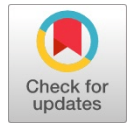


# Reduced Complexity Trellis for Coded Modulation on Channels Subjected to ISI

Vanaja Shivakumar



**Abstract**--The objective in this paper is to analyze the structural complexity of the non-binary coded modulation schemes for transmission over the digital communication channels which are band-limited and subjected to Intersymbol Interference (ISI). The design of reduced complexity M-QAM Coded Modulation schemes are analyzed algebraically. The performance is demonstrated through simulation.

**Index Terms**—M-QAM; Reduced-Complexity; Structural-Complexity; non-binary; Analyze.

## I. INTRODUCTION

High data-rate and high-gain modulation schemes are essential for advanced digital transmission systems. As a result, Trellis Coded Modulation (TCM) has evolved. It provides increased power efficiency and bandwidth efficiency over the un-coded modulation scheme, without demanding for the extra transmission power or bandwidth. Some of its applications are: wired and wireless land communication systems, satellite communications, Spatial Modulation, MIMO, High density Subscriber Loops, WiMAX, magnetic recording and memory writing. Digital technologies of the current era enhanced the applications of TCM either as a component code in concatenated coding systems or directly. Hence newer inventions towards the TCM schemes and its decoding strategies are an essential of the current research scenario. This paper has been organized as follows: Section I contain introduction, Section II explains the Trellis Code generator. In section III, combined trellis structure due to intersymbol interference (ISI) effect explained. Section IV explains Truncated Trellis. In section V decoding strategies are explained. Section VI covers results and conclusions.

## II. TRELLIS CODE GENERATOR/MODULATOR

Design structure of trellis coded modulation reveals that it is a non-binary modulation technique integrated with mapping, shown in Fig.1. For 'u' information bits under transmission, there are 'k' number of redundant bits added by the encoder in a predefined pattern so that  $2^{u+k}$  number of signals are available for transmission. A convolutional encoder is one of the essential components internally, receiving  $\tilde{u}$  input bits and generates  $(\tilde{u}+k)$  coded bits. There are 'γ' number of delay elements which decides the encoder states,  $\delta_n$ .

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\*Correspondence Author(s)

**Vanaja Shivakumar\***, department of Electronics and Communication Engineering, CMR Institute of Technology, Bengaluru, India. Email: vanajaskb@gmail.com

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The Trellis Encoder output sequence enters the signal mapper which does the job of generating an integer 'α' from the possible  $2^{u+k}$  numbers which is the index of one of the signal points of the constellation used for transmission.

$$\text{Number of Encoder States } N_{\delta} = 2^{\gamma} \quad (1)$$

$$= \text{Number of Trellis States}$$

$$\text{Constraint Length of the encoder} = \gamma \quad (2)$$

$$\text{Trellis Coded sequence } Y_n = \{y_n^0, y_n^1, \dots, y_n^{u+k-1}\} \quad (3)$$

$$\text{Signal Constellation size} = M = 2^{u+k} \quad (4)$$

The encoded bits define the subset of the signal constellation from which one signal point is selected for transmission by the un-coded bits combination. This process is called set-partitioning, shown in Fig.3. It is an inherent procedure in TCM due to which the Euclidean distance among the signal points of a subset is maximized. The mapped signal  $a_n$  and the encoder state transitions at any discrete time 'n' is a function of the input sequence and the encoder state.

The encoder has memory and hence will have graphical representation, called trellis. The states are represented as a node in the trellis.

From each node there are  $2^{\tilde{u}+k}$  transitions, each carry one subset of signals. It says that there are  $2^{\tilde{u}-\tilde{u}}$  parallel paths each transition carry. The Fig.2 shows the trellis of 4-state encoder for 16 number of trellis encoded quadrature amplitude modulated signals transmission.

## III. TRANSMISSION ON BAND-LIMITED ISI CHANNEL

When the communication channel is a band-limited ISI channel, equivalently it can be represented as a linear system. The resulting discrete time model of the communication system as a linear filter is shown in Fig.4.

Assuming the channel memory length as L, each of the encoder states is associated with  $2^L$  number of ISI states. The trellis of the system is the combination of the encoder trellis and the ISI states trellis, called Super Trellis,  $\lambda$ . The number of states of super trellis,  $N_{\delta, \lambda}$ , is, as in:

$$N_{\delta, \lambda} = N_{\delta} (M/2)^L = 2^{uL} \quad (5)$$

$$\text{and } \lambda_n = (y_{n-L}, y_{n-L+1}, \dots, y_{n-1}; \delta_n) \quad (6)$$

In reality the decoder structure becomes complicated. An alternate solution is the truncated ISI channel memory length approach.

In the following analysis, input is 3-bits per symbol interval transmission, channel ISI length is 2, and M=16. At any discrete time 'n', the input sequence  $X_n$  is, as in:

$$\text{Input } X_n = (x_n^1, x_n^2, x_n^3), \quad (7)$$

Encoder next state is, as in:



$$\mathfrak{s}_{n+1} = f_1(X_n, \mathfrak{s}_n) \quad (8)$$

$$\text{Let } \mathfrak{s}_{n+1} = (x_{n-1}^1, x_n^1)$$

Now, next state of the combined ISI-Code trellis

$$\lambda_{n+1} = f_2(\mathfrak{s}_n; a(n)) \quad (9)$$

The trellis code of  $a(n)$  is  $(y_n^3, y_n^2, y_n^1, y_n^0)$ , where, as in:

$$y_n^3 = x_n^3; y_n^2 = x_n^2; y_n^1 = x_n^1 \oplus x_{n-2}^1; y_n^0 = x_n^1 \quad (10)$$

Therefore, in terms of  $X_n$ , the next state is, as in:

$$\lambda_{n+1} = (x_{n-2}^1, x_{n-1}^1, x_n^1, x_n^2, x_n^3) \quad (11)$$

## IV. TRUNCATED COMBINED ISI-CODE TRELLIS

When the channel memory length  $Q$  to  $L$  is treated as less significant, where  $0 \leq Q \leq L$ , the number of ISI states associated with each state of the encoder decreases, accordingly, the complexity of the receiver decreases Fig.5. Now the number of states in the combined trellis are  $N_{\mathfrak{s},Q}$ , where

$$N_{\mathfrak{s},Q} = N_{\mathfrak{s}}(M/2)^Q \leq N_{\mathfrak{s}}(M/2)^L \quad (12)$$

The reduced states of the truncated trellis

$$\lambda_{n,Q} = [\mathfrak{s}_n; Y_{n-1}(u_i), Y_{n-2}(u_i), \dots, Y_{n-Q}(u_i)],$$

$$\text{for } \tilde{u} \leq u_i \leq u \quad (13)$$

$$\begin{aligned} \text{where } Y_n(u_1 = 1) &= (y_n^1 y_n^0) \\ Y_n(2) &= (y_n^2 y_n^1 y_n^0) \end{aligned} \quad (14)$$

## V. DECODING STRATEGIES

Decoding of the noisy TCM signals received on AWGN channel is done by the Soft Decision Viterbi Algorithm (SOVA) for the Maximum-Likelihood Estimation of the signal sequence (MLSE). The SOVA is an iterative process, in each iteration it traces the trellis structure of the encoder/modulator to search for the optimum transmitted signal sequence path, called survivor path. It computes the distance between the received noisy signal sequence and the reference as per the predefined structure of the encoder design. The computations become complicated as  $M$  value increases and the encoder states.

In reality, reduced complexity trellis structures are considered for transmission, and, accordingly decoding algorithm becomes less complicated.

If  $M_n(\cdot, a_n)$  is the metric computation, the iterative computation of the algorithm during  $n^{\text{th}}$  interval is given by

$$D_n(\cdot, a_n) = D_{n-1}(\cdot, a_{n-1}) + |z_n - a_n|^2 \quad (15)$$

$$D_n(\cdot, a_n) = D_{n-1}(\cdot, a_{n-1}) + |z_n - \sum_{i=1}^L g_{n-i} a_{n-i} - a_n|^2 \quad (16)$$

$$D_n(\cdot, a_n) = D_{n-1}(\cdot, a_{n-1}) + |z_n - \sum_{i=Q+1}^L g_{n-i} a_{n-i} - \sum_{i=1}^Q g_{n-i} a_{n-i} - a_n|^2 \quad (17)$$

The computational complexity in equation (15) is valid if only Gaussian noise affect the channel. The complexity increases as given by the equation (16) due to cancellation of ISI. When truncated channel trellis is considered the ISI cancellation in the second term of the equation (17) is executed for  $Q$  intervals only. And the previous symbols ISI effect for the length  $Q$  to 1 is cancelled from the history of symbol detection.

## VI. RESULTS AND CONCLUSIONS

Basic design of Ungerboeck's TCM has been studied. The truncated combined ISI-Code trellis structure is analyzed. While the band-limited ISI channels are considered for transmission, the complexity of the MLSE decoding algorithm varies according to the super trellis. This is practically unrealizable. As an alternate, truncated trellis of the system are defined for various possibilities of channel memory length, and near optimum decoding strategies are developed to minimize the computational complexity. As a result, various design structures emerged. The graph 1-2, shown in Fig.6, shows the error event probability characteristic of TCM scheme over bandlimited channel affected by ISI. Various decoding algorithms performances as a function of SNR are plotted. On the observation from left in the graphs, performance improvement towards ISI free characteristic is observed in the curve 2 from left, in comparison with the curve 3, for un-coded 8-QAM scheme, plotted for different channel conditions. Innovative approaches can be introduced to improve the performance further.

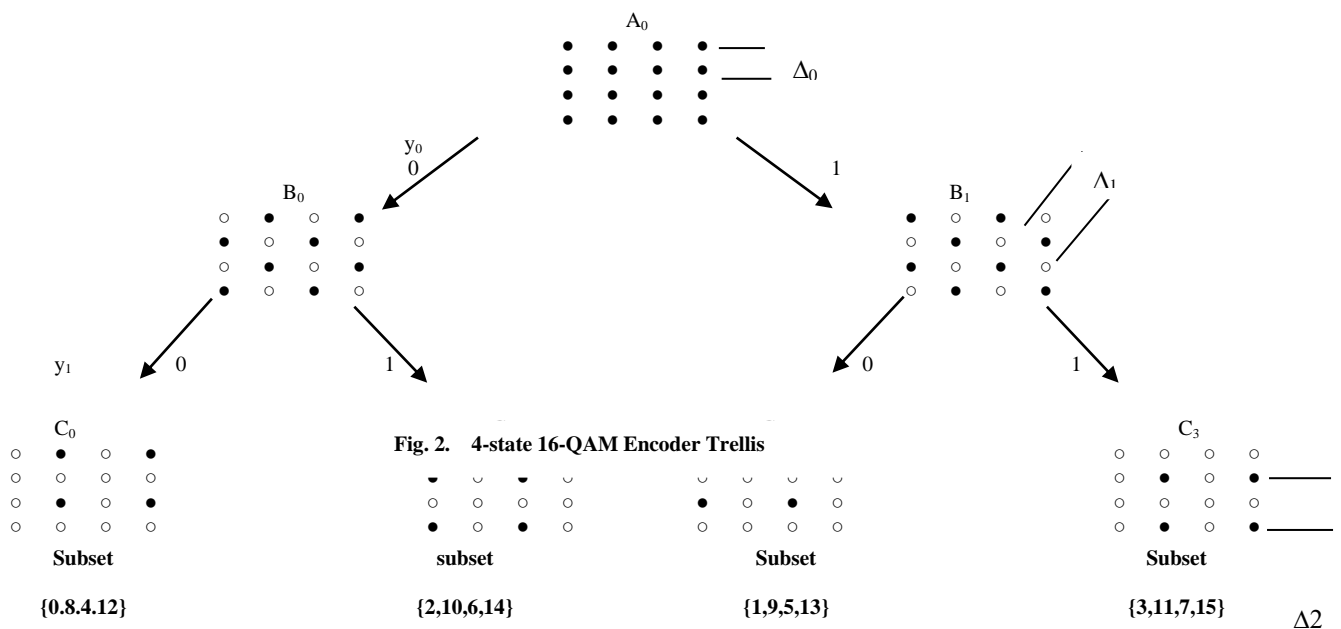
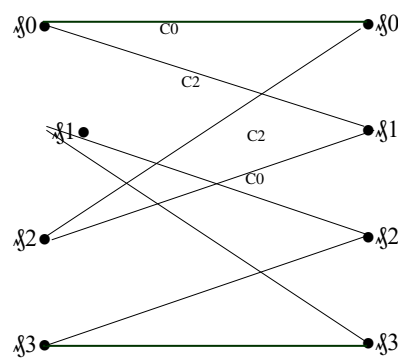
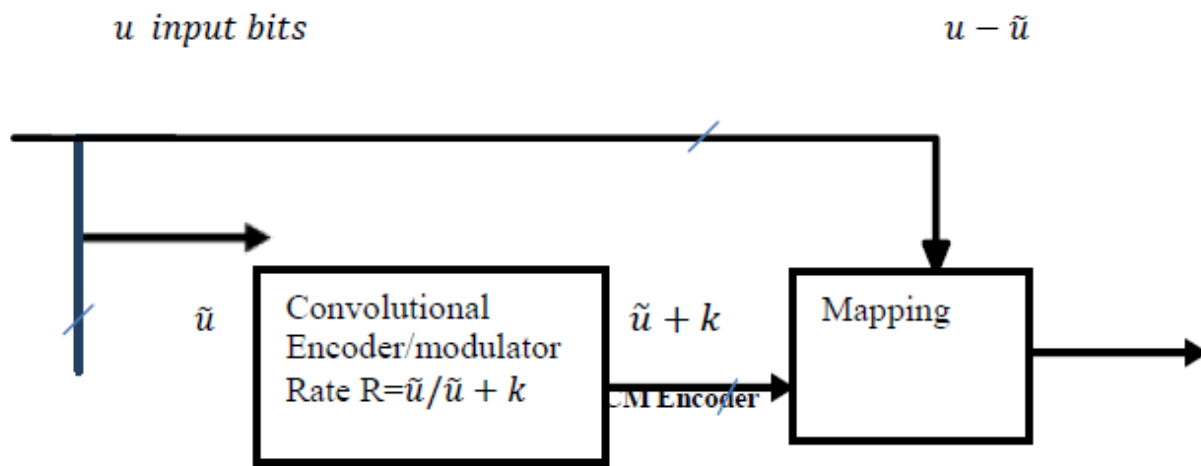


Fig. 2. 4-state 16-QAM Encoder Trellis

Fig. 3 Set-Partitioning of 4-State 16-QAM TCM Scheme

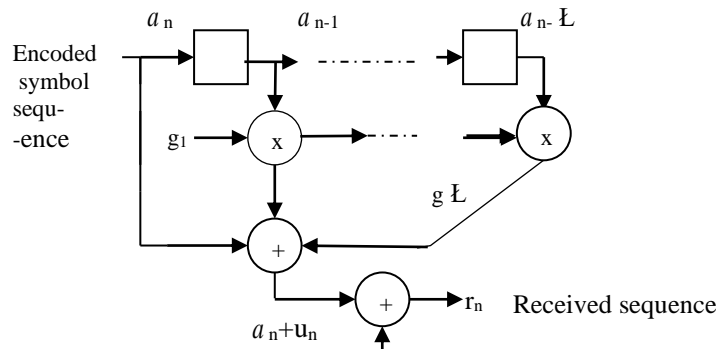


Fig. 4 Combined Encoder-channel model(FSM)

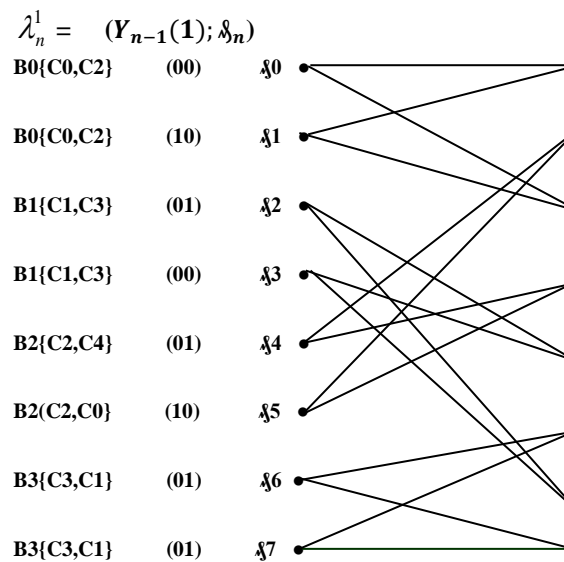


Fig.5 8-state Truncated Combined ISI- Code trellis for 4-state 16-QAM TCM , channel memory length L=2, truncated length Q=1

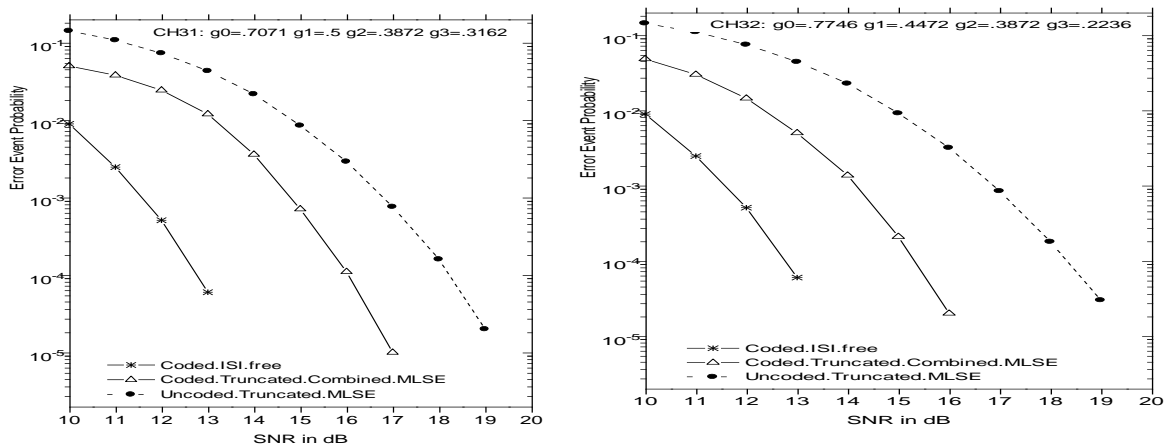


Fig. 6. Error Event Probability Vs SNR for 4-state 16-QAM TCM, on band-limited ISI channels, channel memory length 3, truncated length 1  $g_0, g_1, g_2, g_3$  are channel coefficients

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## AUTHORS PROFILE



**Vanaja Shivakumar** obtained her B.E. degree in Electrical and Electronics Engineering in the year 1987 from Govt. BDT College of Engineering, Davangere, under Mysore University, Karnataka-state, India. She obtained her Master Degree in Digital Electronics from SDMCE, Dharwad, under Karnatak University,

Karnataka-state, India, in 1994. She completed her Ph.D. in the year 2012, from Kuvempu University, Karnataka-state, India, in the area Electronics and Communication Engineering. She served at various positions in various engineering colleges and university. Currently, she is working as Professor, and, Head for Innovation and Entrepreneurship cell at CMR Institute of Technology, Bengaluru, Karnataka-State, India. Her Research interest is: Information Theory and Coding, Signal Processing and Embedded Applications.