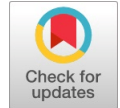


Application of Monte Carlo Method in Hydraulic Structures Safety Assessment in Vietnam Conditions



Lan Huong Nguyen, Quang Hung Nguyen

Abstract: Vietnam is a country with many dams but most of these works are designed according to the Determined design method and analysis of safety factors, while the Random design method and the reliability analysis have become popular in the world for many years. The paper introduces the method of calculating reliability for hydraulic structures according to the Monte Carlo method or the level III random design method. Based on the method of calculating the reliability of dams according to the Monte Carlo, the authors applied to calculate the reliability of the main dam of Phu Ninh - Quang Nam irrigation reservoirs. Compare with the reliability calculations at level II and have recommendations applied to all levels of construction in Vietnam conditions: with important dams will be calculated according to the problem of level III, with less important works of regions, calculated according to level II.

Keywords: Monte Carlo method, failure probability, random variable, confidence function, water overtopping mechanism.

I. INTRODUCTION

By the end of 2017, Vietnam has about 6886 irrigation reservoirs and hydropower plants, of which most dams for irrigation reservoirs are earth dams [1], [2]. Dams are mainly designed according to the deterministic method, in which the safety indicators used for evaluation are safety factors. The safety level of the systems is assessed through problems of hydraulics, stability and durability, in which the specifications of works are simulated through the ability to remove flood water, load capacity of the public, but the influence of random variables and construction components on systems have not been considered. Currently, many advanced countries in the world: Russia, China, Japan and some European countries have used random design methods (level II, level III) and standards of reliability to evaluate confidence of works. The problems of reliability calculation at level II: using the central limit theorem of probability theory with the destructive function is a linear function; random variables that are not necessarily the same and independent, but each of these variables does not play a large role in the function. Calculation of dam safety probabilities has many cases that are inconsistent with this assumption. In confidence functions, there are random variables that cannot be considered as independent and distributed variables that

are not normal distributions. Moreover, some confidence functions Z are nonlinear functions, so it is difficult to find probability distribution law of Z . Probability statement of function Z and also cannot consider Z as a function of many random variables with normal distribution. In addition, the monitoring status as well as the statistics of reservoirs still have many shortcomings, many reservoirs do not have monitoring equipment or have but perform intermittent measurements. Analysis of monitoring data of some lakes with many monitoring data such as Hoa Binh, Vinh Son, Phu Ninh, Tri An and Yen Lap shows that most of the observation lists of the data are not long enough to meet the level exactly according to the statistical problem. In addition, the results of statistical analysis of water level monitoring data, water column permeability and displacement in 2 directions (vertical and horizontal) in the 5 above mentioned reservoirs show that these variables hardly follow the standard distribution law. Therefore, the application of reliability theory to implement probability problems at level II for dam safety calculations in Vietnam conditions still have many limitations, it is necessary to continue research. Calculations at level III are to find the reliability of works in these cases, and problems can use analytical method or Monte Carlo method. In principle, it is possible to calculate the safety probability by analytic method, but very limited because this method has many difficulties in determining the combined probability density function of random variables. Therefore, in the calculation of level III, Monte Carlo method is often used to determine safety probability [2]. Therefore, the study introduced how to calculate the reliability of earth dams according to the approach of Monte Carlo method. Use the Monte Carlo method to solve the relationship function of a failure mechanism (Z) by creating N times the random variables of the Z function according to probability distribution law of random variables, thereby finding the N values of Z function, then use the statistical tool to find the failure probability or the safety probability of works. Calculations are applied to the main dam of Phu Ninh reservoir - Quang Nam province, the reliability of Water overtopping mechanism is calculated according to the level II reliability theory and Monte Carlo method.

II. THEORY

A. Establishment of Confidence Function of Various Problems

Confidence Function F would be established for each of destructive mechanisms to represent the relationship between load bearing and load on the works: [2-3].

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$$Z = R(x_i) - N(y_j) \quad (1)$$

In which:

$N(y_i) = N(y_1, y_2, \dots, y_n)$ is load function.

$R(x_i) = R(x_1, x_2, \dots, x_n)$ is load bearing function.

x_i : basic random variables, including impacts caused by water environment, soils and stones environment, works environment that constitute resistance against destruction for the works.

y_i : basic random variables, including impacts caused by water environment, effects of ground environment through soil and stone properties as well as various loads of works environment through properties of construction materials.

$x_i, y_i, R(x_i), N(y_i), Z$ are random variables and random functions which probability distribution law shall be determined in B.

B. Check of probability distribution law of random variables

Using square standards (χ^2) for checking the probability distribution law of random variables [2,3].

- It's assumed that (H_0) is probability distribution law $F(x)$ of random variables.

- Parameters of probability density function are determined with moment method or maximum estimation, number of parameters is s .

- Number of intervals: k ($6 \leq k \leq 20$), width of interval d :

$$d = \frac{(x_{\max} - x_{\min}(0))}{k} \quad (2)$$

In which: X_{\max} – Maximum value of observed data series n of random variables;

X_{\min} – Minimum value of observed data series n of random variables.

- Counting the number of observations of each interval H .

- Calculating probability p_j of interval j (a_{j-1}, a_j), $j = 1 \rightarrow k$

$$p_j = F(x = a_j) - F(x = a_{j-1}) \quad (3)$$

- Calculating value U :

$$U = \sum_{j=1}^k \frac{(H_j - n \cdot p_j)^2}{n \cdot p_j} \quad (4)$$

- Looking up the table when squaring (χ^2) for determination

of value: $\chi_{f,q}^2$;

In which:

$$f = k - s - 1; q = 1 - \alpha \quad (4)$$

with

α : Incorrect probability;

q : Level of guarantee.

- Checking condition:

$$U \geq \chi_{f,q}^2 \quad (6)$$

If condition (6) is satisfactory: the assumption H_0 is rejected, i.e., the random variable does not meet the selected probability distribution law [2], [3].

If condition (6) is not satisfactory: The random variable with theoretical probability distribution law $F(x)$ which incorrect probability α or level of guarantee q [2,3].

C. Presenting random variables according to probability distribution law

Using Monte Carlo method to create many random variables (ξ) of confidence function Z according to probability distribution law of those variables. Random variable ξ is created in the inverse ($F^{-1}(R)$) of $F(x)$. [4,5].

$$\xi = F^{-1}(R) \quad (7)$$

In which: R is random number with even distribution in the interval (0,1). there are many methods to create R , some Some programming languages (C, Pascal, ...) were established to create functions of random variables R , with excel, $R(x)$ is used to create random number.

Because the probability distribution law and the statistical characteristics of random variables allow the extension of observed data to that random variable, the study mentioned the extension of some probability distribution laws that are common for irrigation reservoir headworks in Vietnam: standard distribution law, standard logarithms, exponent, Rayleigh, Weibull and gamma.

1) Random variables law:

(a) $X(X_1, X_2, X_3, \dots, X_n)$ follow standard distribution law:

$N(\mu_X, \sigma_X^2)$ which expectation is $EX = \mu_X$, variance $DX = \sigma_X^2$.

In which:

$(X_1, X_2, X_3, \dots, X_n)$ is n observations of random variable X

Function of standard distribution density is [2,4-5]:

$$f(x) = \frac{1}{\sigma_X \cdot \sqrt{2\pi}} \cdot e^{-\frac{(x-\mu_X)^2}{2\sigma_X^2}} \quad (8)$$

Random variable X :

$$\xi = \sigma_X \cdot (R_1 + R_2 + \dots + R_{12} - 6) + \mu_X \quad (9)$$

In which:

Expectation of random variable X :

$$\mu_X = \bar{X} = \frac{1}{n} \cdot \sum_{i=1}^n X_i \quad (10)$$

Standard deviation of random variables X :

$$\sigma_X = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (X_i - \bar{X})^2} \quad (11)$$

(b) Random variables $X(X_1, X_2, X_3, \dots, X_n)$ follow standard logarithmic distribution law

Density function is [4]:



$$f(x) = \frac{1}{x \cdot \sigma_Y \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\ln x - \mu_Y)^2}{2 \cdot \sigma_Y^2}} \quad (12)$$

- Changing n observations of random variable X into n observations of random variable Y:

$$\ln X_i = Y_i \quad (13)$$

- Calculating expectation of Y:

$$\mu_Y = \bar{Y} = \frac{1}{n} \cdot \sum_{i=1}^n Y_i \quad (14)$$

- Calculating standard deviation Y:

$$\sigma_Y = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (Y_i - \bar{Y})^2} \quad (15)$$

Presenting random variable Y according to (9) then re-calculating the real value of variable X with formula (13).

(c) Random variables $X(X_1, X_2, X_3, \dots, X_n)$ follow Weibull distribution law

Probability density function of Weibull distribution [4]:

$$f(x) = \frac{c}{b} \cdot \left(\frac{x-a}{b}\right)^{c-1} \cdot \exp\left[-\left(\frac{x-a}{b}\right)^c\right] \quad (16)$$

with $a \leq x \leq \infty; b, c > 0$

In which, parameters: a – Location parameter; b – Ratio parameter; c – Shape parameter determined by linear regression method.

Case 1: $a = 0; c = 1$ Weibull function changes into exponent distribution

- Probability density function of exponent distribution [4]:

$$f(x) = \lambda \cdot e^{-\lambda x} \quad (x > 0, \lambda > 0) \quad (17)$$

- In which: Parameter of function is λ :

$$\lambda = \frac{1}{\bar{X}} \quad (18); \bar{X} \text{ is calculated as (9)}$$

- Random variable X:

$$\xi = \frac{1}{\lambda} \cdot |\ln R| = \bar{X} \cdot |\ln R| \quad (18)$$

Case 2: $a = 0; c = 2$ Weibull function changes into Rayleigh distribution.

Density function of Raileigh distribution [4]:

$$f(x) = \lambda \cdot c \cdot x^{c-1} \cdot e^{-\lambda x^c} \quad (\lambda, c, x > 0) \quad (19)$$

Random variables:

$$\xi = \left(-\frac{1}{\lambda} \cdot \ln R\right)^{\frac{1}{c}} = \left(-\left(\frac{\bar{X}}{0,89622}\right)^2 \cdot \ln R\right)^{\frac{1}{2}} \quad (20)$$

Case 3: $c > 2$, Weibull function is as (16)

Random variables:

$$\xi = a + (-b^c \cdot \ln R)^{\frac{1}{c}} \quad (21)$$

4. Random variables follow gamma distribution law

Probability density function f(x) [2], [4]:

$$f(x) = \frac{b^c}{\Gamma(c)} \cdot (x - a)^{c-1} \cdot \exp[-b \cdot (x - a)]$$

$$\text{with } a \leq x \leq \infty; b, c > 0 \quad (22)$$

In which:

parameters: a – Location parameter;

b – Ratio parameter;

c – Shape parameter determined by moment method.

Gama function:

$$\Gamma(c) = (c - 1)! \quad (23)$$

Random variables:

$$\xi = a + \theta \cdot \left(\rho - \sum_{i=1}^{[k]} \ln R\right) = a + \frac{1}{b} \cdot \left(\rho - \sum_{i=1}^{[c]} \ln R\right) \quad (24)$$

In which:

[c]- Integere part c;

R - Random numbers are evenly distributed in the range (0, 1);

ρ - Random numbers are found by the following algorithm:

Algorithm for finding ρ [4]

Step 1: Cho $m = 1$.

Step 2: Random number $R_{3m-2}, R_{3m-1}, R_{3m}$ (lần 1: R_1, R_2, R_3).

Step 3: Calculating $v = \frac{\delta}{e + \delta}$; in which: δ – fractional part of c.

$$\text{If } R_{3m-2} \leq v, \rho_m = R_{3m-1}^{\frac{1}{\delta}} \text{ and } \eta_m = R_{3m} \cdot \rho_m^{\delta-1}$$

$$\text{If } R_{3m-2} > v, \rho_m = 1 - \ln R_{3m-1} \text{ and } \eta_m = R_{3m} \cdot e^{-\rho_m}$$

Step 4: If $\eta_m > \rho_m^{\delta-1} \cdot e^{-\rho_m}$, increasing m and re-doing step1 to 4;

$$\text{If } \eta_m \leq \rho_m^{\delta-1} \cdot e^{-\rho_m}, \rho = \rho_m$$

D. Confidence of works

Calculating values of confidence function Z according to (1). In order to have stable and converged results, the number of simulations (N) on the confidence function is calculated according to (25) [5]

$$N = \frac{1}{\alpha_{\mu}^2}; N = \frac{2 + \alpha_{\sigma}^2}{\alpha_{\sigma}^2} \quad (25)$$

In which: (α_{μ}) : Relative error of mathematical expectation on the number of tests N; (α_{σ}) : Relative error of variance on the number of tests N.

Safety probability (confidence) [3]:

$$P_{at} = \frac{m}{N} \quad (26)$$

Probability of problem [3],[5]:

$$P_{sc} = 1 - \frac{m}{N} \quad (27)$$

III. RESULTS AND DISCUSSION

The study is going to calculate the confidence of overtopping according to the theory of confidence level II and the Monte Carlo method (level III) under different water levels in the reservoir. Calculations on the confidence of the main dam at level II have identified the mechanism of problem that is likely to cause the greatest impact on the dam failure to be overtopping, thus it's going to calculate the confidence of the main dam by level III when the dam is considered of problem which main cause is overflow [6], [7].

A. Introductions to main dam of Phu Ninh Irrigation Scheme

Phu Ninh reservoir is of grade I, built on Tam Ky river in Tam Ky city, Quang Nam province. Phu Ninh reservoir is responsible for water supply, power generation, aquaculture, ecotourism and flood reduction for the downstream Tam Ky. The main works of the reservoir headworks include 1 main dam, 5 secondary dams, 3 spillways, 3 culverts and hydropower plant. The reservoir's capacity is $500.10^6 m^3$, that provides water for irrigation of 23000ha agricultural lands in Quang Nam province; supplies water for industrial and domestic purposes at about $1.6m^3/s$; supplies water for hydropower plant at 2000kW; supplies water for aquaculture and ecological tourism. Particularly in flood season, the reservoir also involves in flood retention, it's approximately 34.5% flood volume, reducing significantly floods for Tam Ky city and downstream areas. Main dam is Homogenous fill with a Height of dam 40,1m; Length of dam crest 620m; Dam crest elevation +37,5; Height of parapet 1,27m and Width of dam crest 6m [6], [7].

B. Water overtopping Problem

1) Water overtopping mechanism

In order to prevent overflow during the reservoir's operation, the crest of dam (Y_d) shall be always higher than «the highest calculated water level in the reservoir» (Y_{in}). This water level is calculated upon the statistic water level (Y_{mn}) plus height of backwater (h_d) and height of wave (h_{si}). Thus the elevation of the dam crest is considered as the load bearing, while the "highest calculated water level in the reservoir" is considered the load function. Load and load-bearing variables are shown in Fig. 1 [6], [8].

The function of the relationship between the load and the load bearing of overflow mechanism is confidence function Z of overflow mechanism. (28).

$$Z = Y_d - (Y_{mn} + h_d + h_{si}) \\ = Y_d - \left(Y_{mn} + \frac{2 \cdot 10^{-6} \cdot V^2 \cdot D}{g \cdot (Y_{mn} - Y_o)} + h_{si} \right) \quad (28)$$

In Z: h_d is calculated according to (29), h_{si} is calculated according to (30)

$$h_d = 2 \cdot 10^{-6} \cdot \frac{V^2 \cdot D}{g \cdot (Y_{mn} - Y_o) \cdot \cos \alpha} = \frac{V^2 \cdot D}{g \cdot H \cdot \cos \alpha} \quad (29)$$

$$h_{si} = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_\alpha \cdot h_{s1\%} = K \cdot h_{s1\%} \quad (30)$$

In which:

3) Calculation results

H: depth of water head in front of the dam;

$g = 9.81 \left(\frac{m^2}{s} \right)$: gravity.

K: is the coefficient depending on the relative roughness, properties of reinforcing materials the of dam slope, wind speed and upstream slope; h : height of wave which probability is 1%.

Y_d : is determined with dam crest settlement monitoring data.

Y_{mn} : is determined with reservoir water level monitoring data in many years.

Calculating probability of overflow: P_{sc}

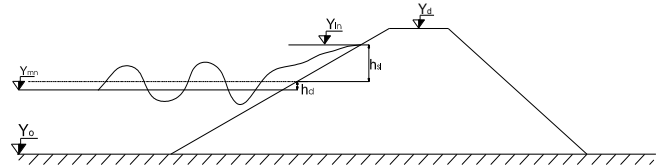


Fig 1. Water overtopping mechanism

2) Calculation input data

The number of observations of random variables in Table 2, is analyzed with statistic data, monitoring records and surveys in existing works. Of the 4 random variables in the Z_1 function, there are only two random variables: Y_d and Y_{mn} have direct monitoring data, so the probability distribution law and the statistical characteristics of these two variables will be calculated directly from statistic data of the reservoir [6].

h_{si} is calculated with random variables (Y_{mn} , m , D , Y_o , Δ) with formula (30).

h_d is calculated with random variables (Y_{mn} , D , Y_o , α^o , V) with formula (29), in which h_d is function of random variable: V .

Other variables are calculated in average.

Tab 2: Observations of random variables [7].

No	Name of variables	Symbol of variables	Number of observations of random variables
1	Dam crest	Y_d (m)	15
2	Reservoir water level	Y_{mn} (m)	29
3	Wind velocity	V (m/s)	15
4	Upstream slope	m	12
5	Wave momentum	D (m)	15
6	Elevation of dam bed	Y_o (m)	10
7	Roughness of stones of upstream slope protection works	Δ m	17
8	The angle of the wind direction is perpendicular to the works alignment.	α^o (degree)	13

The probability distribution law and the statistical characteristics of random variables in function Z are shown in Table 3
Safety probability and problem probability of Water overtopping mechanism are calculated as shown in Table 4
The calculated probability of Water overtopping mechanism according to level II and Monte Carlo method are shown in Fig.

2

Tab 3: Probability distribution law and statistical characteristics of the random variables in the function Z of Phu Ninh main dam

No	Variable	Symbol	Unit	Expectation: μ	Standard deviation σ	Probability distribution law	Parameter of probability distribution function		
							a	b	c
1	Wind velocity	V	m/s	24.48	72.764	gamma	8,169	0,308	5,026
2	Reservoir water level	Y_{mn}	m	32,50	1,631	weibull	0	32,8	14,320
3	Height of wave	h_{sl}	m	25.674	0.2021	Standard logarithm	-	-	-
4	Dam crest	Y_d	m	37,5	0,2	Standard	-	-	-
5	Wave momentum	D	m	3050	-	deterministic	-	-	-
6	Depth of water head in front of the dam	H	m	31.7	-	deterministic	-	-	-
7	Gravity	g	m/s ²	9.81	-	deterministic	-	-	-

Tab 4: Calculated probability of problem and probability of safety against overflow Phú Ninh main dam according to Monte Carlo

TT	m	N	P_{sc}	P_{at}	$\alpha_{\mu}\%$	$\alpha_{\sigma}\%$
1	2	100	0,0100	0,9900	10,00	14,21
2	9	500	0,0180	0,9820	4,47	6,33
3	23	1000	0,0180	0,9820	3,16	4,47
4	125	5001	0,0216	0,9784	1,41	2,00
5	257	10000	0,0205	0,9795	1,00	1,41
6	468	20001	0,0214	0,9786	0,70	1,00
7	700	30000	0,0222	0,9778	0,58	0,82
8	918	40000	0,0219	0,9781	0,50	0,71
9	1164	50000	0,0222	0,9778	0,45	0,63
10	1396	60001	0,0220	0,9780	0,41	0,58
11	1508	65000	0,0220	0,9780	0,39	0,56

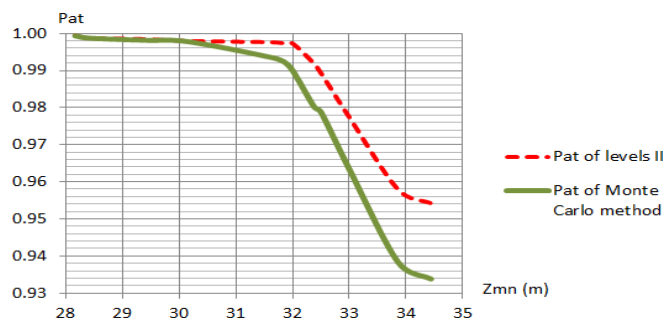


Fig 2. Probability of safety of Phu Ninh main dam according to theory of confidence levels II and III (Monte Carlo method) [7]

4) Analysis of results

The calculated probability of safety against overflow depends on the times of creating random variables, the larger the number of times of creating random variables, the smaller the error of expectation and variance of random variables.

Therefore, in order to minimize these errors or increase the accuracy of calculation results, it is necessary to increase the number of times of creating random variables until the result is stable (converged). According to the results in Table 4: with the number of calculations is 60001 times, the relative error of expectation and the variance of random variables is from $0.41 \div 0.58$, the probability of overflow is $P_{sc} = 0.022$. When solving the problem of overflow according to the theory of confidence level II, the probability of problem $P_{sc} = 0.018$. Thus, with the above data, the error in calculating the probability of overflow at level II and Monte Carlo method is 18.18%. When the water level in the reservoir varies $Y_{mn} = +33.00$ up $Y_{mn} = +34.44$, the confidence of the main dam as calculated by level II is higher than that of level III but not much, the probability of safety of the main dam at level II is 1% to 2% greater than that at level III. Probability calculation by Monte Carlo method can solve the nonlinear relation function and does not require probability distribution law of that function. At the same time, the random variables in the function can be dependent or independent, calculated according to the law of probability distribution without conversion or assumption of normal distribution. Thus, the approach of Monte Carlo method can solve problems with complex functions and random variables that are dependent each on other and have any distribution laws.



With the Water overtopping problem: When calculating with level II, considering the independent random variables, the reliable function is nonlinear but the approximate change in linear function; With the Level III problem, random variables are interdependent and interdependent, the confidence function is a nonlinear function, Random variables reflect closely with reality. Therefore, the calculation of Safety Probability according to level III is very expensive because works are calculated very carefully including key parameters and non-key points. In addition, the provision of sufficient monitoring data to serve calculations at level III is always a difficult issue for Vietnamese irrigation reservoirs under current conditions. Thence, with important dams will be calculated according to the problem of level III, with less important works of regions, calculated according to level II [7]

IV. CONCLUSION

The research has built the reliable function, verification of the probability distribution law of random variables, extending random variables according to the law of probability distribution and calculating the safety probability of works according to statistical formulas to calculate the safety reliability of the Monte Carlo method, and this is also calculation of the level III. To simulate the calculation method, the authors analyzed the reliability of the water overtopping mechanism of the main dam of the Phu Ninh reservoir by Monte Carlo method with the number of simulations ensuring the convergence result up to 65,000 times. Based on comparison with the reliability calculations at level II for the water overtopping mechanism of the main dam and the current status at the dams in Vietnam, The authors have made recommendations on the degree of application of the project reliability calculation under Vietnamese conditions: with important dams will be calculated according to level III, with less important works of regions, then calculated according to level II problem . The article would be good references for dam safety design and management in Vietnam.

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