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Graph of an Intuitionistic Fuzzy Ideal of MΓ Groups in Near Rings

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Abstract: ***The objective of this paper is to establish a relationship between intuitionistic fuzzy (IF) theory, graph theory, intuitionistic fuzzy graph theory with the ideal algebraic structure of a near ring in MΓ group. The idea of graph of an intuitionistic fuzzy (IF) ideal, the regular graph of an intuitionistic fuzzy (IF) ideal and their isomorphism in MΓ group of near rings are discussed. Also, few properties of theirs are studied here as theorems***

Keywords: ***Graph of an intuitionistic fuzzy (IF) ideal, regular graph of an intuitionistic fuzzy (IF) ideal, isomorphism of graphs of intuitionistic fuzzy (IF) ideals in MΓ group of near rings.***

I. INTRODUCTION

The concept of near ring has been applied in geometry, topology, differential equation and automation has been introduced by Pilz et al. [10] with various properties and characteris- tics are derived . In 1996, Zadeh et al. [15] introduced fuzzy sets by defining membership function for a crisp set. As an expansion of fuzzy set by including non-membership to fuzzy set, Atanassov [1] introduced (IF) set with different operations and their character is- tics are analysed. In 2005, Jianming et al.[2] merged ideals of near rings in intuitionistic fuzzy set.

As near ring has their application in automation, topology and ideal in order theory and graph theory, a graph is extended to fuzzy graph and discussed their applications in decision process also, by Rosenfeld et al. [11] in 1975. Sathyanarayana and Prasad et al. [12] expanded near rings with ideals of fuzzy and theoretical idea of graph theory by explaining theorems and representing ideals into graphs.

In 1996 Kim and Kim et al.[4] introduced fuzzy ideals of NR and Jun et al.[3] implemented the fuzzy ideals in ΓNR. The theory of IF ideals of near rings was initiated by Zhan et al. [16] and some related properties were obtained. Fuzzy ideals in gamma rings have been characterized by Palaniappan et al.[9] as extension of ideals of gamma rings in IF sets. Karunambikai et al. [5,6] introduced IF graph and discussed vari- ous characteristics by classifying strongest arc and weakest arc ,strongest path and weakest path , alpha strong ,beta strong ,delta weak .

IF ideals of M gamma groups in near rings has been introduced and discussed its few properties by Mala and Shanmugapriya [7]. Shanmugapriya et al. [13] discussed homo- morphism in Q- intuitionistic L fuzzy sub near rings of a near ring. The algebraic structure of semi group with fuzzy theory and graph theory was connecte6d as fuzzy graph of semi group. The notion of fuzzy ideal graph of semi group was generalized by Murali Krishna Rao et al. [8]. Later, intuitionistic fuzzy representations of intuitionistic fuzzy groups were discussed in detail by Sharma [14].

II. METHODOLOGY

IF ideal is extended to a graph in MΓ group of a near ring. Examples are discussed to explain their order, size, total degree along with theorems by proving them. Isomorphism of graphs of an IF ideal is defined and few theorems are derived using their characters. This is extended and proved for the complement graph also.

A. GRAPH OF THE IF IDEAL

Let $G(V_I, E_I)$ be a graph of an ideal $I(\mu_I, \gamma_I)$ of a (N,·)m of near ring if

- $\mu_I(xy) \geq max\{\mu_I(x), \mu_I(y)\}$
- $\gamma_I(xy) \leq min\{\mu_I(x), \mu_I(y)\}$ for$\{xy\} \in E_I$.

Then$G(V_I, E_I)$ is called graph of the IF ideal $I(\mu_I, \gamma_I)$ and denoted as$G(V_I, E_I, \mu_I, \gamma_I)$.

B. ORDER, SIZE OF THE GRAPH OF THE IF IDEAL

Let $G(V_I, E_I, \mu_I, \gamma_I)$ be a graph of the IF ideal I. The order of G_1 is specified as ($\mu_I(p), \gamma_I(p)$) and it is represented by O(G).

Let $G(V_I, E_I, \mu_I, \gamma_I)$ be a graph of the IF ideal I. The size of G is defined as ($\mu_I(xy), \gamma_I(xy)$),

where $\{xy\} \in E_I$ and it is represented by S(G).

C. DEGREE &TOTAL DEGREE OF A GRAPH OF AN IF IDEAL

The degree of the vertex v of the $G(V_I, E_I, \mu_I, \gamma_I)$ is defined as ($\mu_I(uv)$,$\gamma_I(uv)$) where

$u \in$ v and it is defined by D(v). Let $G_1(V_I, E_I, \mu_I, \gamma_I)$ be an graph of the IF ideal I. Total degree of the vertex $v \in V_I$ is specified as $(D\mu(u) + \mu_I(u), D\gamma(u) + \gamma_I(u))$.It is denoted by TD (u).

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D. EXAMPLES OF GRAPH OF AN IF IDEAL

In the near ring $N = Z_3 \cdot I_1 = \{0\}$ and $I_2 = \{0,1,2\} = Z_3$ are the ideals $G(V_{I_1}, E_{I_1}, \mu_{I_1}, \gamma_{I_1})$ and $G(V_{I_2}, E_{I_2}, \mu_{I_2}, \gamma_{I_2})$ are the graphs of IF ideals I_1 and I_2 with respect to the operation multiplication are represented as follows.

Order of $G(V_{I1}, E_{I1}, \gamma_{I1})$=(1.5,0.6); Order of $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ =(1.5,0.6)
Size of $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (O.7, 0.3)$; Size of $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2}) = (1.0, 0.4)$
Degree of the vertex 1,vertex 0 and vertex 2 in figure 1 are given as D(1) in $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (0.3, 0.2)$

D (0) in $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (0.7, 0.3)$

D (2) in $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (0.4, 0.1)$

Degree of the vertex 1, vertex 0 and vertex 2 in figure 2 are given as

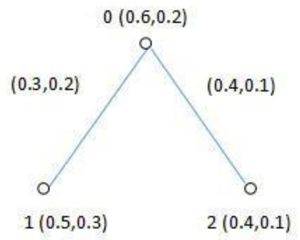

Figure 1 : Graph

D (1) in $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (0.7, 0.2)$

D (0) in $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (0.7, 0.3)$
D (2) in $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (0.6, 0.3)$

E. COMPLETE GRAPH, REGULAR GRAPH & TOTALLY REGULAR GRAPH OF THE IF IDEAL

Let $G(V_I, E_I)$ be a complete graph. Then the graph of the IF ideal $G(V_I, E_I, \mu_I, \gamma_I)$ is called the complete graph of an IF ideal I in the near ring.Let $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ be a graph of the IF ideal I_1.If $D(v) = (k_1, k_2)$ for all $v \in V_{I1}$ then$G(V_I, E_I, \mu_I, \gamma_I)$ is called as regular IF graph (IFG).If each vertex of $G(V_I, E_I, \mu_I, \gamma_I)$ has the same total degree (k_1, k_2) the $G(V_I, E_I, \mu_I, \gamma_I)$ is called as totally regular IF graph (TRIFG) of total degree .

F. EXAMPLE FOR TOTAL DEGREE OF THE GRAPH OF AN IF IDEAL

Considering the figure, No. 1.

$TD(1) = (\mu_{I1}(10) + \mu_{I1}(1), \gamma_{I1}(10) + \gamma_{I1}(1))$
$TD(1) = (0.5+0.3, 0.3+0.2) = (0.8, 0.5)$

$TD(0) = (\mu_{I1}(01) + \mu_{I1}(02) + \mu_{I1}(0), \gamma_{I1}(01) + \gamma_{I1}(0) + \gamma_{I1}(02)) = (1.3, 0.5)$
$TD(2) = (\mu_{I1}(20) + \mu_{I1}(2), \gamma_{I1}(20) + \gamma_{I1}(2)) = (0.8, 0.2)$

Considering the figure, No 2.

$TD(1) = (1.3, 0.4)$

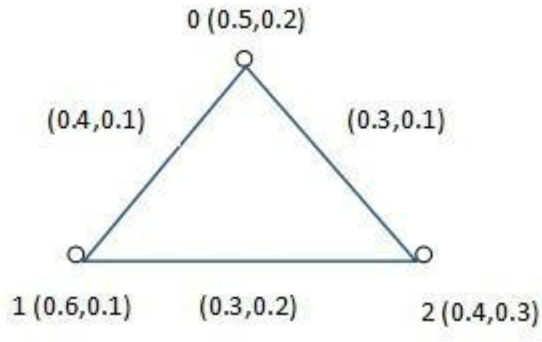

Figure 2: Graph

$TD(2) = (1.0, 0.6)$

$TD(0) = (1.2, 0.4)$

G. THEOREM OF REGULAR GRAPH OF AN IF IDEAL

The size of (k_1, k_2) regular graph for an IF ideal $G(V_I, E_I, \mu_I, \gamma_I)$ is $|v|/2(k_1, k_2)$.

Proof:By definition, the size of $G(V_I, E_I, \mu_I, \gamma_I)$ is defined as $(\mu_I(xy), \gamma_I(xy))$ where $\{xy\} \in E_1$.

Then in this IF graph, $D(v) = 2(\mu_I(uv), \gamma_I(uv))$ where $\{uv\} \in E_1$.
= 2 size of $G(V_I, E_I, \mu_I, \gamma_I)$.

But $D(v) = |v|(k_1, k_2)$ where $|v|$ is the no. of vertices in $G(V_I, E_I, \mu_I, \gamma_I)$. Therefore $2S(G) = |v|(k_1, k_2)$.

Hence $S(G) = |v|/2(k_1, k_2)$.

H. THEOREM ON DEGREE OF THE GRAPH

Let $G(V_I, E_I, \mu_I, \gamma_I)$ be a graph of an IF Ideal I with $V_I = \{v_1, v_2, \ldots, v_n\}$. Then

$D(v) = (k_1, k_2)$ where $v \in V_I$ and $k_1 \geq d(v_i)\mu(v_j)$ and $k_2 \geq d(v_i)\gamma(v_j)$ where v_i Ç $v_j \{v_i v_j\} \in E_I$.

Proof:

Assume $G(V_I, E_I, \mu_I, \gamma_I)$ is the graph of an IF ideal I and $V_I = \{v_1, v_2, \ldots, v_n\}$.
Then $D(v_i) = (\mu(v_i v_j), \gamma(v_i v_j))$ where v_i Ç v_j and $\{v_i v_j\} \in E_I$
But $\mu(v_i v_j) \geq d(v_i)\mu(v_j)$ and $\gamma(v_i v_j) \geq d(v_i)\gamma(v_j)$

Hence $D(v_i) \geq (d(v_i)\mu(v_j), d(v_i)\gamma(v_j))$

I. THEOREM ON CONSTANT FUNCTION

Let $G(V_I, E_I, \mu_I, \gamma_I)$ be a graph of an IF ideal I. Then μ_I and γ_I are the constant function if and only if the (i) is equivalent to (ii).

(i) The graph of an IF ideal is regular.

(ii) The graph of an IF ideal is totally regular.

I. *Proof:*

Let $G(V_I, E_I, \mu_I, \gamma_I)$ be a graph of an IF ideal I and μ_I, γ_I are the constant functions.

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(i) ⇒ (ii).

Assume $G(V_I, E_I, \mu_I, \gamma_I)$ is a graph of an IF ideal I. Then $D(u) = (k_1, k_2)$ for all $u \in V_I$ and

$\mu_I(u) = c_1$ and $\gamma_I(u) = c_2$, constants.

Thus $TD(u) = (D_\mu(u) + \mu(u), D_\gamma(u) + \gamma(u)) = (k_1 + c_1, k_2 + c_2)$ for

all $u \in v_I$. Hence this graph of an IF ideal is totally regular.

(iii) ==⇒ (i)

Assume $G(V_I, E_I, \mu_I, \gamma_I)$ be a totally regular graph of an ideal I and $TD(u) = (p_1, p_2)$ for all $u \in V_I$.

J. ISOMORPHISM OF GRAPHS OF IF IDEAL

Let $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ be the graphs of IF ideals I_1 and I_2. If there exists a map h from I_1 to I_2 such that

(i) h is an isomorphism of ideals.

(ii) $\mu_{I1}(u_1) = \mu_{I2}(h(v_1))$ *and* $\gamma_{I1}(u_1) = \gamma_{I2}(h(u_1))$ for all $u_1 \in I_1$.

(iii)$\mu_{I1}(uv) = \mu_{I2}(h(u)h(v))$ and $\gamma_{I1}(uv) = \gamma_{I2}(h(u)h(v))$ for all $\{uv\} \in E_1$ and $\{h(u)h(v)\} \in E_2$.

Then h is said to be an isomorphism of graphs of IF ideals and is denoted by

$G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) \sim G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$

K. THEOREM ON ISOMORPHISM OF GRAPHS OF IF IDEALS

Let $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ be isomorphic graphs of the IF ideals I_1 and

I_2 respectively then their orders and sizes are same.

Proof:

Suppose$G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ are isomorphic graphs of the IF the ide- als I_1 and I_2 respectively. Then by Def: 3.16 there exists an isomorphism $g : I_1 \to I_2$ satisfying the conditions.

Therefore order of $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (\mu_{I1}(v), \gamma_{I1}(v)) = (\mu_{I2}(g(v)).\gamma_{I2}(g(v)))$

= order of $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$.
Size of $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1}) = (\mu_{I1}(uv), \gamma_{I1}(uv)) = \mu_{I2}(g(u)g(v)), \gamma_{I2}(g(u)g(v)))$

Hence the order and size are same in I_1 and I_2.

L. THEOREM ON ISOMORPHISM OF REGULAR GRAPHS OF IF IDEALS

Let $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ be isomorphic graphs of the IF ideals I_1 and

I_2 respectively. If $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ is regular graph of an IF the ideal I_1 then $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ is a regular graph of the IF ideal I_2.

Proof:

Suppose $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ is a regular graph of an IF ideals I_1 and Φ is the isomorphism of graphs of IF ideals I_1 and I_2 from $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ to $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$.Then there exists an isomorphism $\Phi : I_1 \to I_2$ satisfying def.3.16. Thus

$D(u_1) = (\mu_{I1}(u_1v_1), \gamma_{I1}(u_1v_1))$ where u and $\{u_1v_1\} \in E_{I1}$

$= (\mu_{I2}(\Phi(u_1)\Phi(v_1)), \gamma_{I2}(\Phi(u_1)\Phi(v_1)))$where u_1 Ç v_1 and $\{\Phi(u_1)\Phi(v_1)\} \in E_{I2} = D(\Phi(u_1))$

Hence $G(VI_2, EI_2, \mu I_2, \gamma I_2)$ is a regular graph of the IF ideal I_2.

M. THEOREM ON ISOMORPHISM OF TOTALLY REGULAR GRAPHS

Let $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$be isomorphic graphs of the IF the ideals I_1 and I_2 respectively. If $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ is a totally regular TR and to graph of an IF ideal I_1, then $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ is a totally regular graph of an IF of an ideal I_2.

N. THEOREM ON COMPLEMENT GRAPHS OF IF IDEALS

Let $G(V_I, E_I, \mu_I, \gamma_I)$ be a graph of an IF ideal I. Then the complement of

$G(V_I, E_I, \mu_I, \gamma_I)$ is defined as $G^j(V_I, E_I, \mu_1^j, \gamma_1^j)$ where $\mu_I^j(uv) = \mu_I(uv) - max\{\mu_I(u),$

$\mu_I(v)\}$and $\gamma_I^j(uv) = \gamma_I(uv) - min\{\gamma_I(u), \gamma_I(v)\}$ for all $\{uv\} \in E_1$

O. THEOREM ON COMPLEMENT GRAPHS OF IF IDEALS

Let $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ be graphs of the IF ideals I_1 and I_2 then they are isomorphic iff their complements are isomorphic.

Proof:

Suppose $G(V_{I1}, E_{I1}, \mu_{I1}, \gamma_{I1})$ and $G(V_{I2}, E_{I2}, \mu_{I2}, \gamma_{I2})$ are isomorphic graphs of the IF deals I_1 and I_2 respectively. Then there exists an isomorphism $g : I_1 \to I_2$ such that

$\mu_{I1}(u) = \mu_{I2}(g(u))$ and $\gamma_{I1}(u) = \gamma_{I2}(g(u))$ for all $u \in V_I$ and $g(u) \in V \in I_2$.

Also $\mu_{I1}(uv) = \mu_{I2}(g(u)g(v))$ and $\gamma_{I1}(uv) = \mu_{I2}(g(u)g(v))$ for all $\{uv\} \in E_1$ and $\{g(u)g(v)\} \in E_2$.

Thus $\mu^j_{I1}(uv) = \mu_{I1}(uv) - max\{\mu_{I1}(u), \mu_{I1}(v)\}$.
$= \mu_{I2}(g(u), g(v)) - max\{\mu_{I2}\, g(u), \mu_{I2}\, g(v)\}$.
$= \mu_{I2}(g(u), g(v))$ for $\{uv\} \in E_1$.
$\gamma^j_{I1}(uv) = \gamma_{I1}(uv) - min\{\gamma_{I1}(u), \gamma_{I2}(v)\}$.
$= \gamma_{I2}(g(u)g(v)) - min\{\gamma_{I2}\, g(u), \gamma_{I2}\, g(v)\}$.
$= \gamma^j_{I2}(g(u)g(v))$ for all$\{uv\} \in E_{I1}$.
Hence $G^j(V_{I1}, E_{I1}, \mu^j_{I1}, \gamma_{I1}) \div G^j(V_{I2}, E_{I2}, \mu^j_{I2}, \gamma^j_{I2})$.

III RESULTS AND DISCUSSIONS

There is an introduction of graph of an IF ideal in NR,

regular graph of an intuitionistic fuzzy ideal and isomorphism between graphs of the intuitionistic fuzzy ideals in MΓ group of near rings. Throughout this paper I_1, I_2 denote the IF ideal of a near ring N.

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IV CONCLUSION AND FUTURE SCOPE

The conception of graph of an IF ideals in near ring was presented. The concept of isomorphic and regular graph for IF ideal in near rings NR were discussed, where in IF ideal as a generalized from fuzzy graph of an ideal in near rings. Also, their properties as theorems in graphs of IF ideals were studied. Further the comparison of IF graph of an ideal and graph of IF ideal can be extended.

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