r < R) \quad (7)$$

$$\frac{\partial \varphi_{r1}}{\partial r} = \begin{cases} 0, & -l_1 < z < -l_2, r = R_b \\ 1, & -l_3 < z < -l_4, r = R \end{cases} \quad (8)$$

$$\lim_{r \rightarrow \infty} \sqrt{kr} \left(\frac{\partial \varphi_{r1}}{\partial r} - ik\varphi_{r1} \right) = 0, \quad (9)$$

Since the fluid region as indicate in Figure 1, we divide whole fluid region into four sub-regions. Let us consider the radiated velocity potentials are $\varphi_{r1}^I, \varphi_{r1}^{II}, \varphi_{r1}^{III}, \varphi_{r1}^{IV}$ and φ_{r1}^V in the respected sub-regions, namely *I, II, III, IV* and *V*.

III. MATCHING CONDITIONS

Since matching conditions between the regions follows to preserve the continuity of fluid flows. Therefore, along the boundary $r = R_b$, the conditions are given by

$$\varphi_{r1}^I = \varphi_{r1}^{II} \quad (-l_2 \leq z \leq 0) \quad (10)$$

$$\varphi_{r1}^I = \varphi_{r1}^V \quad (-H \leq z \leq -l_1) \quad (11)$$

$$\frac{\partial \varphi_{r1}^I}{\partial r} = \begin{cases} \frac{\partial \varphi_{r1}^{II}}{\partial r} & (-l_2 \leq z \leq 0) \\ 0 & (-l_1 \leq z \leq -l_2) \\ \frac{\partial \varphi_{r1}^V}{\partial r} & (-H \leq z \leq -l_1) \end{cases} \quad (12)$$

Along the boundary $r = R$, i.e. the regions between *II* and *III* and between *II* and *IV*, the conditions are given by

$$\varphi_{r1}^{II} = \begin{cases} \varphi_{r1}^{III} & (-l_4 \leq z \leq 0) \\ \varphi_{r1}^{IV} & (-l_2 \leq z \leq -l_3) \end{cases} \quad (13)$$

$$\frac{\partial \varphi_{r1}^{II}}{\partial r} = \begin{cases} \frac{\partial \varphi_{r1}^{III}}{\partial r} & (-l_4 \leq z \leq 0) \\ 1 & (-l_3 \leq z \leq -l_4) \\ \frac{\partial \varphi_{r1}^{IV}}{\partial r} & (-l_2 \leq z \leq -l_3) \end{cases} \quad (14)$$

IV. SOLUTION TO THE PROBLEM

Since the fluid region as indicate in Figure 1, we divide whole region into four sub-regions, namely *I, II, III, IV* and *V* and we apply variables separation method for every sub-region to calculate the expression of radiated potential. Therefore the radiated potential in each region described by Wu et. al. (2004) is given by

$$\varphi_{r1}^I = \sum_{n=0}^{\infty} A_n \frac{U_1(\alpha_n r)}{U_1(\alpha_n R_b)} \cos[\alpha_n (z + H)], \quad (15)$$

$$\varphi_{r1}^{II} = \sum_{n=0}^{\infty} \left[B_n \frac{V_1(\beta_n r)}{V_1(\beta_n R)} + C_n \frac{W_1(\beta_n r)}{W_1(\beta_n R)} \right] \cos[\beta_n (z + l_2)] \quad (16)$$

$$\varphi_{r1}^{III} = \sum_{n=0}^{\infty} D_n \frac{R_1(\gamma_n r)}{R_1(\gamma_n R)} \cos[\gamma_n (z + l_4)], \quad (17)$$

$$\varphi_{r1}^{IV} = \left[E_0 r + \sum_{n=1}^{\infty} E_n \frac{I_1(\delta_n r)}{I_1(\delta_n R)} \cos[\delta_n (z + l_2)] \right], \quad (18)$$

$$\varphi_{r1}^V = F_0 r + \sum_{n=1}^{\infty} F_n \frac{I_1(\lambda_n r)}{I_1(\lambda_n R_b)} \cos[\lambda_n (z + H)] \quad (19)$$

where A_n, B_n, C_n, D_n, E_n and F_n are the unknown constants and $\alpha_n, \beta_n, \gamma_n, \delta_n$ and λ_n are the eigen values which are calculated by using the following dispersion relation:

$$\begin{cases} \alpha_n = -ik & \omega^2 = gk \tanh(kH), n = 0 \\ \omega^2 = -g\alpha_n \tan(\alpha_n H) & n = 1, 2, \dots \end{cases} \quad (20)$$

$$\begin{cases} \beta_n = -ik_1 & \omega^2 = gk_1 \tanh(k_1 l_2), n = 0 \\ \omega^2 = -g\beta_n \tan(\beta_n l_2) & n = 1, 2, \dots \end{cases} \quad (21)$$

$$\begin{cases} \gamma_n = -ik_2 & \omega^2 = gk_2 \tanh(kl_4), n = 0 \\ \omega^2 = -g\gamma_n \tan(\gamma_n l_4) & n = 1, 2, \dots \end{cases} \quad (22)$$

$$\delta_n = \frac{n\pi}{l_2 - l_3} \quad n = 0, 1, 2, \dots \quad (23)$$

$$\lambda_n = \frac{n\pi}{H - l_1} \quad n = 0, 1, 2, \dots \quad (24)$$

where k, k_1 and k_2 are the wave numbers in respected regions *I*, *II* and *III*.

The Parameters $U_1(\cdot), V_1(\cdot), W_1(\cdot)$ and $R_1(\cdot)$ are given by

$$U_1(\alpha_n r) = H_1^{(1)}(kr) = H_1^{(1)}(i\alpha_0 r), \quad n = 0 \quad (25)$$

$$U_1(\alpha_n r) = K_1(\alpha_n r), \quad n = 1, 2, \dots \quad (26)$$

$$V_1(\beta_n r) = H_1^{(1)}(k_1 r), \quad n = 0 \quad (27)$$

$$V_1(\beta_n r) = K_1(\beta_n r), \quad n = 1, 2, \dots \quad (28)$$

$$W_1(\beta_n r) = H_1^{(2)}(k_1 r), \quad n = 0 \quad (29)$$

$$W_1(\beta_n r) = I_1(\beta_n r), \quad n = 1, 2, \dots \quad (30)$$

$$R_1(\gamma_n r) = J_1(k_2 r), \quad n = 0 \quad (31)$$

$$R_1(\gamma_n r) = I_1(\gamma_n r), \quad n = 1, 2, \dots \quad (32)$$

V. METHOD TO FIND THE UNKNOWN CONSTANTS

To find the unknown constants appearing in above expression of potentials, first we use equations (10) - (14) followed by multiplication of both sides by a set of eigenfunction. Hence we use the property of orthogonality of eigenfunction, we get the following equations:

$$\int_{-h_3}^0 \phi_d'(R_b, \theta, z) \cdot \cos[\alpha_l(z + h_3)] dz = \int_{-h_3}^0 \phi_d''(R_b, \theta, z) \cdot \cos[\alpha_l(z + h_3)] dz \quad (33)$$

$$\int_{-H}^{-l_1} \phi_{r1}'(R_b, \theta, z) \cdot \cos[\delta_l(z + H)] dz = \int_{-H}^{-l_1} \phi_{r1}''(R_b, \theta, z) \cdot \cos[\delta_l(z + H)] dz \quad (34)$$

$$\int_{-H}^0 \frac{\partial \phi_{r1}'(R_b, \theta, z)}{\partial r} \cdot \cos[\alpha_m(z + H)] dz = \int_{-l_2}^0 \frac{\partial \phi_{r1}''(R_b, \theta, z)}{\partial r} \cdot \cos[\alpha_m(z + H)] dz + \int_{-H}^{-l_1} \frac{\partial \phi_{r1}^V(R_b, \theta, z)}{\partial r} \cdot \cos[\alpha_m(z + H)] dz \quad (35)$$

$$\int_{-l_4}^0 \phi_{r1}''(R, \theta, z) \cdot \cos[\gamma_m(z + l_4)] dz = \int_{-l_4}^0 \phi_{r1}'''(R, \theta, z) \cdot \cos[\gamma_m(z + l_4)] dz \quad (36)$$

$$\int_{-l_4}^0 \phi_{r1}''(R, \theta, z) \cdot \cos[\gamma_m(z + l_4)] dz = \int_{-l_4}^0 \phi_{r1}'''(R, \theta, z) \cdot \cos[\gamma_m(z + l_4)] dz \quad (37)$$

$$\int_{-l_2}^0 \frac{\partial \phi_{r1}''(R, \theta, z)}{\partial r} \cdot \cos[\beta_m(z + l_2)] dz = \int_{-l_2}^{-l_1} \frac{\partial \phi_{r2}'''(R, \theta, z)}{\partial r} \cdot \cos[\beta_m(z + l_2)] dz + \int_{-l_4}^{-l_3} \cos[\beta_m(z + l_2)] dz + \int_{-l_4}^0 \frac{\partial \phi_{r1}^{IV}(R, \theta, z)}{\partial r} \cdot \cos[\beta_m(z + l_2)] dz \quad (38)$$

Again, let us define the following functions

$$M(x_n, y_n, a_1, a_2, z_1, z_2) = \int_{z_1}^{z_2} \cos[x_n(z + a_1)] \cdot \cos[y_n(z + a_2)] dz, \quad (39)$$

$$N(x_n, a_1, z_1, z_2) = \int_{z_1}^{z_2} \cos^2[x_n(z + a_1)] dz. \quad (40)$$

Applying equations (39) and (40) to equations (33)–(38), we get

$$\sum_{n=0}^{\infty} A_n M(\alpha_n, \beta_m, H, l_2, -l_2, 0) = [B_m S'_m + C_m T'_m] \times N(\beta_m, l_2, -l_2, 0) \quad (41)$$

$$A_m N(\alpha_m, H, -H, 0) = \sum_{n=0}^{\infty} [B_n Q_n + C_n S_n] \times M(\alpha_m, \beta_n, H, l_2, -l_2, 0) \quad (42)$$

$$\sum_{n=0}^{\infty} (B_n + C_n) M(\beta_n, \gamma_m, l_2, l_4, -l_4, 0) = D_m N(\gamma_m, l_4, -l_4, 0) \quad (43)$$

$$\sum_{n=0}^{\infty} (B_n + C_n) M(\beta_n, \delta_m, l_2, l_2, -l_2, -l_3) = E_m L_m \times N(\delta_m, l_2, -l_2, -l_3) \quad (44)$$

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$$[B_m O_m + C_m T_m] N(\beta_m, l_2, -l_2, 0) = \sum_{n=0}^{\infty} D_n X_n M(\gamma_n, \beta_m, l_2, l_4, -l_4, 0) + \int_{-l_3}^{-l_2} \cos[\beta_m(z+l_2)] dz + \sum_{n=0}^{\infty} E_n X_n M(\delta_n, \beta_m, l_2, l_2, -l_2, -l_3) \quad (46)$$

where

$$S'_m = \frac{V_1(\beta_m R_b)}{V_1(\beta_m R)}, T'_m = \frac{W_1(\beta_m R_b)}{W_1(\beta_m R)}$$

$$P'_m = \frac{\alpha_m U'_1(\alpha_m R_b)}{U_1(\alpha_m R)}, Q'_n = \frac{\beta_n V'_1(\beta_n R_b)}{V_1(\beta_n R)}$$

$$S_n = \frac{\beta_n W'_1(\beta_n R_b)}{W_1(\beta_n R)}, L_m = \begin{cases} R & m=0 \\ 1 & m=1, 2, \dots \end{cases}$$

$$O_m = \frac{\beta_m V'_1(\beta_m R)}{V_1(\beta_m R)}, T_m = \frac{\beta_m W'_1(\beta_m R)}{W_1(\beta_m R)}$$

$$X_n = \begin{cases} 1 & n=0 \\ \frac{\beta_n I'_1(\beta_n R)}{I_1(\beta_n R)} & n=1, 2, \dots \end{cases}$$

Since each expression of velocity potential is an infinite series, therefore to compute the value of unknown constants we truncate the series to a limited number of term. Then the unknown constants A_n, B_n, C_n, D_n and E_n obtained by solving the system of equations (41)-(46).

VI. ADDED MASS AND DAMPING COEFFICIENTS

The fluid pressure for surge motion to be determined by using Bernoulli's equation and it is given by

$$p = -\rho \frac{\partial \phi_{rad}}{\partial t} \quad (47)$$

The radiated force can be determined by using the potential which radiated by the structure. Therefore the radiated force due to surge motion by oscillating submerged cylinder is given by

$$F_r = -\iint_W p n_x ds, \quad (48)$$

where n_x is the unit outward normal vector on the surface of the body in the direction of x and W is wetted surface of the structure in water. Hence from equations (3), (47) and (48), we get

$$F_r = -\rho \omega \iint_W \varphi(R, z) \cos \theta n_x ds. \quad (49)$$

As in Rahman and Bhatta (sec 8.7 in [8]), we have radiated force in terms of hydrodynamic coefficients i. e. added mass and damping coefficients which is given by

$$F_r = \mu_1 + i \frac{\xi_1}{\omega}, \quad (50)$$

where, μ_1 is called added mass and ξ_1 is called damping coefficient due to surge motion. Therefore, applying equations, we get the expression of added mass and damping coefficient and these are given by

$$\mu_1 + i \frac{\xi_1}{\omega} = -\rho \iint_W (\varphi_{r1}^{II} - \varphi_{r1}^{III} - \varphi_{r1}^{IV}) \cos \theta n_x ds, \quad (51)$$

or,

$$\mu_1 + i \frac{\xi_1}{\omega} = -\pi \rho R \sum_{n=0}^{\infty} (B_n + C_n) \frac{\sin \beta_n (l_2 - l_4) - \sin \beta_n (l_2 - l_3)}{\beta_n} - \pi \rho R \sum_{n=0}^{\infty} D_n \frac{\sin \gamma_n (l_4 - l_3)}{\gamma_n} + \pi \rho R^2 E_1 (l_4 - l_3) + \pi \rho R \sum_{n=0}^{\infty} E_n \frac{\sin \delta_n (l_2 - l_4) - \sin \delta_n (l_2 - l_3)}{\delta_n}. \quad (52)$$

VII. RESULT AND DISCUSSION

From the above discussions, we have seen that to get hydrodynamic forces, we have to determine the associated unknown coefficients appearing in the above expressions of radiated potentials given by the equations (15-19) with the help of matching conditions (10-14). As every expression of potential is found to be a series with infinite terms, therefore to calculate the corresponding values, we must be truncated appropriately after some terms. For our convenience, let us truncate all the infinite series into a limited number of terms by assuming M=30. Hence we arrived at a system of linear equations with the unknown coefficients. Then the solutions of these unknown coefficients corresponding to above system of linear equations are obtained by using MATLAB programming. Let us consider the values of the parameters throughout our calculation as

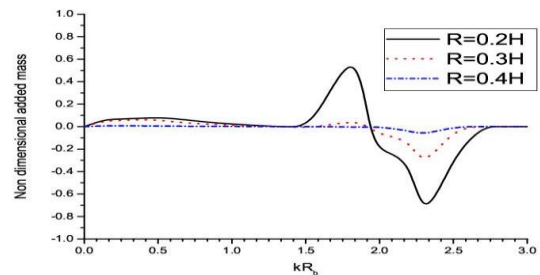


Fig. 2: Non dimensional added mass versus non dimensional wave number for different radii R.

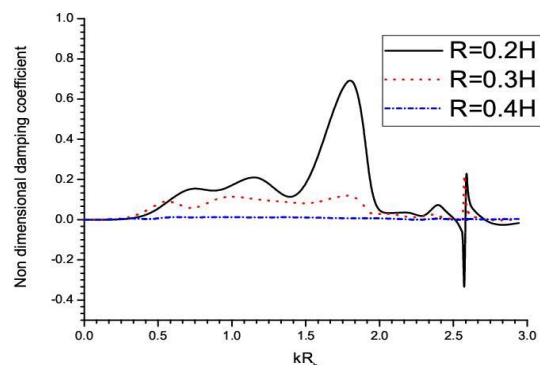


Fig.3: Non dimensional damping coefficient versus non dimensional wave number for different radii R.

$H = 3m, g = 9.8m/s^2, l_2 = 0.75m, H = 3m$. In Fig. 2 we plot non-dimensional added mass against non-dimensional wave number for different radius of submerged cylinder i.e. for different values of R by taking $R = 0.4H, 0.6H, 0.8H$. From the figure we observed that there is no significant change for all value of radius except at some particular wave number (also for some particular frequencies as there is a relation between wave number and frequency which is given by dispersion relation). Therefore for all frequency range 1.5-2.9, from the figure we observed the value of added mass behave highly oscillated which we can consider as a resonance situation and also the added mass almost diminish to zero for higher value of wave number. Again in Fig. 3, we plot non-dimensional damping coefficient against non dimensional wave number for different radius of upper cylinder R . From the figure we conclude that the damping coefficient increases with decreases of radii which happen only within the lower range of wave number and for higher values of wave number the value of damping coefficients almost tends to zero.

VIII. CONCLUSION

In this paper, we approach theoretically based on the method of eigenfunction expansion and separation of variables to obtained an analytical expression for the radiated velocity potential of the problem of radiation of water wave due to surge motion of submerged cylinder placed above a circular plat in water of finite depth. Set of hydrodynamic coefficients due to the oscillation submerged cylinder have also derived theoretically from the expressions of potentials. Then we introduced the method to calculate the unknown constants appearing in the expression, we have applied the appropriate matching conditions along the physical and virtual boundaries between the regions. Finally we plot numerical result by graphically and also we conclude that the added mass shows oscillating behavior at some particular frequencies and damping coefficient increases with decreases the value of radii of submerged cylinder which is occurs within lower range of frequency. Therefore our device can be consider as a wave energy device to extract energy from the water.

NOTATIONS

- l_1 : Uniform water depth
 ω : Angular frequency
 g : Gravitational acceleration
 ρ : Fluid Density
 p : Fluid Pressure
 $J_1(.)$: Bessel function of the first kind of order 1
 $H_1^{(1)}(.)$: Hankel function of first kind of order 1
 $H_1^{(2)}(.)$: Hankel function of second kind of order 1
 $I_1(.)$: Modified Bessel function of first kind of order 1
 $K_1(.)$: Modified Bessel function of second kind of order 1

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AUTHORS PROFILE



Pankaj Borah received his B.Sc. in Mathematics from Jorhat Science College, India in 2014 and M.Sc. from Tezpur University, India in 2016. He is currently pursuing Ph.D. degree in Mathematics at NERIST, India. His main interested area of research work is in the field of fluid dynamic and water wave theory.



Nijara Konch born in 1993. She graduate in Mathematics from Jorhat Science College, India and She finished Master degree in Mathematics at Tezpur University. Currently she serves as a Assistant Professor at B. Borooah College, India.