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M/M/C Queue under Monte Carlo Simulation for a Restaurant Model

Vijeta Iyer, S. Aruna devi

Abstract: ***In this paper we present a stochastic queuing model for a restaurant which captures the stationary density flow relations. The performance of controlling the heterogeneous crowd in a restaurant under Monte Carlo simulation with various service distributions has been discussed. Using this analysis in future the waiting time of the customers can be reduced and the profit of the management can also be increased. The future behaviour of a restaurant networks both in simulation and analytical methods have been analysed.***

Keywords: Inter arrival pattern, Service pattern, M/M/C Queue, Monte Carlo Simulation.

I. INTRODUCTION

A queue is a line of people or things waiting in an order to get the service [1]. Queuing theory was introduced by A.K.Erlang in 1909.He published various articles in telephone traffic [2].Queueing Theory is mainly a branch of applied probability theory. It has many applications in different fields, namely, communication networks, computer systems, machine plants etc. Consider a service providing centre and a population of customers, which at some time intervals enter the service centre in order to get the service. Mostly, the service provider can only serve a limited number of customers. If a new customer arrives and the service is exhausted, he needs to stand in waiting line or queue until the service provider becomes free. So main three elements of a service providing centre are: a population of customers, the service facility and the waiting line. Also it can be considered that the several service providers are arranged in a network and a single customer can walk through this network at a specific path, visiting several service centres. With the help of Queueing Theory, many questions can be answered e.g. the mean waiting time in the queue, the mean system response time (waiting time in the queue plus service times), distribution of the number of customers in the queue, mean utilization of the service facility, distribution of the number of customers in the system etc.

II. SIMULATION

Monte Carlo simulation is a computerized mathematical technique that allows people to account for risk in quantitative analysis and decision making. Monte Carlo simulation helps the decision-maker with a range of possible outcomes and the probabilities they will occur for any choice of action.The technique was first used by scientists working on the atom bomb; it was named for Monte Carlo, the Monaco resort town renowned for its casinos. Since its introduction in World War II, Monte Carlo simulation has been used to model a variety of physical and conceptual systems. Monte Carlo simulation performs risk analysis by building models of possible results by uncertainty. It then calculates results over and over, each time using a different set of random values from the probability functions. Depending upon the number of uncertainties and the ranges specified for them, a Monte Carlo simulation could involve thousands or tens of thousands of recalculations before it is complete. Monte Carlo simulation produces distributions of possible outcome values. By using probability distributions, variables can have different probabilities of different outcomes occurring. Probability distributions are a much more realistic way of describing uncertainty in variables of a risk analysis.

III. MODEL DESCRIPTION

In this paper we discuss the application of simulation in M/M/C queueing model on the arrival and serving of customers in a restaurant. Chi-square test has been used to verify whether the arrival and service process satisfies their distributions. The simulation table helps in tracking the system over time. The main aim of this paper is to show that the service time taken by the servers to serve the customers can be reduced. Also the paper aims in finding the waiting time of the customers, idle time of the server and Queue length. Simulation and analytic solutions are also compared.

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***Correspondence Author(s)**
Dr. Vijeta Iyer, Department of Science and Humanities (Mathematics), Kuaraguru College of Technology, Coimbatore, India.
Ms. S Arunadevi, Department of Science and Humanities (Mathematics), Kuaraguru College of Technology, Coimbatore, India.

Time (Hrs)	No.of customers Arrived	No.of tables allotted	No. of orders collected	No. of services done
Lunch Break (12.00 noon – 1.00 pm)				
<12.00noon	0	0	0	0
12.00-12.05	0	0	0	0
12.06-12.10	2	0	0	0
12.11-12.15	10	4	0	0
12.16-12.20	2	8	4	0
12.21- 12.25	4	2	10	0
12.26-12.30	9	8	6	2
12.31-12.35	7	9	6	2
12.36-12.40	4	5	4	2
12.41-12.45	4	2	8	4
12.46-12.50	0	2	0	5
12.51-12.55	0	0	0	2
12.56-1.00	4	2	4	2
>1.00pm	4	8	8	31
Total	**50**			

In this paper section1 gives Introduction and model description. Section 2 gives Chi-Square test. Section 3 gives calculation of simulation and analytical methods. Section 4 describes numerical study and section 5 gives the conclusion. The details of the restaurant are presented on left [3]:

Total Number of customers – 50

IV. CALCULATION

At a restaurant the customer's arrival is a random phenomenon and the time between the arrivals in Lunch hours varies from 12.00 noon to 1.00pm.and the table allotted hours, order collected hours and service time for the customers, varies from 12.00 noon to 1.00pm. For some customers the service is rendered even after 1.00pm The frequency distributions with their probabilities are given below:

Table 1: Arrival Distribution

Time (Hrs)	No.of customers Arrived	P(X)	No.of tables allocated	P(X)	No. of orders collected	P(X)	No. of services done	P(X)
Lunch Break (12.00 noon – 1.00 pm)								
<12.00 noon	0	0	0	0	0	0	0	0
12.00-12.05	0	0.06	0	0.05	0	0.06	0	0.07
12.06-12.10	2	0.06	0	0.08	0	0.06	0	0.07
12.11-12.15	10	0.09	4	0.05	0	0.07	0	0.07
12.16-12.20	2	0.05	8	0.05	4	0.06	0	0.06
12.21- 12.25	4	0.12	2	0.14	10	0.13	0	0.06
12.26-12.30	9	0.15	8	0.12	6	0.13	2	0.12
12.31-12.35	7	0.14	9	0.13	6	0.12	2	0.12
12.36-12.40	4	0.09	5	0.13	4	0.12	2	0.11
12.41-12.45	4	0.07	2	0.07	8	0.06	4	0.06
12.46-12.50	0	0.06	2	0.08	0	0.07	5	0.06
12.51-12.55	0	0.06	0	0.05	0	0.06	2	0.07
12.56-1.00	4	0.02	2	0.02	4	0.02	2	0.02
>1.00pm	4	0.03	8	0.03	8	0.04	31	0.05
Total	**50**							

Table 2: CHI-SQUARE TEST FOR ARRIVAL

We are having a set of observed frequencies and we want to test if the experimental results support the hypothesis by finding the difference between observed frequencies and expected frequencies. This test is known as χ^2 test of goodness of fit, developed by Karl Pearson.

Null Hypothesis H_0: The Poisson distribution fits well into the data.

Alternative Hypothesis H_1: The Poisson distribution does not fit well into the data.

Level of Significance at $\alpha = 0.01$

Test statistic under H_0 is $\chi_e^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ = 30.39

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = 21.16 \qquad P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Degrees of freedom = (r-1)(s-1) = 12

Tabulated value for 12 degrees of freedom at 1% level of significance is (χ_0^2).

Hence $\chi_e^2 < \chi_0^2$.

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We accept H_0 and conclude that the Poisson distribution is a good fit for the given data.

Time (Hrs)	X	P(X)	Observed Frequencies (No of customers arrived)	Expected Frequency	Chi-square value
<12.00 noon	0	0	0	0	0
12.00-12.05	1	0.06	0	3	3
12.06-12.10	2	0.06	2	3	0.33
12.11-12.15	3	0.09	10	4.5	6.7
12.16-12.20	4	0.05	2	2.5	0.1
12.21- 12.25	5	0.12	4	6	0.67
12.26-12.30	6	0.15	9	7.5	0.3
12.31-12.35	7	0.14	7	7	0
12.36-12.40	8	0.09	4	4.5	0.05
12.41-12.45	9	0.07	4	3.5	0.07
12.46-12.50	10	0.06	0	3	3
12.51-12.55	11	0.06	0	3	3
12.56-1.00	12	0.02	4	1	9
>1.00pm	13	0.03	4	1.5	4.17
Total			**50**		

Table 3: Tag Number for Arrival Distribution

Time (Hrs)	X	P(X)	Obs Freq (No of customers arrived)	Cumulative Probability	Tag No.
<12.00 noon	0	0	0	0	0
12.00-12.05	1	0.06	0	0.06	0-5
12.06-12.10	2	0.06	2	0.12	6-11
12.11-12.15	3	0.09	10	0.21	12-20
12.16-12.20	4	0.05	2	0.26	21-25
12.21- 12.25	5	0.12	4	0.38	26-37
12.26-12.30	6	0.15	9	0.53	38-52
12.31-12.35	7	0.14	7	0.67	53-66
12.36-12.40	8	0.09	4	0.76	67-75
12.41-12.45	9	0.07	4	0.83	76-82
12.46-12.50	10	0.06	0	0.89	83-88
12.51-12.55	11	0.06	0	0.95	89-94
12.56-1.00	12	0.02	4	0.97	95-96
>1.00pm	13	0.03	4	1	97-100
Total			**50**		

Table 4: Tag Number for Table Allocation Distribution

Time (Mins)	X	P(X)	Observed Frequencies (No of tables allocated)	Cumulative Probability	Tag Number
0-1	16	0.32	8.33	0.32	0-31
1-2	16	0.32	8.33	0.64	32-63
2-3	2	0.04	8.33	0.68	64-67
3-4	8	0.16	8.33	0.84	68-83
4-5	4	0.08	8.33	0.92	84-91
5-6	4	0.08	8.33	1	92-100
Total	**50**		**50**		

Service Distribution of Customers arrived

The exponential distribution fits the service distribution using the Chi-Square test.

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Table 5: Tag Number for Order Collection

Time (Hrs)	X	P(X)	Obs Freq (No of orders collected)	Cumulative Probability	Tag Number
0-2	6	0.13	6	13	0-12
4-Feb	12	0.25	6	38	13-37
6-Apr	12	0.25	6	63	38-62
8-Jun	8	0.17	6	80	63-79
10-Aug	5	0.1	6	90	80-89
12-Oct	1	0.02	6	92	90-91
14-Dec	0	0	6	92	91-92
14-16	4	0.08	6	100	93-100
Total	**48**		**48**		

Table 6: Tag Number for Service Distributionx

Time (Hrs)	X	P(X)	Expected Freq	Cumulative Prob	Tag No.
0-5	0	0	6.8		0
10-Jun	3	0.06	6.8	6	0-5
15-Nov	4	0.08	6.8	14	13-Jun
16-20	14	0.28	6.8	42	14-41
21-25	18	0.36	6.8	78	42-77
26-30	7	0.14	6.8	92	78-91
31-35	4	0.08	6.8	100	92-100

Table 7: Simulation for Multi Server Model

S.No	R1	I	A	R2	S1	R3	S2	SP	WT	IT
1	7	6	12.06	27	12.08	94	0.35	12.43	0.37	0.08
2	12	11	12.11	16	12.13	10	0.15	12.28	0.17	0.13
3	9	6	12.06	59	12.1	41	0.2	12.3	0.24	0.1
4	37	21	12.21	94	12.35	2	0.1	12.45	0.24	0.35
5	85	46	12.46	96	12.6	31	0.2	12.81	0.35	0.6
6	43	26	12.26	30	12.28	21	0.2	12.48	0.22	0.28
7	76	41	12.41	41	12.45	14	0.2	12.65	0.24	0.45
8	26	21	12.21	44	12.25	43	0.25	12.5	0.29	0.25
9	95	56	12.56	86	12.6	20	0.20	12.6	0.04	0.6
10	8	6	12.06	88	12.14	55	0.25	12.39	0.33	0.14
11	29	21	12.21	48	12.25	56	0.25	12.5	0.29	0.25
12	83	46	12.46	17	12.48	35	0.2	12.68	0.22	0.48
13	41	26	12.26	82	12.34	6	0.15	12.49	0.23	0.34

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14	70	36	12.36	45	12.4	44	0.2	12.6	0.24	0.4
15	66	31	12.31	67	12.37	10	0.15	12.52	0.21	0.37
16	86	46	12.46	35	12.48	7	0.15	12.63	0.17	0.48
17	78	41	12.41	69	12.47	95	0.35	12.82	0.41	0.47
18	47	26	12.26	34	12.28	44	0.25	12.53	0.27	0.28
19	91	51	12.51	89	12.59	66	0.2	12.79	0.28	0.59
20	82	41	12.41	1	12.41	69	0.25	12.66	0.25	0.41
21	64	31	12.31	19	12.33	71	0.25	12.58	0.27	0.33
22	58	31	12.31	43	12.35	13	0.15	12.5	0.19	0.35
23	34	21	12.21	16	12.23	24	0.2	12.43	0.22	0.23
24	64	31	12.31	18	12.33	22	0.2	12.53	0.22	0.33
25	26	21	12.21	54	12.25	25	0.2	12.45	0.24	0.25
26	88	46	12.46	58	12.5	8	0.15	12.65	0.19	0.5
27	53	31	12.31	89	12.41	23	0.2	12.61	0.3	0.41
28	38	26	12.26	15	12.28	60	0.25	12.53	0.27	0.28
29	14	11	12.11	28	12.13	8	0.15	12.28	0.17	0.13
30	100	1	12.01	83	12.09	47	0.25	12.34	0.33	0.09
31	44	26	12.26	75	12.32	67	0.25	12.57	0.31	0.32
32	49	26	12.26	11	12.26	30	0.2	12.46	0.2	0.26
33	20	11	12.11	89	12.19	12	0.15	12.34	0.23	0.19
34	35	21	12.21	43	12.25	85	0.3	12.55	0.34	0.25
35	99	1	12.01	15	12.11	23	0.2	12.31	0.3	0.11
36	10	6	12.06	90	12.16	67	0.25	12.41	0.35	0.16
37	37	21	12.21	51	12.25	46	0.25	12.5	0.29	0.25
38	19	11	12.11	52	12.15	31	0.2	12.35	0.24	0.15
39	18	11	12.11	71	12.17	28	0.2	12.37	0.26	0.17
40	65	31	12.31	58	12.35	50	0.25	12.6	0.29	0.35
41	50	26	12.26	100	12.4	29	0.2	12.6	0.34	0.4
42	78	41	12.41	3	12.41	34	0.2	12.61	0.2	0.41
43	4	0	12	59	12.04	29	0.2	12.24	0.24	0.04
44	31	21	12.21	84	12.29	48	0.25	12.54	0.33	0.29
45	53	31	12.31	26	12.33	97	0.35	12.68	0.37	0.33
46	51	26	12.26	38	12.3	48	0.25	12.55	0.29	0.3
47	49	26	12.26	41	12.3	88	0.3	12.6	0.34	0.3
48	43	26	12.26	37	12.26	97	0.35	12.61	0.35	0.26
49	44	26	12.26	64	12.32	55	0.25	12.57	0.31	0.32
50	83	46	12.46	55	12.5	51	0.25	12.75	0.29	0.5

R1, R2,R3 - Random Numbers IT– Idle time of the server (mins) A – Actual Arrival time (mins) WT- Waiting time of the Customers

I-Inter arrival time (mins) SP- Actual Service provided S1,S2 – Servers

Simulation Calculation

Average Arrival Time = 12.33

Average Service Time = 17min

Waiting time of the customers = 25mins

Idle time of the server = 10mins

Analytical calculation

Average Arrival Time = 12.30

Average Service Time = 20mins

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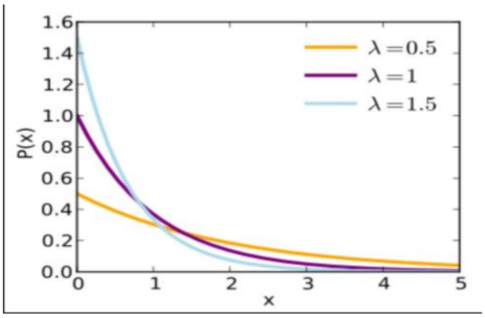
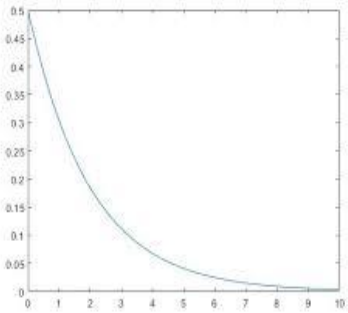
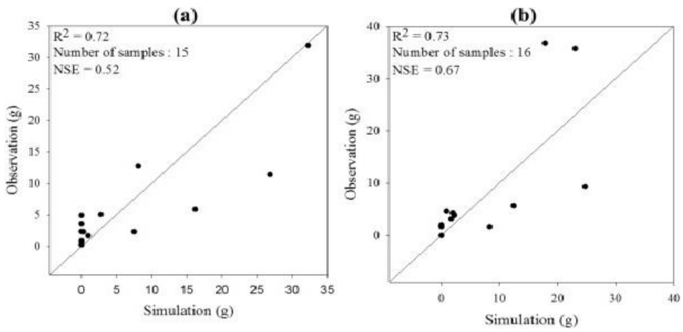
Waiting time of the customers = 25mins
Idle time of the server = 8mins

V. NUMERICAL STUDY







VI. CONCLUSION

In this paper work, a simulation table for queuing system with a multiple service station has been presented. Here we have developed a model for serving the food to the customers in a restaurant and decided to reduce the waiting time of the customers to get the service done. Numerical examples are given to show the similarity between numerical and analytical calculation. Hence in future this reduction in waiting time can be used to attract more customers and hence to increase the profit of the management.

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AUTHORS PROFILE

Vijeta Iyer is a faculty of Mathematics with teaching experience of 12 years, currently working as Assistant Professor at Kumaraguru College of Technology, Coimbatore. She has pursued her Ph.D and was accredited for the same in March 2014 from Barkatullah University, Bhopal. The title of her thesis was "Some contributions to chains, order and connectedness in Topology". Her fundamental research interests are Queueing Theory, Topology and Fuzzy sets. She has published 11 papers at international level and 2 papers at national level. She has presented many research papers in national and international conferences also.

S.Aruna devi is working as an Assistant Professor at Kumaraguru College of Technology, Coimbatore, since 2015. She is doing her Ph.D., (Part-Time) in the Department of Physical and Computational Sciences at Avinashilingam Institute of Higher Education for women, Coimbatore. She completed her bachelor degree in Mathematics branch from Bharathiar University, Coimbatore, India in 2003. She completed her master's degree in Applied Mathematics branch from Anna University, Chennai, India in 2005. She completed her M.Phil in Mathematics from Bharathiar University, Coimbatore, India in 2008.Her fundamental research interests are Queueing Theory, Optimization and Fuzzy topology.

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