Computational Performance of a Single Server State Dependent Queue

R.Sakthi, V.Vidhya

Abstract: In this article we consider a single server state dependent queuing system with the service rate varying according to types of customers like normal, tagged and heavy tailed. We develop a state dependent queuing model in which the service rate depends on customer type arrive to the system with quasi-birth-death environment structure and using matrix geometric method we develop the system performance measures. Also, we use these performance measures by utilizing the maximum potential of server and perform sensitivity analysis with numerical illustrations.

Index Terms: Quasi-birth-death process, state dependent, stationary probability, tagged customer.
AMS Subject Classification: 60K25, 90B22, 68M20.

I. INTRODUCTION

Queuing systems is an unavoidable fertile in modern society human environment, communication network, production process etc. In communication system during transfer of data the packets gets congested that results in a block up of resources which leads to loss of time, excess cost, overload etc. In most of the queuing system the server's rate of service has variations due to the different type of customers, especially customer with high importance are given preference compared with normal customer. A common situation is observed in service facility of n (≥1) class of customers in a single queue with some customer having significant importance. Among the available methods in queuing theory, the matrix geometric method is a special approach to provide efficient and stable numerical solutions to acontinuous time Markov chain of complex queuing model where the closed form solutions are obtained specifically in case of various phase and multistate queueing system.

In this paper we develop a single server M/M/1 state dependent queuing model with the following novelty: Consider a queuing system with the customer arrival having two types normal and tagged customers. The normal customer enters to the system gets service at the rate μ_1 till it reaches a tagged customer. When the server attains a moment to serve a tagged customer denoted by a pair of numbers (n,m), where n stands for the position of the tagged customer in the queue, and where m denotes the number of tagged customers behind it, the server increases its speed and finish the service as early as possible because of high importance.
After the service completion the server observes the heavy tail queue, in order to maintain the system stability the server continues with the same rate of service that has been rendered to tagged customer till the heavy tailed reduces its length.Many researchers like Bekker [2],Delasay [4], George [7], Hopp [8] contributed their work related to control of queue with dependent service rate. Queuing model with system load increase that lead to either speedup or slowdown the service rate effect was investigated recently by Batt [1]. Speedup effects have been studied in many contexts refer George [7] in which the joint probability distribution of the queue length is obtained in a network with Markovian routing and each station has Markovian service rate that depends on the length of the queue at that station. These research work assert that server speedup is optimal in certain situation particularly George [7] obtained the optimal single server service rate with increase in queue length. The Markov chain is a quasi-birth-death (QBD) process with a special structure expressed in terms of explicit rate matrix was investigated by Leeuwaarden [14]. The most extensive study of quasi birth death using matrix geometric invariant vectors was done by Neuts [15] and the methodology is based on the fact that the sub vectors of steady state probability vector are connected with one another in matrix geometric structure. The rate matrix is formulated in terms of matrix geometric structure which is obtained as the minimal positive solution of a non-linear matrix equation. Computation of the steady state distribution using matrix geometric solution of state dependent queue can be referred in Yue [16], in which the various performance measures of $M/E_k/1$ are obtained. Single server state dependent service of finite queueing system was analyzed by Kalyanaraman [10]. Recently in Delasay [4] the authors developed a state dependent queuing system in which the service rate depends on the system such as load and overwork, where overwork refers to system that has been with heavy load for an extended period of time in quasi-birth-death structure. In this work we develop the single server queuing model with state dependent service rate described as in the next section to obtain the performance measure and the state with maximum efficiency of the server. do not underline.

II. MODEL DESCRIPTION

A. Model

Consider a single server queuing system with two types of customers as normal type of customer and tagged type. Assume that the normal type of customers arrive according to a Poisson process with rate λ_1, the arrival rate of tagged customer as λ_2 and if the arrival rate heavy tailed as λ_3. The arrival rate of tagged customer with high priority is denoted as $\alpha\lambda_1$ and for heavy tailed denoted as λ_2 . Customers arrive to the system in Poisson arrival rate are served with service rate μ_1, the service time are exponentially distributed. When a tagged customer enters the queue the server immediately turn to high rate setup, starts serving tagged customer with increased service rate μ_2 .

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Once the tagged customer are served the server turns to the waiting customer and if the waiting customer queue is heavy tailed then it starts serving with rate μ_3. Otherwise, continuing in the scenario with normal service rate after serving tagged customer, the queue may grow larger and the system will lead to get instability. To avoid the instability the server will switch to continue the service with the rate of service μ_3 from μ_2 that has been rendering service to tagged customer. As soon as the heavy tail decreases the server returns to the normal state with service rate μ_1 and this assumption of model can be observed from Figure 1.

B. Figure

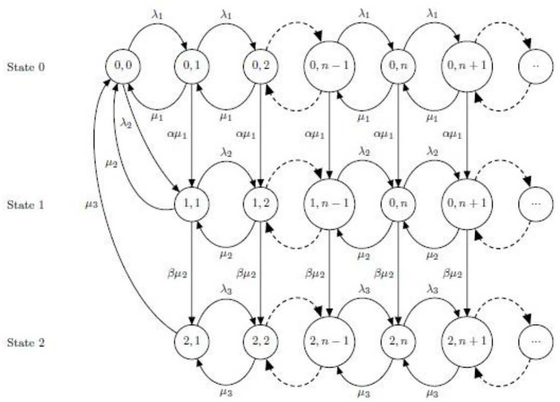

Figure 1 Transition state diagram

III. ASSUMPTION

Let

$$S = \{(L_n(t), S_n(t)) : n \geq 1\}$$

represent the two dimensional Markov chain with state space

$$S = \{(0,0) \cup (j,k) : j = 0,1,2 \text{ and } k \geq 1\}$$

The arrival rate λ is assumed to be Poisson process, the inter arrival time and service time are mutually independent. The server providing service has two service rates represented as μ_1 for normal customer and the service rate represented as μ_2 ($0 < \mu_1 < \mu_2$) the effective rate for tagged customer or at the moment of heavy tail. Formulating the model we define the steady state probability as follows:

$P_{0,n}$: Probability that server is rendering service for normal customer with rate μ_1

$P_{1,n}$: Probability that server is rendering service for tagged customer with rate μ_2

$P_{2,n}$: Probability that server is rendering service to heavy tailed customers with rate μ_3

The steady state equations are

$$(\lambda_1 + \lambda_2)P_{0,0} = \mu_1 P_{1,0} + q\mu_2 P_{1,1}$$
$$(\alpha\lambda_1 + \mu_1)P_{0,1} = \alpha\lambda_1 P_{0,0} + \mu_1 P_{0,2}$$
$$(\alpha\lambda_1 + \mu_1)P_{0,1} = \mu_1 P_{0,n+1} + \alpha\lambda_1 P_{0,n-1}, n \geq 1 \qquad (1)$$
$$(\beta\lambda_2 + \mu_2)P_{1,1} = \beta\lambda_2 P_{0,0} + \mu_2 P_{1,2}$$
$$(\beta\lambda_2 + \mu_2)P_{1,n} = \beta\lambda_2 P_{1,n-1} + \mu_2 P_{1,n+1}, n \geq 1 \qquad (2)$$

$$(\lambda_3 + \mu_3)P_{2,1} = \beta P_{1,n} + \lambda_3 P_{2,n-1}$$
$$(\lambda_3 + \mu_3)P_{2,n} = \beta P_{1,n} + \lambda_3 P_{2,n-1} + \mu_2 P_{2,n+1}, n \geq 1 \ (3)$$

The infinitesimal generator of the QBD is given by

$$Q = \begin{pmatrix} L_{00} & F_{00} & & & & \\ B_{10} & L_{10} & F_0 & & & \\ & B_2 & L_1 & F_0 & & \\ & \dots & \dots & \dots & & \\ & & & B_2 & L_1 & F_0 \end{pmatrix}$$

$$L_{00} = -(\lambda_1), \quad F_{00} = (\lambda_1 \quad 0 \quad 0), \qquad (5)$$

$$B_{10} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ 0 \end{pmatrix}, F_0 = \begin{pmatrix} \alpha\lambda_1 & 0 & 0 \\ 0 & \beta\lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$L_{10} = \begin{pmatrix} -(\alpha\lambda_1 + \mu_1) & 0 & 0 \\ 0 & -(\beta\lambda_2 + \mu_2) & 0 \\ 0 & 0 & -\lambda_3 \end{pmatrix} (6)$$

$$L_1 = \begin{pmatrix} -(\alpha\lambda_1 + \mu_1) & 0 & 0 \\ 0 & -(\beta\lambda_2 + \mu_2) & 0 \\ 0 & 0 & -(\lambda_3 + \mu_3) \end{pmatrix},$$

$$B_2 = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} \qquad (7)$$

Let $\prod$ be the steady state probability vector of Q. By partitioning the vector as $\prod = [\prod_0, \prod_1, \dots \prod_n]$ where $\prod_0 = [P_{0,0}]$ is a non-negative real number and $\prod_n = [\pi_{0,n}, \pi_{1,n}, \pi_{2,n,}]$, $n \geq 1$ is a row vector of dimension 3. The vector $\prod$ satisfies $\prod Q = 0$ and $\prod e = 1$, where e is a column vector of an appropriate dimension having each element as 1. It is apparently visible that the stability condition is satisfied, the sub vectors of $\prod$, corresponding to the various levels are given by $\prod_j = \prod_1 R^{j-1} = \prod_{j-1} R$, where $j \geq 1$. R is the minimal non-negative solution of the matrix of quadratic equation given by:

$$R^2 B_2 + RL_1 + F_0 = 0 \qquad (8)$$

$$\textstyle\prod_0 L_{00} + \prod_1 RB_{10} = 0 \qquad (9)$$

$$\textstyle\prod_0 F_{00} + \prod_1 L_1 + \prod_2 B_2 = 0 \qquad (10)$$

$$\textstyle\prod_{n-1} F_0 + \prod_n L_1 + \prod_{n+1} B_2 = 0, n \geq 2 \qquad (11)$$

normalizing the condition

$$\textstyle\prod_0 + \prod_1 (I - R)^{-1} e = 1 \qquad (12)$$

under the boundary state equation

$$\textstyle\prod_1 (L_1 + RB_2) = 0 \qquad (13)$$

In general, it's cumbersome to express the exact value of R analytically, but at the same time the matrix R can be approximated as follows:

(i) $R(0) = 0$,

(ii) $R(n+1) = -(F_0 + R^2(n)B_2)(L_1^{-1}), n \geq 0$

This iterative algorithm is convergent, i.e $R = \lim_{n\to\infty} R(n)$ refer Neuts [15].

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IV. PERFORMANCE MEASURES

This section aims to predict the various performance measures using the probability vector obtained in the previous section. The various performance measures are given as:

The expected number of normal customer in the server is given by

$$E(N) = \sum_{n=0}^{\infty} n\pi_{0,n} \quad (14)$$

The expected number of tagged customer in the server is given by

$$E(T) = \sum_{n=1}^{\infty} n\pi_{1,n} \quad (15)$$

The expected number of heavy tailed customer in the server is given by

$$E(H) = \sum_{n=1}^{\infty} n\pi_{2,n} \quad (16)$$

The expected number of customers in the system is given by

$$E(S) = E(N) + E(T) = E(H) \quad (17)$$

The waiting time of normal customer, tagged customer, heavy tailed customer in the system are calculated by using Little's theorem refer George [7]

$$W(N) = \frac{E(N)}{\lambda_1}, W(T) = \frac{E(N)}{\lambda_2}, W(H) = \frac{E(H)}{\lambda_3} \quad (18)$$

V. NUMERICAL RESULTS

To verify the analytical results the computational values for the model under the study, we encode a computer program with MATLAB software and perform numerical experiments. To illustrate the assumption of this work, the probability vectors and various performance measures are obtained by fixing the some parameters for the system. First we vary λ_1 from 0.5, 1.1 and 1.5 and fix λ_2 = 0.6, λ_3 = 0.9, μ_1 = 1.5, μ_2= 2.0, μ_3 = 2.5, α = 0.05, β = 0.7. The expected number of customer gradually increase as the arrival rate increases correspondingly there is change in waiting time which can be observed in Table 1. In the Table 2 we fix λ_1 and vary the parameter α correspondingly arrival rate λ_2 and µ2, so that when there is an increasing trend in arrival rate the service rate is increases in order to reduce the queue length at the same time waiting time is also reduced. In the Table 3 we fix λ_1 and λ_2, the parameter β change from 0.1, 0.5, 0.7 correspondingly the arrival rate λ_3 and μ_3 are changed accordingly 0.2, 0.7, 1.7 and 3.0, 5.0, 7.0 respectively, so that when there is an increasing trend in arrival rate the service rate is increases in order to reduce the queue length at the same time waiting time also got reduced.

Table 1 – λ_1 versus average customer and waiting time

λ_1	E(N)	E(T)	E(H)	W(N)	W(T)	W(H)
0.5	0.5175	0.7763	0.9316	0.0086	0.0129	0.4658
1.1	1.5091	2.2637	1.2347	0.0635	0.0516	1.3582
1.4	2.0034	3.0051	1.2879	0.1624	0.0814	1.8031

Table 2 – α with λ_2, μ_2

α	λ_2	μ_2	E(N)	E(T)	E(H)	W(N)	W(T)	W(H)
0.05	0.8	2.5	0.513	0.57	0.92	0.006	0.012	0.462
0.05	1.1	3.5	0.512	0.41	0.92	0.004	0.009	0.461
0.05	1.3	5.0	0.516	0.35	0.93	0.003	0.007	0.565

Table 3 – β with λ_3, μ_3

β	λ_3	μ_3	E(N)	E(T)	E(H)	W(N)	W(T)	W(H)
0.1	0.2	3.0	5.44	1.81	2.17	0.009	0.08	1.08
0.5	0.7	5.0	1.50	1.75	2.10	0.014	0.04	1.05
0.7	1.7	7.0	0.60	1.71	2.05	0.012	0.03	1.02

VI. CONCLUSION

In this work we have investigated the single server state dependent queuing system with the service rate varying according to types of customers like normal, tagged and heavy tailed. Numerical examples are carried out to observe the trend of the mean number of customers and mean waiting time of queue, system respectively for varying parametric values.

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