

# FSO Cooperative System Analysis for DF based Relay

Sanya Anees

**Abstract**— This paper presents performance and information theoretic analysis for cascaded free space optical (FSO) communication system using decode-and-forward relaying protocol. Optical links experience Gamma-Gamma distributed atmospheric turbulence, path loss, and misalignment losses. Derived closed-form statistical characteristics of the end-to-end SNR of the system are used for outage analysis, bit-error-rate analysis, and information theoretic analysis of the proposed system, for various modulation schemes and adaptive transmission protocols, respectively. In the performance analysis, impact of misalignment losses and scintillations is seen on various performance parameters and the adaptive modulation schemes using capacity of proposed systems.

**Index Terms**— Bit error rate, decode-and-forward relaying, Gamma-Gamma turbulence, free space optical (FSO) communication, path loss, pointing error.

## I. INTRODUCTION

Optical wireless communication (OWC) provide numerous benefits like cost effectiveness, interference free to electromagnetic radiations, license-free spectrum, and fast speed communications; providing researchers an alternative to radio-frequency (RF) communication systems. It can be used from space to terrestrial to underwater communication [1], [2]. Terrestrial OWC also known as free space optical communication (OWC) can be employed for various applications like last mile access, disaster management, high-definition transmission, and back-haul transmission. However, atmospheric conditions, turbulence, and misalignment losses affect the range, performance and reliability of FSO. Scintillations or atmospheric turbulence is a result of temperature and pressure fluctuations in the atmosphere causing variations in the refractive index in atmosphere. Misalignment losses also called pointing errors are caused due to the misalignment between transmitter and the receiver apertures due to the earthquakes, wind, and/or thermal expansion).

### A. Literature Review

In order to overcome these challenges, cooperative communication has been considered as an effective solution, either as mixed RF/FSO systems or as serial FSO systems [3]–[11] using either amplify-and-forward (AF) or decode-and-forward (DF) relaying scheme. In [3], for FSO based relay systems, outage analysis is performed using both AF and DF relaying techniques, binary pulse position modulation, and direct detection, where optical links are characterized by Log-normal distributed irradiance and path loss. Error rate analysis is studied for MIMO (multiple input multiple output) based FSO system employing a simple modulation technique such as on-off keying (OOK) and direct type of detection [12],

where the FSO links are K-distributed. The paper in [13] presents an FSO based dual-hop cooperative system using OOK. This system used AF relaying scheme and the optical links are modeled using Gamma-Gamma distributed irradiance, path loss, and misalignment losses. For this system, using accurate approximations average capacity is analyzed. The shortcoming of OOK scheme is its design difficulty which occurs as the threshold set for bit detection requires channel information. This is not a concern in case of either subcarrier intensity modulation (SIM) [14] or phase shift techniques (PSK) [15]. In [4], SIM technique is very well explained for a FSO based cooperative system using DF relaying scheme, differential modulation over Gamma-Gamma FSO links. An AF based SIM-PSK based multi-hop FSO system is analyzed for outage and error rate analysis in [16], where the optical links are characterized by Gamma-Gamma distributed atmospheric turbulence.

### B. Motivation

FSO is a feasible technology to address radio spectrum scarcity problem and also enjoys fiber-like transmission speed. However, it performs poorly for long-haul communication. As pointed out in [3], the fading variance in FSO is dependent on the link range which will allow FSO cooperative systems to not only increase the coverage area but also bring out great performance enhancement despite the fading effects.



**Fig. 1. FSO based cooperative system model using a DF relay.**

Thus, it would be beneficial to perform a generalized study on the FSO cooperative system using DF relays, in terms of error rate, outage, and capacity.

### C. Contribution

In this paper, we perform a detailed study of FSO based cooperative system using DF relaying scheme, SIM scheme, where an RF subcarrier pre-modulated with the information then correspondingly change optical source's intensity by giving sufficient bias [14].

In the considered system, optical links are modeled for atmospheric turbulence and misalignment losses. Using the derived expressions of statistical characteristics of SNR of the system, i.e. cumulative distribution function (CDF) and probability density function (PDF), novel closed-form analytical expressions for outage analysis, bit error rate (BER), and average (avg.) capacity are obtained. Impact of atmospheric turbulence and misalignment losses on above mentioned system



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Sanya Anees, Electronics and Communication Department, Indian Institute of Information Technology Guwahati, Guwahati, Assam, India.

evaluation parameters is observed.

## II. SYSTEM & CHANNEL MODEL

This section discusses a DF relay based FSO communication system, where source (S) transmits information using optical signals to destination (D) using a DF relay (R), as given in the Fig 1. Both optical links are identically but independently faded and are modeled using Gamma-Gamma distributed turbulence and Rayleigh distributed pointing errors. For optical signal modulation, SIM technique is used and for detection process, direct type of detection is used. After opto-electric conversion, the electrical signal at R is given by

$$y_{s,r} = \eta h_{s,r} x + e_{s,r}, \quad (1)$$

where  $\eta_{s,r}$  is the opto-electric transformation coefficient,  $x$  is the transmitted signal,  $I_{s,r}$  is the real-valued irradiance of the considered link, where  $I_{s,r} = I_{t,s,r} I_{p,s,r} I_{e,s,r}$  [17], where  $I_{t,s,r}$  represents irradiance effect,  $I_{p,s,r}$  represents path loss effect, and  $I_{e,s,r}$  represents misalignment loss effect in the first link of the proposed system. At D, the optical to electrical converted received signal is given by

$$y_{r,d} = \eta_{r,d} I_{r,d} \hat{x} + e_{r,d}, \quad (2)$$

where  $\eta_{r,d}$  is the opto-electric transformation coefficient,  $\hat{x}$  is the decoded-estimate of  $x$  at R,  $e_{r,d}$  is the additive white Gaussian noise (AWGN)  $N(0, \sigma_{r,d}^2)$ , and  $I_{r,d}$  denotes irradiance, where  $I_{r,d} = I_{t,s,d} I_{p,r,d} I_{e,r,d}$  [17], where  $I_{t,s,r}$  represents irradiance effect,  $I_{p,s,r}$  represents path loss effect, and  $I_{e,s,r}$  represents misalignment loss effect in the second link of the proposed system. Though both S-R and R-D links are independently faded but they undergo same distribution for irradiance, i.e., Gamma-Gamma distribution, characterizing moderate to strong fading. In case of DF relayed communication, symbol-by-symbol decoding is done at the each relay.

Thus, for a general single relayed communication system using DF relay, maximum transmission rate will be minimum of (1) maximum reliable decoding-information rate at the relay and (2) maximum reliable decoding information rate at the destination. [18]. Therefore, for our proposed communication system, end-to-end SNR,  $\gamma_z$ , can be expressed as [18], [19]

$$\gamma_z = \min(\gamma_{s,r}, \gamma_{r,d}) \quad (3)$$

From (1), (2), expressing transmitter as  $p \in \{s, r\}$  and receiver as  $q \in \{s, r\}$ , where  $p \neq q$ , we get  $\gamma_{p,q} = \frac{\eta_{p,q}^2 I_{p,q}^2 P_t}{\sigma^2} = \bar{\gamma}_{p,q} I_{p,q}^2$ . Average electrical SNR, obtained from the received signal after opto-electric coefficient,  $\bar{\gamma}_{p,q} = \eta_{p,q}^2 P_t / \sigma_{p,q}^2$  with  $P_t$  being the power of the transmitted signal. Average SNR of the optical signal,  $\bar{\gamma}_{a,p,q}$ , can be obtained from the electrical SNR using relation,  $\bar{\gamma}_{a,p,q} = \frac{(a_{p,q}+1)(b_{p,q}+1)}{a_{p,q}b_{p,q}} \bar{\gamma}_{p,q}$ , where  $a_{p,q}$  and  $b_{p,q}$  are the irradiance parameters [20].

### A. Impact of Path loss and Irradiance

In considered scenario, the instantaneous system SNR,  $\gamma_{p,q}$  has following PDF in terms of modified Bessel function of the second kind ( $K_v(\cdot)$ ) as given by

$$f_{\gamma_{p,q}}(\gamma) = \frac{(b_{p,q} a_{p,q})^{\frac{z_{1p,q}}{2}} \gamma^{\frac{z_{1p,q}}{4}-1}}{\Gamma(b_{p,q}) \Gamma(a_{p,q}) \bar{\gamma}_{p,q}^{\frac{z_{1p,q}}{4}}} \times K_{z_{2p,q}} \left( \sqrt{b_{p,q} a_{p,q}} \sqrt{\frac{\gamma}{\bar{\gamma}_{p,q}}} \right) \quad (4)$$

where,  $z_{1p,q} = b_{p,q} + a_{p,q}$ , and  $z_{2p,q} = -b_{p,q} + a_{p,q}$ .

The CDF of  $\bar{\gamma}_{p,q}$ , is written as

$$F_{\gamma_{p,q}}(\gamma) = \frac{(b_{p,q} a_{p,q})^{\frac{z_{1p,q}}{2}} \gamma^{\frac{z_{1p,q}}{4}-1}}{\Gamma(b_{p,q}) \Gamma(a_{p,q}) \bar{\gamma}_{p,q}^{\frac{z_{1p,q}}{4}}} \times G_{1,3}^{2,1} \left( b_{p,q} a_{p,q} \sqrt{\frac{\gamma}{\bar{\gamma}_{p,q}}} \left| \begin{matrix} 1 - \frac{z_{1p,q}}{4} \\ \frac{z_{2p,q}}{2}, -\frac{z_{2p,q}}{2}, \frac{z_{2p,q}}{2} \end{matrix} \right. \right), \quad (5)$$

where,  $G_{p,q}^{m,n} \{ \cdot | \cdot \}$  is the Meijer-G function [21, Chapter 2.24].

### B. Impact of Path loss, Irradiance and Misalignment Error

In considered scenario, the instantaneous system SNR,  $\gamma_{p,q}$ , has following PDF in terms of Meijer-G function as given by

$$f_{\gamma_{p,q}}(\gamma) = \frac{(b_{p,q} a_{p,q})^{\frac{z_{1p,q}}{2}} \gamma^{\frac{z_{1p,q}}{4}-1}}{2\gamma \Gamma(b_{p,q}) \Gamma(a_{p,q})} \times G_{1,3}^{3,0} \left( (f_{p,q} b_{p,q} a_{p,q} \sqrt{\frac{\gamma}{\bar{\gamma}_{p,q}}})^{\frac{\xi_{p,q}^2+1}{2}} \right), \quad (6)$$

where  $\gamma = \frac{\bar{\gamma}_{p,q}}{h_t A_{0p,q} f_{p,q}}$ ,  $f_{p,q} = \frac{\varepsilon_{p,q}^2}{\varepsilon_{p,q}^2 + 1}$

$A_{0p,q} = |\text{erf}(v_{p,q})|^2$ ,  $v_{p,q} = \frac{\sqrt{\frac{\pi}{2}} R_{p,q}}{w_{ep,q}}$  is the ratio between equivalent beamwaist,  $w_{ep,q}$ , and misalignment error variation,  $\sigma_{sp,q}$ , at  $q$ .

For this scenario, distribution function of end-to-end SNR is obtained using [21, Eq(1.16.2.1), Eq (8.2.2.19)] and (6) as

$$F_{\gamma_{p,q}}(\gamma) = \frac{2^{z_{1p,q}-2} \xi_{p,q}^2}{2\pi \Gamma(a_{p,q}) \Gamma(b_{p,q})} G_{3,7}^{6,1} \left( W_1 \gamma \left| \begin{matrix} 1, \varepsilon_{p,q}^2+1, \varepsilon_{p,q}^2+2 \\ \mathcal{K}_{1,0} \end{matrix} \right. \right), \quad (7)$$

where

$$W_1 = \frac{(f_{p,q} a_{p,q} b_{p,q})^2}{16 \bar{\gamma}_{p,q}} \text{ and } K_1 = \frac{\xi_{p,q}^2}{2}, \frac{\xi_{p,q}^2+1}{2}, \frac{a_{p,q}}{2}, \frac{a_{p,q}+1}{2}, \frac{b_{p,q}}{2}, \frac{b_{p,q}+1}{2}.$$

## III. STATISTICAL PROPERTIES OF END-TO-END SYSTEM SNR

Statistical Properties of the SNR for the considered DF based two-hop FSO systems are derived in this section.

### A. Impact of Path Loss and Irradiance

- 1) CDF: From (3), distribution function of  $\gamma_z$ , for the proposed system can be written as

in which  $F_{\gamma_{p,q}} = 1 - (1 - F_{\gamma_{s,r}})(1 - F_{\gamma_{r,d}})$ ,  
SNR of the  $p$ - $q$  link.

Putting (5) in (8), distribution function of  $\gamma_z$  is given by

$$F_{\gamma_z} = 1 - \left( \left( 1 - \mathcal{P}_1 \gamma^{\frac{z_{1s,r}}{4}} G_{1,3}^{2,1} \left( \mathcal{W}_2 \sqrt{\gamma} \left| \frac{1 - \frac{z_{1s,r}}{2}}{\frac{z_{2r,d}}{2}}, \frac{-z_{2r,d}}{2}, \frac{-z_{1r,d}}{2} \right| \right) \right) \right)$$

where,

$$\mathcal{P}_1 = \frac{1}{\Gamma(b_{p,q})\Gamma(a_{p,q})} \left( \frac{a_{s,r}b_{s,r}}{\sqrt{\gamma_{r,d}}} \right)^{\frac{z_{1s,r}}{2}}, \mathcal{W}_2 = \text{NR for proposed system, using (3) can be written as}$$

$$f_{\gamma_z}(\gamma) = f_{\gamma_{s,r}}(\gamma)(1 - F_{\gamma_{r,d}}) + f_{\gamma_{r,d}}(\gamma)(1 - F_{\gamma_{s,r}}(\gamma)). \quad (10)$$

Using (4) and (5) in (10), the PDF of  $\gamma_z$  can be obtained as

$$f_{\gamma_z}(\gamma) = \left( \mathcal{P}_1 \gamma^{\frac{z_{1s,r}}{4}-1} K_{z_{2s,r}} \left( 2\sqrt{\mathcal{W}_2 \sqrt{\gamma}} \right) \right) \times \left( 1 - \mathcal{P}_1 \gamma^{\frac{z_{1s,r}}{4}} G_{1,3}^{2,1} \left( \mathcal{W}_3 \sqrt{\gamma} \left| \frac{1 - \frac{z_{1r,d}}{2}}{\frac{z_{2p,q}}{2}}, \frac{-z_{2p,q}}{2}, \frac{-z_{1p,q}}{2} \right| \right) \right) \quad (11)$$

From the PDF, we proceed to calculate the MGF of the  $\gamma_z$ .

3) MGF: The MGF is defined in terms of PDF as [22, Eq. (5-96)]

Putting (11) in (12), further applying [23, Eq. (5.6.3.1)] and

$$\mathcal{M}_{\gamma}(s) \triangleq \int_{-\infty}^{\infty} f_{\gamma_s}(\gamma) e^{-s\gamma} d\gamma. \quad (12)$$

$$\mathcal{M}_{\gamma_z}(s) = \frac{\mathcal{P}_1}{\frac{z_{1s,r}}{4}} G_{1,4}^{4,1} \left( \frac{\mathcal{W}_2^2}{16s} \left| \frac{1 - \frac{z_{1s,r}}{4}}{\mathcal{K}'_1} \right. \right) + \frac{\mathcal{P}_2}{\frac{z_{1r,d}}{4}} G_{1,4}^{4,1} \left( \frac{\mathcal{W}_3^2}{16s} \left| \frac{1 - \frac{z_{1r,d}}{4}}{\mathcal{K}''_1} \right. \right) - \frac{\mathcal{P}_1 \mathcal{P}_2}{(4\pi)^2 s^{\frac{z_3}{4}}} S \left( \frac{\mathcal{W}_2^2}{16s}, \frac{\mathcal{W}_3^2}{16s} \left| \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right| \frac{z_3}{4} \left| \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \mathcal{K}_1 \right| \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \mathcal{K}_3 \right), \quad (13)$$

$$z_3 = z_{1s,r} + z_{1r,d}, \mathcal{K}'_1 = \frac{z_{2s,r}}{2}, \frac{z_{2s,r}+2}{4}, \frac{-z_{2s,r}}{4}, \frac{-z_{2s,r}+2}{4},$$

$$\mathcal{K}''_1 = \frac{z_{2r,d}}{2}, \frac{z_{2r,d}+2}{4}, \frac{-z_{2r,d}}{4}, \frac{-z_{2r,d}+2}{4},$$

$$\mathcal{K}_2 = \frac{-z_{1s,r} + 2}{4}, \frac{-z_{1s,r}}{4} \quad \mathcal{K}_3 = \frac{z_{2s,r}}{4}, \frac{z_{2s,r} + 2}{4}, \frac{-z_{2s,r}}{4}$$

$$\frac{-z_{2s,r}}{4}, \frac{-z_{1s,r}}{4}, \frac{-z_{2s,r} + 2}{4}, \mathcal{K}_4 = \frac{-z_{1r,d} + 2}{4}, \frac{-z_{1r,d}}{4}$$

$$\mathcal{K}_5 = \frac{z_{2r,d}}{4}, \frac{z_{2r,d} + 2}{4}, \frac{-z_{2r,d}}{4}, \frac{-z_{2r,d} + 2}{4}, \frac{-z_{1r,d}}{4}, \frac{-z_{1r,d} + 2}{4}.$$

Also,  $S(\cdot)$  stands for Extended Generalized Meijer-G function (EGMGF) [24], [25]

## B. Impact of Path loss, Irradiance and Misalignment Error

For optical links modeled using Gamma-Gamma irradiance, path loss, and misalignment errors, statistical properties of the SNR is discussed in this section.

1) CDF: Using (7) in (8), we get

$$F_{\gamma_z} = 1 - \left( 1 - \mathcal{P}_3 G_{3,7}^{6,1} \left( \mathcal{W}_{4\gamma} \left| \begin{matrix} 1, \mathcal{K}_6 \\ \mathcal{K}_{7,0} \end{matrix} \right. \right) \right) \left( 1 - \mathcal{P}_4 G_{4,7}^{6,0} \left( \mathcal{W}_{5\gamma} \left| \begin{matrix} \mathcal{K}_8 \\ \mathcal{K}_9 \end{matrix} \right. \right) \right)$$

where,

$$\mathcal{P}_3 = \frac{2^{z_{1s,r}-2} \xi_{s,r}^2}{2\pi\Gamma(a_{p,q})\Gamma(b_{p,q})}, \quad \mathcal{P}_4 = \frac{2^{z_{1r,d}-2} \xi_{r,d}^2}{2\pi\Gamma(a_{r,d})\Gamma(b_{r,d})}$$

$$\mathcal{W}_4 = \frac{(f_{s,r} a_{s,r} b_{s,r})^2}{16\bar{\gamma}_{s,r}},$$

$$\mathcal{K}_6 = \frac{\xi_{s,r}^2 + 1}{2}, \frac{\xi_{s,r}^2 + 2}{2} \quad \mathcal{K}_7 = \frac{\xi_{s,r}^2}{2}, \frac{\xi_{s,r}^2 + 2}{2}, \frac{a_{s,r}}{2}, \frac{a_{s,r} + 1}{2}$$

$$\frac{b_{s,r}}{2}, \frac{b_{s,r} + 1}{2}, \quad \mathcal{K}_8 = \frac{\xi_{r,d}^2 + 1}{2}, \frac{\xi_{r,d}^2 + 2}{2}$$

and

$$\mathcal{K}_9 = \frac{\xi_{r,d}^2}{2}, \frac{\xi_{r,d}^2 + 1}{2}, \frac{a_{r,d}}{2}, \frac{a_{r,d} + 1}{2}, \frac{b_{r,d}}{2}, \frac{b_{r,d} + 1}{2}$$

2) PDF: Substituting (6), (7), and [21, Eq. (8.1.2.19)] in (10), the density function is given by

$$f_{\gamma_z}(\gamma) = \mathcal{P}_3 \gamma^{-1} G_{2,6}^{6,0} \left( \mathcal{W}_4 \gamma \left| \begin{matrix} \mathcal{K}_6 \\ \mathcal{K}_7 \end{matrix} \right. \right) \left( 1 - \mathcal{P}_4 G_{4,7}^{6,0} \left( \mathcal{W}_5 \gamma \left| \begin{matrix} \mathcal{K}_8 \\ \mathcal{K}_9 \end{matrix} \right. \right) \right) \quad (15)$$

TABLE I:

BINARY MODULATION PARAMETERS FOR BER CALCULATION

Modulation Techniques	p	q
CBPSK	0.5	1
CBFSK	0.5	0.5
DBPSK	1	1
NBFSK	1	0.5

3) MGF: Substituting (15) in (12) and then using [23, Eq. (5.6.3.1)], the MGF for the considered system with significant misalignment losses is given by

$$\mathcal{M}_{\gamma_z}(s) = \frac{\mathcal{P}_3}{s} G_{4,7}^{6,2} \left( \frac{\mathcal{W}_4}{s} \left| \begin{matrix} 0, 1, \mathcal{K}_6 \\ \mathcal{K}_{7,0} \end{matrix} \right. \right) - \mathcal{P}_3 \mathcal{P}_4 S \left( \frac{\mathcal{W}_4}{s}, \frac{\mathcal{W}_5}{s} \left| \begin{matrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{matrix} \right. \right) \left( \begin{matrix} 6 & 0 & \mathcal{K}_6 \\ 2 & 6 & \mathcal{K}_7 \end{matrix} \right) \left( \begin{matrix} 6 & 1 \\ 3 & 7 \end{matrix} \right) \frac{1, \mathcal{K}_8}{\mathcal{K}_9, 0}$$

$$-\mathcal{P}_3\mathcal{P}_4S\left(\frac{\mathcal{W}_5}{s},\frac{\mathcal{W}_4}{s}\left|\begin{bmatrix}1 & 0 \\ 1 & 0\end{bmatrix}\right|\begin{bmatrix}6 & 0 \\ 2 & 6\end{bmatrix}\mathcal{K}_8\right)\left|\begin{bmatrix}6 & 1 \\ 3 & 7\end{bmatrix}\mathcal{K}_9,0\right) \quad (16)$$

#### IV. PERFORMANCE ANALYSIS

This section discusses outage analysis, error analysis, and information theoretic analysis for the proposed FSO based cooperative system.

##### A. Impact of Path loss and Irradiance

1) *Outage Probability*: Outage probability of the considered system for a threshold of  $\gamma_{th}$ , using (9), is given by

$$\times \left(1 - \mathcal{P}_{2th}^{\frac{z_{1r,d}}{4}} G_{1,3}^{2,1} \left( \mathcal{W}_3 \sqrt{\gamma_{th}} \left| \begin{bmatrix} 1 - \frac{z_{1s,r}}{2} \\ \frac{z_{2r,d}}{2} - \frac{z_{2r,d}}{2} - \frac{z_{1r,d}}{2} \end{bmatrix} \right| \right) \right) \quad (17)$$

2) *Average BER*: BER analytical analysis of the proposed system is derived for different modulation schemes under the impact of path loss and irradiance.

a) *Binary Modulation Techniques*: Applying [26, Eq. (12)], BER for binary modulation schemes can be written as

$$b_j = (2j - 1) \sqrt{\frac{3}{M-1}} \text{ and } \varphi_M = \frac{1}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right). \quad (18)$$

Where, p and q are the binary modulation parameters described in [26] given in Table I. Putting (9) in (18), further applying [23, Eq. (5.6.3.1)], BER can be derived as

$$\mathcal{P}_{eb} = \frac{P_1 q^{\frac{-z_{1s,r}}{4}}}{8\pi\Gamma(p)} G_{3,7}^{6,1} \left( \frac{\mathcal{W}_2^2}{16q} \left| 1 - p - \frac{z_{1s,r}}{4}, \mathcal{K}_2 \right. \right) + \frac{P_2 q^{\frac{-z_{1r,d}}{4}}}{8\pi\Gamma(p)} G_{3,6}^{4,3} \left( \frac{\mathcal{W}_3^2}{16q} \left| 1 - p - \frac{z_{1r,d}}{4}, \mathcal{K}_4 \right. \right)$$

b) *M-PSK*: Instantaneous BER FOR M-PSK scheme is given by [27]

$$P_{ep}(\gamma) \cong \frac{2}{\zeta_M} \sum_{j=1}^{\max(\frac{M}{4},1)} Q(a_j \sqrt{2\gamma}), \quad (20)$$

where,  $\zeta_M = \max(\log_2 M, 2)$ ,

$Q(\cdot)$  is the Q-function, and  $a_j = \sin\left(\frac{(2j-1)\pi}{M}\right)$ .

Using (20), BER can be written as

$$+ \frac{\mathcal{P}_4}{s} G_{4,7}^{6,2} \left( \frac{\mathcal{W}_4}{s} \left| \begin{bmatrix} 0,1,\mathcal{K}_8 \\ \mathcal{K}_9,0 \end{bmatrix} \right. \right) \quad , \quad = - \int_0^\infty F_{\gamma_Z}(\gamma) dP_e(\gamma) \quad (21)$$

Putting (9) and (20) in (21), further applying [23, Eq. (5.6.3.1)], BER for M-PSK constellation can be given by

$$P_{ep} = \sum_{j=1}^{\max(\frac{M}{4},1)} \frac{\mathcal{P}_2 a_j^{\frac{-z_{1s,r}}{4}}}{4\pi^2 \zeta_M} G_{3,6}^{4,3} \left( \frac{\mathcal{W}_2^2}{16a_j^2} \left| \frac{1}{2} - \frac{z_{1s,r}}{4}, \mathcal{K}_2 \right. \right) + \sum_{j=1}^{\max(\frac{M}{4},1)} \frac{\mathcal{P}_2 a_j^{\frac{-z_{1s,r}}{4}}}{4\pi^2 \zeta_M} G_{3,6}^{4,3} \left( \frac{\mathcal{W}_3^2}{16a_j^2} \left| \frac{1}{2} - \frac{z_{1s,r}}{4}, \mathcal{K}_3 \right. \right) \times \frac{a_j^{\frac{-z_3}{4}}}{\pi^2 \zeta_M} S \left( \frac{\mathcal{W}_2^2}{16a_j^2}, \frac{\mathcal{W}_3^2}{16a_j^2} \left| \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right| \frac{1}{2} + \frac{z_3}{4} \left| \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \mathcal{K}_3 \right| \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \mathcal{K}_4 \right) \quad (22)$$

c) *M-QAM*: Instantaneous BER for M-QAM technique is given by [27]

$$P_{ep}(\gamma) \cong 4\psi M \sum_{j=1}^{\sqrt{\frac{M}{2}}} Q(b_j \sqrt{\gamma}) \quad (23)$$

in which

Using (21),(9), (23), and [23, Eq. (5.6.3.1)], BER of M-QAM can be written as

$$P_{eq} = \sum_{j=1}^{\sqrt{M/2}} \frac{\mathcal{P}_1 \psi M}{\pi^{\frac{3}{2}}} \left( \frac{b_j^2}{2} \right)^{\frac{-z_{1s,r}}{4}} G_{3,6}^{4,3} \left( \frac{\mathcal{W}_2^2}{8b_j^2} \left| \frac{1}{2} - \frac{z_{1s,r}}{4}, \mathcal{K}_3 \right. \right) + \sum_{j=1}^{\sqrt{M/2}} \frac{\mathcal{P}_2 \psi M}{2\pi^2} \left( \frac{b_j^2}{2} \right)^{\frac{-z_{1r,d}}{4}} G_{3,6}^{4,3} \left( \frac{\mathcal{W}_3^2}{8b_j^2} \left| \frac{1}{2} - \frac{z_{1r,d}}{4}, \mathcal{K}_5 \right. \right) \times \left( \frac{b_j}{2} \right)^{\frac{-z_3}{4}} S \left( \frac{\mathcal{W}_2^2}{8b_j^2}, \frac{\mathcal{W}_3^2}{8b_j^2} \left| \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right| \frac{1}{2} + \frac{z_3}{4} \left| \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \mathcal{K}_3 \right| \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \mathcal{K}_5 \right). \quad (24)$$

$$- \frac{\mathcal{P}_1 \mathcal{P}_2}{(4\pi)^2 s^4} S \left( \frac{\mathcal{W}_2^2}{16q}, \frac{\mathcal{W}_3^2}{16} \left| \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right| \frac{p+z_3}{4} \left| \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \mathcal{K}_3 \right| \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \mathcal{K}_5 \right), \quad (19)$$

3) *Average Capacity*: Average capacity represents maximum information transmission rate affordable for very less error probability. Here, average capacity is analyzed for proposed system employing different adaptive modulation protocols, namely, ORA, OPRA, and TCIFR.

a) *ORA*: Ergodic capacity under ORA can be expressed as [28]

$$C_{ora} \approx \frac{B}{\log 2} \sum_{j=1}^N v_n C_q(s_n) \left\{ \frac{\delta}{\delta s} M_\gamma(s_n) \right\} s \xrightarrow{yields} s_n \quad (25)$$



where,  $S_n = \tan\left(\frac{\pi}{4} \cos\left(\frac{2n-1}{2N}\pi\right) + \frac{\pi}{4}\right)$ , This is accomplished when system SNR is greater than a written as [29]

$$c_{tcifr} = \frac{B}{\log 2} \log\left(1 + \frac{1}{I}\right) \left(1 - P_{out}(\beta_o)\right) \quad (31)$$

$$v_n = \frac{\pi^2 \sin\left(\frac{2n-1}{2N}\pi\right)}{4N \cos\left(\frac{\pi}{4} \cos\left(\frac{2n-1}{2N}\pi\right) + \frac{\pi}{4}\right)} C_q(S_n) = \frac{\frac{P_2 \gamma_o}{4\pi} G_{1,5}^{5,0} \left(\frac{W_2^2 \gamma_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1'' \end{matrix} \right| \right) - S\left(\frac{W_2^2 \gamma_o}{16}, \frac{W_3^2 \gamma_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1'' \end{matrix} \right| \right)}{\frac{P_2 \gamma_o}{4\pi} G_{1,5}^{5,0} \left(\frac{W_2^2 \gamma_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1'' \end{matrix} \right| \right) - S\left(\frac{W_2^2 \gamma_o}{16}, \frac{W_3^2 \gamma_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1'' \end{matrix} \right| \right)} \quad (30)$$

$$-H_{3,6}^{4,3} \left\{ \frac{1}{s_n^q} \left| \begin{matrix} (1,1), (1,1), (1,q) \\ (1,1), (0,1) \end{matrix} \right| \right\}, B \text{ stands for bandwidth}$$

$q \in \{1,2\}$ , and  $N$  is a positive integer.

Using derivative of MGF written in (13), further putting it in (25), and then applying [23, Eq. (5.6.3.1)], [25, Eq. (4)], and [24, Eq. (2.1)], ergodic capacity is derived as

$$C_{ora} \approx \frac{B}{\log 2} \sum_{j=1}^N v_n C_q(S_n)$$

b) *OPRA*: After setting a constraint on the average transmitted power, the OPRA capacity is computed which can be written as [29]

$$C_{opra} = \frac{B}{\log 2} \int_{-\infty}^{\infty} \ln\left(\frac{\gamma}{\gamma_o}\right) f_{\gamma z}(\gamma) d\gamma \quad (27)$$

where  $\gamma_o$  denotes the optimum cutoff SNR threshold which should comply with [29]

$$\int_{-\infty}^{\infty} \left(-\frac{1}{\gamma} + \frac{1}{\gamma_o}\right) f_{\gamma z}(\gamma) d\gamma = 1 \quad (28)$$

Substituting (11) in (27), and using [30], the capacity can be derived

$$C_{opra} = \frac{B}{\log 2} \left[ \frac{P_1 \gamma_o}{4\pi} G_{2,6}^{6,0} \left( \frac{W_2^2 \gamma_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4}, 1 - \frac{Z_3}{4} \\ \mathcal{K}_1', -\frac{Z_3}{4}, -\frac{Z_3}{4} \end{matrix} \right| \right) - \frac{P_1 P_2}{4(\pi r)^2} \right] \\ \times \gamma_o^{\frac{Z_3}{4}} S \left( \frac{W_2^2 \gamma_o}{16}, \frac{W_3^2 \gamma_o}{16} \left| \begin{matrix} 2 - \frac{Z_3}{4}, 1 - \frac{Z_3}{4} \\ \mathcal{K}_1', -\frac{Z_3}{4} \end{matrix} \right| \right) - \left| \begin{matrix} (4, 0) \\ (2, 6) \end{matrix} \right| \mathcal{K}_4 \mathcal{K}_5 \\ + \frac{P_2 \gamma_o}{4\pi} G_{2,6}^{6,0} \left( \frac{W_2^2 \gamma_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4}, 1 - \frac{Z_3}{4} \\ \mathcal{K}_1', -\frac{Z_3}{4}, -\frac{Z_3}{4} \end{matrix} \right| \right) - \frac{P_1 P_2 \gamma_o^{\frac{Z_3}{4}}}{4(\pi r)^2}$$

The threshold set is numerically computed as given in the beginning of this page. This is done by putting (11) in (28) further applying [30].

c) *TCIFR*: This is a simple transmission strategy where transmitter adapts the transmitted power with change in noise levels so as to always have a constant SNR at receiver.

where  $I = \int_{\beta_o}^{\infty} \frac{f_{yz}(\gamma)}{\gamma} d\gamma$  is determined by numerically  $\frac{\delta c_{tcifr}}{\delta \beta_o} = 0$

By substituting (11) in I, and using [30,

$$\begin{aligned} & \times \left[ -\frac{P_1}{4\pi s_n^{\frac{Z_3}{4}+1}} G_{1,4}^{4,1} \left( \frac{W_2^2}{16 s_n} \left| \begin{matrix} -\frac{Z_3}{4} \\ \mathcal{K}_1' \end{matrix} \right| \right) + \frac{P_1 P_2}{4(\pi r)^2 s_n^{\frac{Z_3}{4}+1}} \right] \\ & \times S \left( \frac{W_2^2}{16 s_n}, \frac{W_3^2}{16 s_n} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1' \end{matrix} \right| \right) - \left| \begin{matrix} (4, 0) \\ (2, 6) \end{matrix} \right| \mathcal{K}_4 \mathcal{K}_5 \\ & - \left[ \frac{P_2}{4\pi s_n^{\frac{Z_3}{4}+1}} G_{1,4}^{4,1} \left( \frac{W_3^2}{16 s_n} \left| \begin{matrix} -\frac{Z_3}{4} \\ \mathcal{K}_1'' \end{matrix} \right| \right) + \frac{P_1 P_2}{4(\pi r)^2 s_n^{\frac{Z_3}{4}+1}} \right] \\ & \times S \left( \frac{W_2^2}{16 s_n}, \frac{W_3^2}{16 s_n} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1'' \end{matrix} \right| \right) - \left| \begin{matrix} (4, 0) \\ (2, 6) \end{matrix} \right| \mathcal{K}_4 \mathcal{K}_5 \\ & I = \frac{P_1 \beta_o}{4\pi} G_{1,5}^{5,0} \left( \frac{W_2^2 \beta_o}{16} \left| \begin{matrix} 2 - \frac{Z_3}{4} \\ \mathcal{K}_1', 1 - \frac{Z_3}{4} \end{matrix} \right| \right) - \frac{P_1 P_2 \beta_o^{\frac{Z_3}{4}-1}}{4(\pi r)^2} \\ & \times S \left( \frac{W_2^2 \beta_o}{16}, \frac{W_3^2 \beta_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1' \end{matrix} \right| \right) - \left| \begin{matrix} (4, 0) \\ (2, 6) \end{matrix} \right| \mathcal{K}_4 \mathcal{K}_5 \\ & + \frac{P_2 \beta_o}{4\pi} G_{1,5}^{5,0} \left( \frac{W_2^2 \beta_o}{16} \left| \begin{matrix} 2 - \frac{Z_3}{4} \\ \mathcal{K}_1'', 1 - \frac{Z_3}{4} \end{matrix} \right| \right) - \frac{P_1 P_2 \beta_o^{\frac{Z_3}{4}-1}}{4(\pi r)^2} \\ & \times S \left( \frac{W_2^2 \beta_o}{16}, \frac{W_3^2 \beta_o}{16} \left| \begin{matrix} 1 - \frac{Z_3}{4} \\ \mathcal{K}_1'' \end{matrix} \right| \right) - \left| \begin{matrix} (4, 0) \\ (2, 6) \end{matrix} \right| \mathcal{K}_4 \mathcal{K}_5 \end{aligned}$$

Average capacity under TCIFR is derived by putting (17) and (32) in (31).

## B. Impact of Path Loss, Irradiance, and Misalignment Error

1) *Outage Probability*: Outage probability for this scenarios is given by applying (14) as

$$P_{out}(\gamma_{th}) = 1 - \left( 1 - P_3 G_{3,7}^{6,1} \left( w_4 \gamma_{th} \left| \begin{matrix} 1, K_6 \\ K_7, 0 \end{matrix} \right| \right) \right) \times \left( 1 - P_4 G_{3,7}^{6,1} \left( w_5 \gamma_{th} \left| \begin{matrix} 1, K_9 \\ K_9, 0 \end{matrix} \right| \right) \right) \quad (33)$$

2) *Average BER*: This subsection discusses BER for various modulation techniques.

a) *Binary Modulation Techniques*: Putting (14) in (1) further applying [23, Eq. (5.6.3.1)], BER can be derived as

$$P_{eb} = \frac{P_3}{2\Gamma(p)} G_{4,7}^{6,0} \left( \frac{W_4}{q} \middle| \begin{matrix} 1-p, 1, \mathcal{K}_6 \\ \mathcal{K}_7, 0 \end{matrix} \right) + \frac{P_4}{2\Gamma(p)} G_{4,7}^{6,2} \left( \frac{W_5}{q} \middle| \begin{matrix} 1-p, 1, \mathcal{K}_8 \\ \mathcal{K}_9, 0 \end{matrix} \right) - \frac{P_3 P_4}{2\Gamma(p)} S \left( \frac{W_4}{q}, \frac{W_5}{q} \middle| \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix} \middle| \begin{matrix} (4 & 2) & 1, \mathcal{K}_6 \\ (2 & 6) & \mathcal{K}_7, 0 \end{matrix} \middle| \begin{matrix} (4 & 2) & 1, \mathcal{K}_8 \\ (2 & 6) & \mathcal{K}_9, 0 \end{matrix} \right) \quad (34)$$

b) *M-PSK*: Using (14), (20), and (21), , further applying [23, Eq. (5.6.3.1)], BER for M-PSK constellation can be derived as

$$P_{ep} = \sum_{j=1}^{\max(\frac{M}{4,1})} \frac{P_3}{\sqrt{\pi \zeta M}} G_{4,7}^{6,2} \left( \frac{2W_4}{b_j^2} \middle| \begin{matrix} \frac{1}{2}, 1, \mathcal{K}_6 \\ \mathcal{K}_7, 0 \end{matrix} \right) + \sum_{j=1}^{\max(\frac{M}{4,1})} \frac{P_4}{\sqrt{\pi \zeta M}} G_{4,7}^{6,2} \left( \frac{2W_5}{a_j^2} \middle| \begin{matrix} \frac{1}{2}, 1, \mathcal{K}_8 \\ \mathcal{K}_9, 0 \end{matrix} \right) - \sum_{j=1}^{\max(\frac{M}{4,1})} \frac{P_3 P_4}{\sqrt{\pi \zeta M}} \times S \left( \frac{2W_4}{b_j^2}, \frac{2W_5}{b_j^2} \middle| \begin{matrix} 1 & 0 \\ 1 & 1 \end{matrix} \middle| \begin{matrix} (4 & 2) & \mathcal{K}_2 \\ (2 & 6) & \mathcal{K}_3 \end{matrix} \middle| \begin{matrix} (4 & 2) & \mathcal{K}_4 \\ (2 & 6) & \mathcal{K}_5 \end{matrix} \right) \quad (35)$$

c) *M-QAM*: : Determining the solution of (21) upon substituting (14) and (23), further applying [23, Eq. (5.6.3.1)], BER for M-QAM constellation can be derived as

$$P_{eq} = \sum_{j=1}^{\sqrt{\frac{M}{2}}} \frac{2P_3 \phi M}{\sqrt{\pi}} G_{4,7}^{6,2} \left( \frac{2W_4}{b_j^2} \middle| \begin{matrix} \frac{1}{2}, 1, \mathcal{K}_6 \\ \mathcal{K}_7, 0 \end{matrix} \right) + \sum_{j=1}^{\sqrt{\frac{M}{2}}} \frac{2P_3 \phi M}{\sqrt{\pi}} G_{4,7}^{6,2} \left( \frac{2W_5}{a_j^2} \middle| \begin{matrix} \frac{1}{2}, 1, \mathcal{K}_8 \\ \mathcal{K}_9, 0 \end{matrix} \right) - \sum_{j=1}^{\sqrt{\frac{M}{2}}} \frac{2P_1 P_2 \phi M}{\sqrt{\pi}} \left[ \frac{P_3}{\gamma_0} G_{3,7}^{7,0} \left( W_{4\gamma_0} \middle| \begin{matrix} 1, \mathcal{K}_6 \\ \mathcal{K}_7, 0 \end{matrix} \right) - \frac{P_3 P_4}{\gamma_0} S \left( W_{4\gamma_0}, W_{5\gamma_0} \middle| \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix} \middle| \begin{matrix} (6 & 0) & \mathcal{K}_6 \\ (2 & 6) & \mathcal{K}_7 \end{matrix} \middle| \begin{matrix} (6 & 1) & 1, \mathcal{K}_8 \\ (3 & 7) & \mathcal{K}_9, 0 \end{matrix} \right) + \frac{P_3}{\gamma_0} G_{3,7}^{7,0} \left( W_{5\gamma_0} \middle| \begin{matrix} 1, \mathcal{K}_8 \\ \mathcal{K}_9, 0 \end{matrix} \right) - \frac{P_3 P_4}{\gamma_0} S \left( W_{5\gamma_0}, W_{4\gamma_0} \middle| \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix} \middle| \begin{matrix} (6 & 0) & \mathcal{K}_8 \\ (2 & 6) & \mathcal{K}_9 \end{matrix} \middle| \begin{matrix} (6 & 1) & 1, \mathcal{K}_6 \\ (3 & 7) & \mathcal{K}_7, 0 \end{matrix} \right) - \frac{P_3}{\gamma_0} G_{3,7}^{7,0} \left( W_{4\gamma_0} \middle| \begin{matrix} 2, \mathcal{K}_6 \\ \mathcal{K}_7, 1 \end{matrix} \right) + \frac{P_3 P_4}{\gamma_0} S \left( W_{4\gamma_0}, W_{5\gamma_0} \middle| \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix} \middle| \begin{matrix} (6 & 0) & \mathcal{K}_6 \\ (2 & 6) & \mathcal{K}_7 \end{matrix} \middle| \begin{matrix} (6 & 1) & 1, \mathcal{K}_8 \\ (3 & 7) & \mathcal{K}_9, 0 \end{matrix} \right) - \frac{P_4}{\gamma_0} G_{4,8}^{8,0} \left( W_{5\gamma_0} \middle| \begin{matrix} 1, \mathcal{K}_8 \\ \mathcal{K}_9, 1 \end{matrix} \right) + \frac{P_3 P_4}{\gamma_0} S \left( W_{5\gamma_0}, W_{4\gamma_0} \middle| \begin{matrix} 2 & 0 \\ 2 & 1 \end{matrix} \middle| \begin{matrix} (6 & 0) & \mathcal{K}_8 \\ (2 & 6) & \mathcal{K}_9 \end{matrix} \middle| \begin{matrix} (6 & 1) & 1, \mathcal{K}_6 \\ (3 & 7) & \mathcal{K}_7, 0 \end{matrix} \right) = 1 \quad (39)$$

$$\times S \left( \frac{2W_4}{b_j^2}, \frac{2W_5}{b_j^2} \middle| \begin{matrix} 1 & 0 \\ 1 & 1 \end{matrix} \middle| \begin{matrix} (4 & 2) & \mathcal{K}_2 \\ (2 & 6) & \mathcal{K}_3 \end{matrix} \middle| \begin{matrix} (4 & 2) & \mathcal{K}_4 \\ (2 & 6) & \mathcal{K}_5 \end{matrix} \right) \quad (36)$$

3) *Average Channel Capacity*:

a) *ORA*: Following similar approach as done in previously, the ORA capacity in this scenario can be derived as

$$C_{avg} = \frac{B}{\log 2} \sum_{j=1}^N v_n C_q(s_n) \times \left[ -\frac{P_3}{s^2} G_{4,7}^{6,2} \left( \frac{W_4}{s} \middle| \begin{matrix} -1, 1, \mathcal{K}_6 \\ \mathcal{K}_7, 0 \end{matrix} \right) + \frac{P_3 P_4}{s} S \left( \frac{W_4}{s}, \frac{W_5}{s} \middle| \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix} \middle| \begin{matrix} (6 & 0) & \mathcal{K}_6 \\ (2 & 6) & \mathcal{K}_7 \end{matrix} \middle| \begin{matrix} (6 & 1) & 1, \mathcal{K}_8 \\ (3 & 7) & \mathcal{K}_9, 0 \end{matrix} \right) - \frac{P_4}{s^2} G_{4,7}^{6,2} \left( \frac{W_5}{s} \middle| \begin{matrix} -1, 1, \mathcal{K}_8 \\ \mathcal{K}_9, 0 \end{matrix} \right) + \frac{P_3 P_4}{s} S \left( \frac{W_5}{s}, \frac{W_4}{s} \middle| \begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix} \middle| \begin{matrix} (6 & 0) & \mathcal{K}_8 \\ (2 & 6) & \mathcal{K}_9 \end{matrix} \middle| \begin{matrix} (6 & 1) & 1, \mathcal{K}_6 \\ (3 & 7) & \mathcal{K}_7, 0 \end{matrix} \right) \quad (37)$$

b) *OPRA*: Following similar approach as done in previously, the ORA capacity in this scenario is given by

$$C_{opra} = \frac{B}{\log 2} \left[ P_3 G_{4,8}^{8,0} \left( W_{4\gamma_0} \middle| \begin{matrix} 1, 1, \mathcal{K}_6 \\ \mathcal{K}_7, 0, 0 \end{matrix} \right) - P_3 P_4 \times S(W_{4\gamma_0}, W_{5\gamma_0} \middle| \begin{matrix} 2, 0 \\ 2, 2 \end{matrix} \middle| \begin{matrix} 1, 1 \\ 0, 0 \end{matrix} \middle| \begin{matrix} (6, 0) & \mathcal{K}_6 \\ (2, 6) & \mathcal{K}_7 \end{matrix} \middle| \begin{matrix} (6, 1) & 1, \mathcal{K}_8 \\ (3, 7) & \mathcal{K}_9, 0 \end{matrix} \right) + P_4 G_{4,8}^{8,0} \left( W_{5\gamma_0} \middle| \begin{matrix} 1, 1, \mathcal{K}_8 \\ \mathcal{K}_9, 0, 0 \end{matrix} \right) - P_3 P_4 \times S(W_{5\gamma_0}, W_{4\gamma_0} \middle| \begin{matrix} 2, 0 \\ 2, 2 \end{matrix} \middle| \begin{matrix} 1, 1 \\ 0, 0 \end{matrix} \middle| \begin{matrix} (6, 0) & \mathcal{K}_8 \\ (2, 6) & \mathcal{K}_9 \end{matrix} \middle| \begin{matrix} (6, 1) & 1, \mathcal{K}_6 \\ (3, 7) & \mathcal{K}_7, 0 \end{matrix} \right) \quad (38)$$

TABLE II  
SIMULATION PARAMETERS OF THE PROPOSED SYSTEM [25-27]

Parameter	Value
Transmit Divergence at	1/e1 mrad
Diameter of receiver	20 cm
Beam width ( $w_b$ )	100 cm
FSO Link distance	1 km
Jitter std. deviation ( $\sigma_s$ )	10 cm

TABLE III  
SKY SCENARIOS [3], [15], [28]

Weather	Visibility	$C_n^2$ range	$\sigma_1^2$	Turbulence Parameters
Light fog	0.5 km	$10^{-15} \text{m}^{-2/3}$	0.6	a = 5.42, b = 3.79
Clear	10 km	$10^{-14} \text{m}^{-2/3}$	2.0	a = 3.99, b = 1.70

where  $\gamma_0$  is numerically solved from the equation given at the top of the page which is obtained by substituting (15) in (28) further applying [30].

c) TCIFR: Putting (15) equation in  $I$ , further applying [30],  $J$  is given by

$$\begin{aligned}
 I &= \frac{\mathcal{P}_3}{\beta_0} \left[ \mathcal{P}_3 G_{3,7}^{7,0} \left( \mathcal{W}_4 \beta_0 \middle| \begin{matrix} 2, \mathcal{K}_6 \\ \mathcal{K}_7, 1 \end{matrix} \right) - \frac{\mathcal{P}_3 \mathcal{P}_4}{\beta_0} \right. \\
 &\times S(\mathcal{W}_4 \beta_0, \mathcal{W}_5 \beta_0) \left[ \begin{matrix} 1,0 \\ 1,1 \end{matrix} \middle| \begin{matrix} (6,0) \mathcal{K}_6 \\ (2,6) \mathcal{K}_7 \end{matrix} \middle| \begin{matrix} (6,1) 1, \mathcal{K}_8 \\ (3,7) \mathcal{K}_9, 0 \end{matrix} \right] \\
 &+ \mathcal{P}_4 G_{4,8}^{8,0} \left( \mathcal{W}_5 \gamma_0 \middle| \begin{matrix} 1,1, \mathcal{K}_8 \\ \mathcal{K}_9, 0,0 \end{matrix} \right) - \mathcal{P}_3 \mathcal{P}_4 \\
 &\left. - S(\mathcal{W}_5 \beta_0, \mathcal{W}_4 \beta_0) \left[ \begin{matrix} 2,0 \\ 2,2 \end{matrix} \middle| \begin{matrix} (6,0) \mathcal{K}_8 \\ (2,6) \mathcal{K}_9 \end{matrix} \middle| \begin{matrix} (6,1) 1, \mathcal{K}_6 \\ (3,7) \mathcal{K}_7, 0 \end{matrix} \right] \right] \quad (40)
 \end{aligned}$$

As done earlier TCIFR capacity is derived by using (33), (40), and (31).

## V. NUMERICAL RESULTS

This sections shows outage analysis, error analysis, and information theory analysis for proposed two-hop FSO system using DF relay are discussed. Table II shows the simulation parameters used in the paper also applicable for various practical FSO communication systems [31]–[33]. In Table III, different weather scenarios are taken into considerations [14], [34], [35], for varying turbulence conditions. Equal SNRs of both optical links are taken. Fig. 2 portrays performance of outage for proposed system for threshold SNR taken as 5 dB. From the figure, it is observed degradation in outage of the proposed system is directly proportional to the degrad-

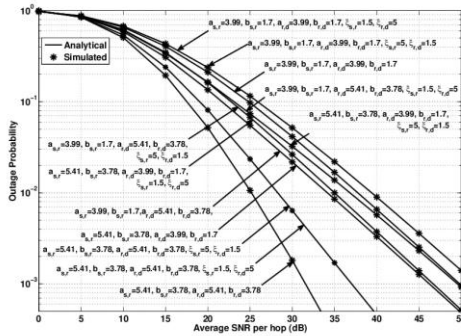


Fig. 2. Probability of outage vs SNR for various irradiance and misalignment scenarios.

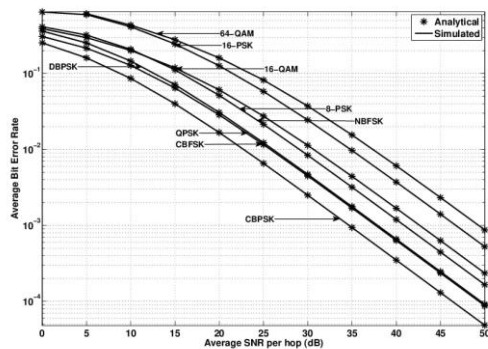


Fig. 3. Avg. BER vs avg. SNR for various modulation schemes at particular channel condition

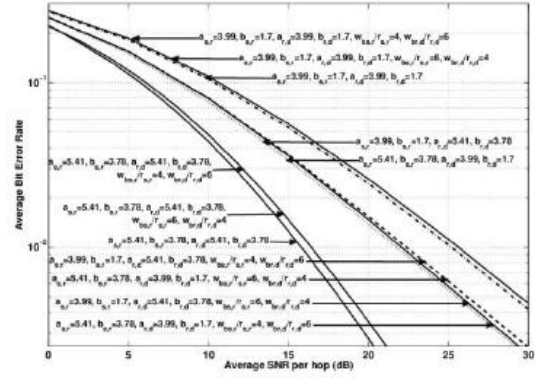


Fig. 4. Av. BER vs. avg. SNR for CBPSK, jitter set to be 1, and various turbulence parameters and different beamwidths.

-ation in the turbulence in the optical links. Introduction of misalignment losses further degrades the outage performance of the system. For ex., considering SNR to be 30dB, and  $a_{s,r}=5.41$ ,  $b_{s,r}=3.78$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$ , the outage probability is  $1.81 \times 10^{-3}$  which rises to  $6.38 \times 10^{-3}$  and  $5.18 \times 10^{-2}$  for  $a_{s,r}=5.41$ ,  $b_{s,r}=3.78$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$ ,  $\xi_{s,r}=5$ ,  $\xi_{r,d}=1.5$  and  $a_{s,r}=3.99$ ,  $b_{s,r}=1.7$ ,  $a_{r,d}=3.99$ ,  $b_{r,d}=1.7$ , respectively. From fig. 2, it can also be inferred that the following conditions will have same outage performance: (i) strong turbulence, weak misalignment losses in S-R link with weak turbulence, strong misalignment losses in RD link (S-W-W-S) and weak turbulence, strong misalignment losses in S-R link with strong turbulence, weak misalignment losses in R-D link (W-S-W-S), (ii) (S-S-W-W) and (W-W-S-S), and (iii) high irradiance in first hop and weak irradiance in second hop and vice-versa. The performance of the system with weak turbulence, strong misalignment losses in one link and strong turbulence, weak misalignment losses in another link will perform better than weak turbulence, weak misalignment losses in one link and strong turbulence

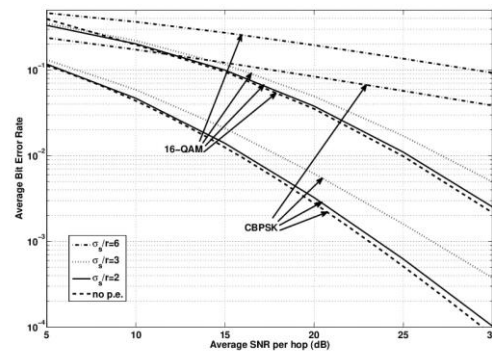


Fig. 5. Avg BER vs. avg SNR for CBPSK and 16-QAM and beamwidths,  $w_{b,s,r} = w_{b,r,d} = 10$



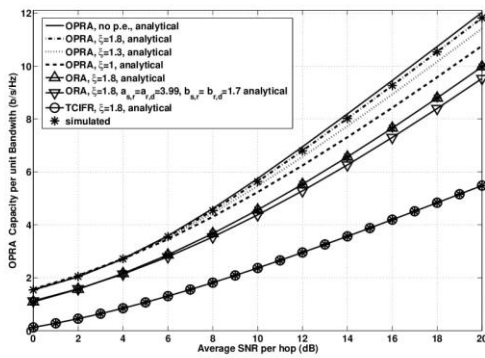


Fig. 6. Avg. capacity vs avg. SNR for  $a_{s,r}=3.99$ ,  $b_{s,r}=1.7$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$ ,  $\xi_{s,r}=\xi_{r,d}=\xi=\{1, 1.3, 1.8\}$  and different adaptive modulation techniques.

pointing errors in one link and strong turbulence, strong misalignment losses in another link. All these observations result because in DF systems, performance gets limited by the weak performing hop.

In Fig. 3 shows BER performance for  $a_{s,r}=5.41$ ,  $b_{s,r}=3.78$ ,  $a_{r,d}=3.99$ ,  $b_{r,d}=1.7$ ,  $\xi_{s,r}=1.5$ ,  $\xi_{r,d}=5$  and all modulation schemes are considered. It is observed CBPSK has the best BER performance as compared to the other modulation techniques. CBFSK and QPSK have similar and same goes for the BER performance of NBFSK and 8PSK. From the figure it is seen that the CBPSK outperforms DBPSK, CBFSK outperforms NBFSK, and 16-QAM outperforms 16-PSK, which are in accordance with all the expected outcomes. In fig. 4, BER for CBPSK is performed, considering different beam width values and fading scenarios but constant jitter value of 1. Figure shows that stronger the impact of turbulence and pointing error, significant will be the deterioration of error performance. Considering SNR = 20dB, for  $a_{s,r}=5.41$ ,  $b_{s,r}=3.78$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$  BER  $2.79 \times 10^{-3}$  and it increases to  $3.34 \times 10^{-3}$  and  $2.44 \times 10^{-2}$  for  $a_{s,r}=5.41$ ,  $b_{s,r}=3.78$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$ ,  $w_{b_{s,r}}=6$ ,  $w_{b_{r,d}}=4$ ,  $a_{s,r}$

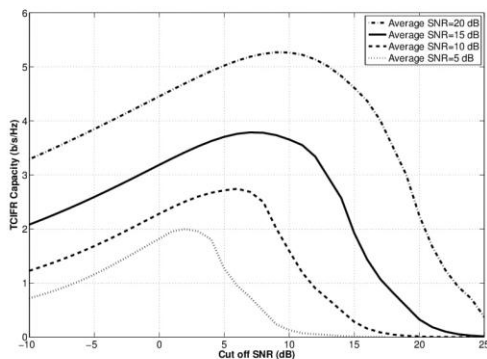


Fig. 7. Avg. capacity vs. threshold SNR for  $a_{s,r}=3.99$ ,  $b_{s,r}=1.7$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$ ,  $\xi_{s,r}=\xi_{r,d}=2.5$

$=3.99$ ,  $b_{s,r}=1.7$ ,  $a_{r,d}=3.99$ ,  $b_{r,d}=1.7$  respectively. When beam width was considered smaller,  $\xi$  was smaller, corresponding to severe impact of misalignment losses, resulting in degraded error performance. Also BER performance patterns observed here are similar to the patterns observed for outage performance in Fig. 2.

In Fig. 5, BER analysis graphs are shown for 16-QAM and CBPSK, varying jitter parameter, keeping fixed fading parameters,  $a_{s,r}=5.41$ ,  $b_{s,r}=3.78$ ,  $a_{r,d}=3.99$ ,  $b_{r,d}=1.7$ ,  $\xi_{s,r}=1.5$ ,  $\xi_{r,d}=5$  and beam widths,  $w_{b_{s,r}}=w_{b_{r,d}}=10$ . Figure shows that increasing jitter, decreases  $\xi$  which accounts to strong effect of misalignment losses and thus leads to poorer BER performance. In plot of CBPSK with no p.e., for SNR = 25 dB, error probability rises from  $3.582 \times 10^{-4}$  to  $6.22 \times 10^{-4}$ ,  $1.60 \times 10^{-3}$ , and  $5.71 \times 10^{-2}$ , when jitter values were 2, 3, and 6, resp. Figure also presents that for lower modulation scheme, misalignment losses severs error performance more than in higher order ones.

Fig. 6 presents average capacity of the proposed system for  $a_{s,r}=\{3.99, 3.99\}$ ,  $b_{s,r}=\{1.7, 1.7\}$ ,  $a_{r,d}=\{5.41, 3.99\}$ ,  $b_{r,d}=\{3.78, 1.7\}$ ,  $\xi_{s,r}=\xi_{r,d}=\xi=\{1, 1.3, 1.8\}$  and various adaptive modulation schemes, e.g., ORA, OPRA, and TCIFR. Following conclusions can be drawn from the figure, (i) higher the scintillations and misalignment losses, poorer is the average capacity and (ii) the capacity of the considered serial FSO system follows decreasing order, i.e., OPRA scheme, ORA scheme, TCIFR scheme. As is observed from Fig. 6, for 18 dB SNR, for  $a_{s,r}=3.99$ ,  $b_{s,r}=1.7$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$ ,  $\xi=1.8$ , we get capacities as 11.82, 9.97, and 5.48, for OPRA, ORA, and TCIFR, respectively.

Fig. 7 shows capacity under truncated channel inversion scheme for  $a_{s,r}=3.99$ ,  $b_{s,r}=1.7$ ,  $a_{r,d}=5.41$ ,  $b_{r,d}=3.78$ ,  $\xi=2.5$ , and various avg. SNR values. For certain value of irradiance and misalignment loss, cut-off SNR is achieved where capacity has maximum value. This cut-off SNR value is directly proportional to the value of the avg. SNR

## VI. CONCLUSION

This paper has presented two-hop serial FSO communication system. Using system SNR statistics, probability of outage, information theoretic analysis, and BER comparison for proposed system is made for ramifications of channel conditions, misalignment losses, and different system parameters.

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## AUTHORS PROFILE



Sanya Anees (M'16) received Masters (W/D) in Communication Engineering from the University of Manchester, U.K., in 2011 and received Ph.D. from the Indian Institute of Teaching Delhi, in 2016. Her research interests include optical wireless communications, MIMO systems, and cooperative communications.

Currently, she is working as an Assistant Professor in the Department of Electronics and Communication Engineering, Indian Institute of Information Technology Guwahati, Assam, India. She is reviewer of IEEE Transactions on Communications, IEEE Transactions on Wireless Communications, IEEE Transactions on Aerospace and Electronic Systems, IEEE Access, IET Communications, and has reviewed various IEEE conference papers such as, ICC, Globecom, VTC, and WCNC. She has also served as TPC Member of IEEE NCC-2018, IEEE ICSC-2018, and IEEE CICT-2017. She was awarded Early Career Research Award by SERB, DST in 2017. In her Graduation, she has been awarded University Gold medal-2010 for being University Topper in Electronics & Communication Engg. Branch, Shri Rawatpura Sarkar Gold medal-2010 for being University Topper amongst students from Electronics & Communication Engineering, Computer Science, and Information Technology, and Prof. S. T. Chakravati Gold medal-2010 for being University Topper amongst students from Electronics & Communication Engineering, Electrical Engineering, Mechanical Engineering, and Civil Engineering.