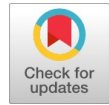


Nonlinear Analysis of Flat Steel Frame Semi rigid Connection Subjected to Dynamic Loads

Hong Son Nguyen, Trung Thanh Pham, Van Quan Tran, Quang Hung Nguyen



Abstract: The paper aims to analyze the plastic frame elasticity with a beam-column joint which is a nonlinear semi-rigid linkage, an ideal plastic elastic material model; set up the elastic elastic equation, using the Newmark method to solve the differential equations of oscillating equations with Newton-Raphson iterative method of improvement and programming to determine the internal force and the movement of the frame by the programming language MATLAB program; limit the scope of research on structural problems subject to earthquake load described by acceleration diagram.

Keywords: elastic nonlinear, flat steel frame, MATLAB, earthquake load.

I. INTRODUCTION

The problem of analyzing the structure of flat bar system made of ideal plastic elastic material has great significance when the structure is subject to earthquake load, because if the model of linear elastic material is considered, it does not reflect the true working of materials [8]. Moreover, for steel frames, considering the semi-rigid connection, it is also necessary to analyze more accurately by using the torque relation characteristic curve and the angle of rotation of the curve as a curve [2]. Thus, when analyzing steel structure subjected to earthquake load, it is necessary to consider at the same time the ideal plastic elastic material and nonlinear semi-rigid linkage. This will reflect the actual behavior of both material and structure compared to conventional analysis (rigid bonding, linear elastic material) with the slender model or floor shear model [6].

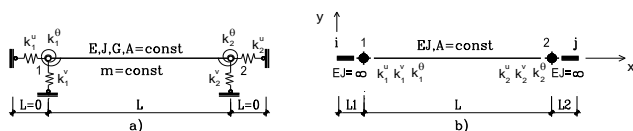


Figure 1. The half-rigid Timoshenko double element bar

Some domestic and foreign authors have used a semi-rigid linkage model with a spring for rotational displacements, including the hard-beam cross-section for analyzing structures with semi-rigid links. The author extends the above link model by considering two more

springs for straight axial displacement and straight-forward displacement perpendicular to the rod axis as shown in Figure 1a, with additional considerations of the column-column intersection hardening as shown in Figure 1 .b [1]. The construction of the stiffness matrix and equivalent mass matrix of the Timoshenko bar element has a semi-rigid link at the two ends, which the author has created can refer to documents [1], [2].

II. THEORY

2.1. Differential equations

When examining the oscillation of the bar structure, the following assumptions are acknowledged:

- Structural work according to planar diagrams, ideal plastic elastic material;
- Resistance to the movement of the system is viscous drag;
- Movement of the ground in horizontal direction as an absolute hard piece with horizontal displacement at all points equally.

The basic equation of the finite element method written for the problem of linear semi-rigid bonding systems subjected to earthquake load, including the external load, takes the form:

$$M\ddot{U} + C\dot{U} + KU = R \quad (1)$$

Inside:

$$K = \sum_{e=1}^n K_e^L = \sum_{e=1}^n L_e^T K_e^L L_e;$$

$$M = \sum_{e=1}^n M_e^L = \sum_{e=1}^n L_e^T M_e^L L_e;$$

$$K_e = T_e^T H_e^T \bar{K}_e^* H_e T_e;$$

$$M_e = T_e^T \bar{M}_e^* T_e,$$

With: \bar{K}_e^* , \bar{M}_e^* - stiffness matrix and equivalent mass matrix of elements bar two semi-rigid ends in the local coordinate system set by the author, can refer to documents [1], [2];

T_e , H_e - the matrix converts coordinates from the local to the global coordinate and the super-element relation matrix with the semi-rigid bar element in the local coordinate system [1], [2];

L_e - Super-element positioning matrix in the matrix of structure;

Manuscript published on 30 August 2019.

*Correspondence Author(s)

Hong Son Nguyen, Hanoi Architectural University, 100000 Ha noi, Vietnam

Trung Thanh Pham, Vietnam Institute for Bulding Science and Technology, Hanoi, Vietnam

Van Quan Tran, University of Transport Technology, 100000 Hanoi, Vietnam

Quang Hung Nguyen, Thuyloi University, Hanoi, Vietnam.

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>.



C -Barrier matrix, assumption is defined as a linear combination of stiffness matrix and mass matrix of structure, determined by relation [6]:

$$\mathbf{C} = \alpha \mathbf{K} + \beta \mathbf{M} \quad (3)$$

The right side of the equation is the external load, the nodal load vector given by the horizontal displacement of the ground soil [6]:

$$\mathbf{R} = -\mathbf{M} \cdot \mathbf{r} \cdot \ddot{\mathbf{u}}_g \quad (4)$$

With: **r** - vector effect coefficient of nodal displacement, denoting displacement at nodes caused by horizontal displacement of the ground by unit,

$\ddot{\mathbf{u}}_g$ - acceleration of horizontal displacement of the ground;

2.2. The matrices of the semi-rigid structure link equations

2.2.1. Function of form of semi-rigid bar element [2]

a) Function of shape along the bar axis.

$$N_1(x) = 1 - k_1^u \bar{k}_{11}^* - \frac{1 - (k_1^u + k_2^u) \bar{k}_{11}^*}{L} x, \quad (5)$$

$$N_4(x) = k_1^u \bar{k}_{44}^* + \frac{1 - (k_1^u + k_2^u) \bar{k}_{44}^*}{L} x, \quad (6)$$

b) Jaw shape in the direction of the axis of the rod.

$$N_2(x) = 1 - k_1^v \bar{k}_{22}^* - k_1^\theta \bar{k}_{23x}^* +$$

$$\left(\frac{k_1^\theta \bar{k}_{23}^* - k_2^\theta \bar{k}_{26}^*}{2L} - \frac{6 - 3L(k_1^\theta \bar{k}_{23}^* + k_2^\theta \bar{k}_{26}^*)}{2L^2(1+\Phi)} + \frac{k_1^v \bar{k}_{22}^* + k_2^v \bar{k}_{25}^*}{4iL(1+\Phi)} \right) x^2 +$$

$$+ \left(\frac{2 - L(k_1^\theta \bar{k}_{23}^* + k_2^\theta \bar{k}_{26}^*)}{L^3(1+\Phi)} - \frac{k_1^v \bar{k}_{22}^* + k_2^v \bar{k}_{25}^*}{6iL^2(1+\Phi)} \right) (x^3 - 0,5\Phi x),$$

$$N_3(x) = -k_1^v \bar{k}_{32}^* + (1 - k_1^\theta \bar{k}_{33}^*) x +$$

$$+ \left(\frac{k_1^\theta \bar{k}_{33}^* - k_2^\theta \bar{k}_{36}^* - 1}{2L} - \frac{3(1 - k_1^\theta \bar{k}_{33}^* - k_2^\theta \bar{k}_{36}^*)}{2L(1+\Phi)} + \frac{k_1^v \bar{k}_{32}^* + k_2^v \bar{k}_{35}^*}{4iL(1+\Phi)} \right) x^2 +$$

$$+ \left(\frac{1 - k_1^\theta \bar{k}_{33}^* - k_2^\theta \bar{k}_{36}^*}{L^2(1+\Phi)} - \frac{k_1^v \bar{k}_{32}^* + k_2^v \bar{k}_{35}^*}{6iL^2(1+\Phi)} \right) (x^3 - 0,5\Phi x),$$

$$N_5(x) = k_1^v \bar{k}_{52}^* - k_1^\theta \bar{k}_{53x}^* +$$

$$+ \left(\frac{k_1^\theta \bar{k}_{53}^* - k_2^\theta \bar{k}_{56}^*}{2L} - \frac{6 + 3L(k_1^\theta \bar{k}_{53}^* + k_2^\theta \bar{k}_{56}^*)}{2L^2(1+\Phi)} - \frac{k_1^v \bar{k}_{52}^* + k_2^v \bar{k}_{55}^*}{4iL(1+\Phi)} \right) x^2 -$$

$$+ \left(\frac{2 + L(k_1^\theta \bar{k}_{53}^* + k_2^\theta \bar{k}_{56}^*)}{L^3(1+\Phi)} + \frac{k_1^v \bar{k}_{52}^* + k_2^v \bar{k}_{55}^*}{6iL^2(1+\Phi)} \right) (x^3 - 0,5\Phi x), \quad (9)$$

$$N_6(x) = k_1^v \bar{k}_{62}^* - k_1^\theta \bar{k}_{63x}^* +$$

$$+ \left(\frac{k_1^\theta \bar{k}_{63}^* - k_2^\theta \bar{k}_{66}^* + 1}{2L} - \frac{3(1 - k_1^\theta \bar{k}_{63}^* - k_2^\theta \bar{k}_{66}^*)}{2L(1+\Phi)} - \frac{k_1^v \bar{k}_{62}^* + k_2^v \bar{k}_{65}^*}{4iL(1+\Phi)} \right) x^2 +$$

$$+ \left(\frac{1 - k_1^\theta \bar{k}_{63}^* - k_2^\theta \bar{k}_{66}^*}{L^2(1+\Phi)} + \frac{k_1^v \bar{k}_{62}^* + k_2^v \bar{k}_{65}^*}{6iL^2(1+\Phi)} \right) (x^3 - 0,5\Phi x).$$

Inside: k_i^u, k_i^v, k_i^θ ($i=1;2$) - softness of straight spring against displacement and rotation position at the first and second ends of the bar;

\bar{k}_{ij}^* ($i,j=1\div 6$) - elements of the stiffness matrix of the bar of two semi-rigid ends in the local coordinate system, can be found in the literature [1].

The form function for the Timoshenko element of the two half-rigid couplings set by the unit displacement method can be found in the literature [2].

2.2.2. Element stiffness matrix bar links half hard.

The stiffness matrix of bar elements has a semi-rigid link of three linear springs built by the unit displacement method, which can be referenced in [1], the elements of this matrix are of the form:

$$\bar{k}_{11}^* = \bar{k}_{44}^* = \frac{EA}{L} \frac{1}{1 + EA(k_1^u + k_2^u)/L};$$

$$\bar{k}_{14}^* = \bar{k}_{41}^* = -\frac{EA}{L} \frac{1}{1 + EA(k_1^u + k_2^u)/L};$$

$$\bar{k}_{22}^* = \bar{k}_{55}^* = \frac{12i}{L^2} \frac{A_1 + A_2}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{23}^* = \bar{k}_{32}^* = \frac{6}{L} \frac{1 + 2ik_2^\theta}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{25}^* = \bar{k}_{52}^* = \frac{12i}{L^2} \frac{-(A_1 + A_2)}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{26}^* = \bar{k}_{62}^* = \frac{6}{L} \frac{1 + 2ik_1^\theta}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{33}^* = \frac{\Phi + 6 + 12ik_2^\theta}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{35}^* = \bar{k}_{53}^* = -\frac{12i}{L} \frac{A_2}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{36}^* = \bar{k}_{63}^* = \frac{-\Phi}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{56}^* = \bar{k}_{65}^* = -\frac{6}{L} \frac{1 + 2ik_1^\theta}{\Phi(A_1 + A_2) + 12iA_1A_2};$$

$$\bar{k}_{66}^* = \frac{\bar{\Phi} + 6 + 12ik_1^0}{\bar{\Phi}(A_1 + A_2) + 12iA_1A_2},$$

Inside : $\bar{\Phi} = \frac{12i}{L^2}(k_1^v + k_2^v) + \frac{12i}{L} \frac{\mu}{GA} - 2$; $A_1 = \frac{1}{2i} + k_1^0$;

$$A_2 = \frac{1}{2i} + k_2^0 ; i = \frac{EJ}{L}$$

2.2.3. Matrix mass element bar links half hard

The mass matrix of linear semi-rigid bar elements in the local coordinate system takes the form:

$$\bar{M}_e^* = [\bar{m}_{ij}^*], (i,j=1\div 6) \tag{12}$$

Inside: \bar{m}_{ij}^* are elements of the equivalent mass matrix of the bar.

For bars with regular cross sections, the above elements are determined according to [7]:

$$\bar{m}_{ij}^* = \rho A \int_0^L N_i(x)N_j(x)dx, \tag{13}$$

Inside: ρ , A respectively material density and element area area.

Performing integrals (13) with Mathematica math software [9] with $N_i(x)$ and $N_j(x)$ are form functions defined from (5) to (10), we get the elements of mass matrix. For example, the element of this matrix with zero displacement straight ($k^u=k^v=0$) takes the form:

$$\begin{aligned} \bar{m}_{22}^* = & +\rho AL \frac{78 + 6i(82k_1^0 + 117k_2^0) + 24i^2[68k_2^0k_2^0 + 4k_1^0k_2^0(41 + 84ik_2^0)]}{210[1 + 4i(k_1^0 + k_2^0) + 12i^2k_1^0k_2^0 + \Phi(1 + ik_1^0 + ik_2^0)]^2} + \\ & +\rho AL \frac{72i^2k_1^0k_1^0[11 + 7ik_2^0(11 + 20ik_2^0)]}{210[1 + 4i(k_1^0 + k_2^0) + 12i^2k_1^0k_2^0 + \Phi(1 + ik_1^0 + ik_2^0)]^2} + \\ & +\rho AL \frac{21\Phi(7 + 22ik_1^0 + 32ik_2^0 + 80i^2k_1^0k_2^0)(1 + ik_1^0 + ik_2^0) + 70\Phi^2(1 + ik_1^0 + ik_2^0)^2}{210[1 + 4i(k_1^0 + k_2^0) + 12i^2k_1^0k_2^0 + \Phi(1 + ik_1^0 + ik_2^0)]^2} \end{aligned}$$

2.3. Method of solving equations of half-rigid bonding motion equations

For linear systems with semi-rigid links, motion equations (1) are linear differential equations. The author used the direct integration method according to Newmark [4.7] to determine the displacement vector of the system. Since then determine the internal force in the bar elements.

In the case of nonlinear semi-rigid link system (considering the torque-angle relationship) after being linearized into a binary or a linear triangle with a flowing shelf [1] taking into account the plasticity of the material, the equation (1) is a nonlinear differential equation, can combine Newmark method and Newton-Raphson iterative method to improve [4].

2.4. A mathematical sign of the link state at either end of a bar element

The problem of nonlinear semi-rigidity refers to the plasticity of the material, the softness coefficients of the link change during the iteration process, depending on the internal force state at the two ends of the bar. After each iteration, update the stiffness matrix and mass matrix when the internal

force is determined at both ends 1 and 2 of the element. Check that the bar link state has a softness of $k^{(1)}$ or $k^{(2)}$ or $k^{(3)}$ through the torque at the top of the rod (M_k) compared to the elastic moment Me and the maximum torque Mu of the link and expressed through the following mathematical expression:

If $|M_k| \leq M_e$ then the half-hard link has a softness of $k^{(1)}$,

If $M_e < |M_k| \leq M_u$ then the half-hard link has a softness of $k^{(2)}$,

If $|M_k| > M_u$ then the half-hard link has a softness of $k^{(3)}$,

Inside: M_k - momen at the top of the bar (M_1, M_2);

The yield condition of the cross section in the bending plane is taken [8]:

$$\frac{|M|}{M_o} = 1 ; \text{ when } 0 \leq \frac{|N|}{N_o} \leq 0,15.$$

$$\text{and } \frac{|M|}{M_o} + 1,18 \left(\frac{|N|}{N_o} - 1 \right) = 0 ; \text{ when } 0,15 < \frac{|N|}{N_o} \leq 1. \tag{17}$$

2.5. Convergence standards

Convergence criteria consider simultaneous displacement of the node and force using the energy-repeating standard on the public change of residual force [4]:

$$\frac{\Delta U^{(i)T}(\mathbf{R}_{t+\Delta t} - \mathbf{F}_{t+\Delta t}^{(i-1)} - \mathbf{M}\ddot{U}_{t+\Delta t}^{(i-1)})}{(\mathbf{U}_{t+\Delta t}^{(i)} - \mathbf{U}_t)^T(\mathbf{R}_{t+\Delta t} - \mathbf{F}_t - \mathbf{M}\ddot{U}_t)} \leq \varepsilon_e \tag{18}$$

Inside ε_e is the accuracy required for the public change of the excess force, this value is taken according to requirements, in the problem in the numerical example below $\varepsilon_e = 10^{-6}$.

2.6. Button internal force of the element

The internal force vector of point $(\mathbf{F}_{t+\Delta t}^{(i)})_e$ at iteration (i) is determined by the formula:

$$(\mathbf{F}_{t+\Delta t}^{(i)})_e = (\mathbf{F}_{t+\Delta t}^{(i-1)})_e + \mathbf{K}_t^e \Delta U^{(i)}, \tag{19}$$

Inside: \mathbf{K}_t^e - element stiffness matrix at time (t);

$\mathbf{F}_{t+\Delta t}^{(i-1)}$ - the nodal force of the element at time (t + Δt) in the second iteration (i-1);

$\Delta U^{(i)}$ - vector displacement node in the second iteration (i).

III. EXAMPLE BY NUMBER

Based on the calculation and algorithm steps as presented, the author has built DASF (*Dynamic Analysis of Steel Frame*) program to analyze the structure of non-linear semi-rigid steel frame affected by earthquakes. to plasticity of materials in MATLAB language.



Nonlinear analysis of flat steel frame semi rigid connection subjected to dynamic loads

Consider the following example: Analysis of a two-stage, semi-rigid steel-frame plastic case, as shown in Figure 2a; affected by Lan Cang earthquake load level 8 described by acceleration diagram [3]; beam-column binding characteristic lines, at nodes 9, 11, 12 and 14 as shown in Figure 2b and column legs at nodes 1 and 5 as Figure 2c [5]; softness for straight displacement zero; critical drag coefficient $\xi_1=\xi_2=0,05$; evenly distributed mass per meter long $\bar{m}=0,5\text{kNs}^2/\text{m/md}$ and concentrated mass $\bar{M}=2,0\text{kNs}^2/\text{m}$; column section with wing plate size (200x8) mm, abdominal plate size (200x6) mm; cross section of beams with wing plate size (200x8)

mm, abdominal plate size (400x6) mm; Steel has elastic modulus $E=2.1.105\text{MPa}$, Poisson's coefficient $\nu=0,3$, $f_y=248\text{MPa}$, ideal plastic elastic material model.

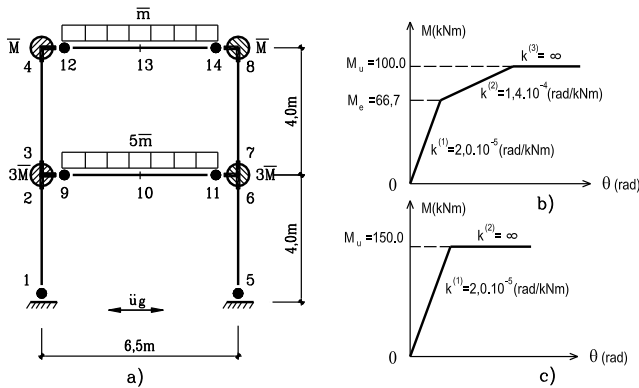


Figure 2. Two-tiered frame diagram, one span and link characteristic

The largest displacement and dynamic internal force results (including resistance), the ideal plastic elastic material with the linkage case is linear or nonlinear is shown in Table 1.

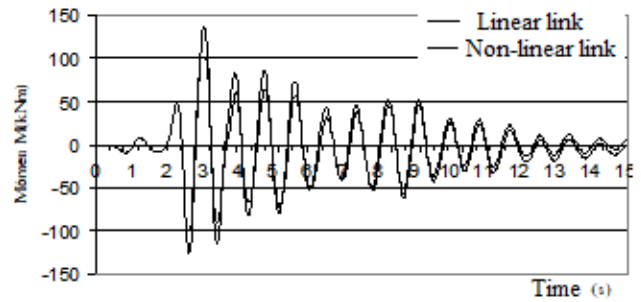


Figure 3. Diagram of bending moment button No. 9, Timoshenko bar

Table 1. Results of calculating the dynamic moment M, the largest displacement $\Delta_{4\max}$ (mm)

Combination		Bar Timoshenko, link			Bar Euler-Bernoulli, link		
Result	Node	Non-linear	Linear	Difference(%)	Non-linear	Linear	Difference(%)
Mômen động (kNm)	1	97.4144	97.4144	0.000	97.4144	97.4144	0.000
	2	97.4144	87.6607	10.013	97.4144	87.9425	9.723
	3	21.5667	20.0029	7.251	24.9185	20.3668	18.266
	4	39.9133	41.0102	-2.748	43.6674	42.1258	3.530
	5	97.4144	97.4144	0.000	97.4144	97.4144	0.000
	6	97.4144	87.6607	10.013	97.4144	87.9425	9.723
	7	21.5667	20.0029	7.251	24.9185	20.3668	18.266
	8	39.9133	41.0102	-2.748	43.6674	42.1258	3.530
	9	128.5994	100.0000	22.239	133.1458	100.0000	24.894
	10	0.0000	0.0001	0.0000	0.0000	0.0001	0.0000
	11	128.5994	100.0000	22.239	133.1458	100.0000	24.894
	12	42.0230	42.8265	-1.912	46.0506	44.0252	4.398
	13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	14	42.0230	42.8265	-1.912	46.0506	44.0252	4.398
Δ_4	13	97.4144	97.4144	0.000	97.4144	97.4144	0.000

Combination	Bar Timoshenko, link	Bar Euler-Bernoulli, link
Note: Dynamic value is determined at the position of the link (bar with hard button).		

IV. CONCLUSION

DASF software was created by the author to analyze the plastic structure of non-linear semi-rigid steel frame, affected by earthquakes described by acceleration diagram;

- This software has overcome the disadvantages of nonlinear structural analysis problem that some popular software like SAP2000 and ETAB have not mentioned;

Nonlinearity of bonding and plasticity of structural materials has a great influence on the displacement state and internal force of the system.

REFERENCES

1. Nguyen Hong Son, Nguyen Tram, "Analysis of plastic structure of non-linear semi-rigid flat steel frame under static load", Proceedings of the national conference on mechanical mechanics of deformed solid material for the eighth time, Thai Nguyen, August 2006.
2. Nguyen Hong Son, Vu Thanh Hai, "Building the equivalent mass matrix of bar elements with elastic mass distribution distribution", Journal of Irrigation and Environment, No. 13, 6/2006, Water Resources University, page 31-34. (2006).
3. Vietnamese design standard TCXDVN, design of shock-resistant construction, (Draft 2000 and 2005)
4. Bathe K.J. Finite Element Procedures, Prentice-Hall International, Inc. (1996)
5. CEN: Eurocode 3, (1992), Design of Steel Structures-Part I-General-Rules and Rules for Building, CEN ENV. (1992)
6. Chopra A. K., Dynamics of Structures, Theory and Application to Earthquake Engineering, Prentice Hall, Upper Saddle River, New Jersey 07458. (1985)
7. Clough R.W., Penzien J. Dynamic of Structures, McGraw-Hill Book Co, New York. (1993)
8. Bruneau M., Uang C., Whittaker A. Ductile Design of Steel Structures, McGraw Hill, New York. (1998)
9. Wolfram Research. Mathematica 5.1, Inc., 100 Trade Center Drive, Champaign, IL, 61820-7237, USA. (2004)