Intuitionistic Fuzzy Digital Continuous Maps and Connected Maps

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Abstract: ***The geometrical and topological possessions among the fragments of a digital picture play a massive role in image processing. Together Intuitionistic fuzzy logic has many applications in image processing. This article aims at such a property of continuity. It introduces intuitionistic fuzzy digital continuous maps between any two intuitionistic fuzzy digital images defined by the intuitionistic fuzzy digital regular sets and to study some of its properties. Besides, the concept of intuitionistic fuzzy digital connected maps is established by giving some restrictions on intuitionistic fuzzy digital continuous maps and derived many interesting results.***

Keywords: IFD connected maps, IFD connected sets, IFD continuous maps, IFD homeomorphisms.

I. INTRODUCTION

The introduction of fuzzy sets by Lofti A. Zadeh [12] lead to many generalizations of fuzzy set. In 1983, K. T. Atanassaov [2] introduced the "Intuitionistic Fuzzy Sets" and many more works were done by him and his colleagues [1,3]. In image processing, Rosenfeld and Kak [9,10,11] preferred to extract fuzzy subsets instead ordinary sets. And it is noted that Intuitionistic fuzzy sets (IFS) are more convenient than the fuzzy set and other types of sets. In this extend, we developed the notions of Intuitionistic fuzzy digital structure convexity, concavity, convex envelopes, simplexes and convex maps [5,6,7,8]. This paper introduces the topological concepts of continuity and connectedness in terms of Intuitionistic fuzzy subsets of the digital plane and develops and some of their basic properties.

II. PRELIMINARIES

In order to understand the concept of this paper, some basic definitions are given in this section. In this paper the Euclidean plane is denoted as *E* and the unit interval [0,1] is denoted by *I*.

Definition 2.1 [10]

Let *P*, *Q* be the points of Σ . Then a path ρ from *P* to *Q* is a sequence of points $P = P_0, P_1, \ldots, P_n = Q$ such that P_i is adjacent (either 4-adjacent or 8-adjacent) to P_{i-1}, $for\ 1 \le i \le n$.

Definition 2.2 [10] Let *S* be any subset of Σ . We say that the points *P*, *Q* are connected in *S* if there is a path from *P* to *Q* consisting entirely of points of *S*.

Definition 2.3 [8]

Let $A_{\sim} = \langle \mu_{A\sim}, \nu_{A\sim} \rangle$ be the intuitionistic fuzzy subset of *E*. Then the intuitionistic fuzzy digital subset (or IFD subset) of the points of *E* is denoted by $A'_{\sim}$, whose degree of membership and the degree of non membership are defined by,

$$\mu_{A'_{\sim}}(P) = max\{\mu_{A_{\sim}}(R) \mid R \in P^*\} \quad and$$

$$\nu_{A'_{\sim}}(P) = min\{\nu_{A_{\sim}}(R) \mid R \in P^*\}.$$

Definition 2.4 [8]

Let $A_{\sim} = \langle \mu_{A\sim}, \nu_{A\sim} \rangle$ be the intuitionistic fuzzy subset of *E*. Then $A_{\sim}$ is said to be intuitionistic fuzzy digital regular (or IFD regular) set if the sets $A_{r\sim}$, defined by $A_{r\sim} = \{R : \mu_{A_{r\sim}}(R) > r \quad and \quad \nu_{A_{r\sim}}(R) < r\}$ are intuitionistic fuzzy regular for all $r \in [0,1)$.

Definition 2.5 [5]

Let $A'_{\sim} = \langle \mu_{A'_{\sim}}, \nu_{A'_{\sim}} \rangle$ be an IFD subset of the points in *E*. Let $\rho : P = P_0, P_1, \ldots, P_n = Q$ be any path between two points of *E*. Then an intuitionistic fuzzy digital strength (or IFD strength) of a path with respect to $A'_{\sim}$ in *E* is defined and denoted by $s^{\rho}_{A'_{\sim}}$ whose degree of membership and degree of non membership
are given by,

$$\mu_{s^{\rho}_{A'_{\sim}}} = \min_{0 \le i \le n} \mu_{A'_{\sim}}(P_i) \quad \text{and} \quad \nu_{s^{\rho}_{A'_{\sim}}} = \max_{0 \le i \le n} \nu_{A'_{\sim}}(P_i)$$

with $\mu_{s^{\rho}_{A'_{\sim}}} + \nu_{s^{\rho}_{A'_{\sim}}} = 1.$

Definition 2.6 [5]

Let $A'_{\sim} = \langle \mu_{A'_{\sim}}, \nu_{A'_{\sim}} \rangle$ be an IFD subset of the points in *E*. Let $\rho : P = P_0, P_1, \ldots, P_n = Q$ be any path between two points of *E*. Then the degree of intuitionistic fuzzy digital connectedness (or IFD connectedness) of the points of a path ρ with respect to $A'_{\sim}$ in Σ is denoted by $\mathcal{C}_{A'_{\sim}}$ whose degree of membership and degree of non membership are given by,

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$\mu_{e_{A'_\sim}}(P,Q) = \max_{\rho} \mu_{s^{\rho}_{A'_\sim}}$ and $\nu_{e_{A'_\sim}}(P,Q) = \min_{\rho} \nu_{s^{\rho}_{A'_\sim}}$

with $\mu_{e_{A_\sim}} + \nu_{e_{A_\sim}} \leq 1.$

Definition 2.7 [5]

Let $A'_\sim = \langle \mu_{A'_\sim}, \nu_{A'_\sim} \rangle$ be an IFD subset of the points in *E*. Then *P* and *Q* are said to be intuitionistic fuzzy digital connected (or IFD connected) in $A'_\sim$ if,

$$\mu_{e_{A'_\sim}}(P,Q) = \min(\mu_{A'_\sim}(P), \mu_{A'_\sim}(Q)) \text{ and}$$
$$\nu_{e_{A'_\sim}}(P,Q) = \max(\nu_{A'_\sim}(P), \nu_{A'_\sim}(Q))$$

Definition 2.8 [8]

Let $A_{r\sim} = \{R : \mu_{A_{r\sim}}(R) > r \ \ and \ \ \nu_{A_{r\sim}}(R) < r\}$ be the intuitionistic fuzzy regular subset of *E*. Then the intuitionistic fuzzy digital image (IFD image) of $A_{r\sim}$ is an IFD set $\mathcal{D}(A_\sim)$, which is defined as

$$\mathcal{D}(A_{r\sim}) = \{R : A_{r\sim} \cap R^* is\, non\ empty\}.$$

III. INTUITIONISTIC FUZZY DIGITAL CONTINUOUS MAPS

In this section we constructed the intuitionistic fuzzy digital continuous maps and obtained some of the results.

Definition 3.1

Let $A_\sim$ and $B_\sim$ be the IFD subsets of *E* and let $\mathcal{D}(A_\sim)$ and $\mathcal{D}(B_\sim)$ be their IFD images respectively. A map $\psi : \mathcal{D}(A_\sim) \to \mathcal{D}(B_\sim)$ is called an intuitionistic fuzzy digital continuous map (or IFD continuous map) if for any two IFD connected points *P* and *Q* in $\mathcal{D}(A_\sim)$ their IFD images $\psi(P)$ and $\psi(Q)$ are IFD connected in $\mathcal{D}(B_\sim)$.

Proposition 3.1

Let $A_\sim$ and $B_\sim$ be the IFD regular subsets of *E* and let $\mathcal{D}(A_\sim)$ and $\mathcal{D}(B_\sim)$ be their IFD images respectively. Any map $\psi : \mathcal{D}(A_\sim) \to \mathcal{D}(B_\sim)$ is IFD continuous iff for every IFD connected set $A_{1\sim}$ in $\mathcal{D}(A_\sim)$, its IFD image $\psi(A_{1\sim})$ is IFD connected in $\mathcal{D}(B_\sim)$.

Proof: Let $\psi : \mathcal{D}(A_\sim) \to \mathcal{D}(B_\sim)$ be an IFD continuous map, $A_{1\sim}$ be an IFD connected set in $\mathcal{D}(A_\sim)$ and let *P* and *Q* be any two IFD connected points in $A_{1\sim}$. From the Definition 2.1, the IFD images $\psi(P)$ and $\psi(Q)$ are IFD connected in $\mathcal{D}(B_\sim)$. Then clearly the set $\{\psi(P) \in \mathcal{D}(B_\sim) : P \in A_{\sim 1}\}$ is IFD connected in $\mathcal{D}(\mathrm{B}_\sim)$.

Conversely, assume *P* and *Q* be any two IFD connected points in $\mathcal{D}(A_\sim)$. Let $A_{1\sim}$ be an IFD connected set containing the points *P* and *Q*. Clearly $A_{1\sim}$ is an IFD connected set and so its IFD image $\psi(A_{1\sim})$ is IFD connected in $\mathcal{D}(\mathrm{B}_\sim)$, which contains only $\psi(P)$ and $\psi(Q)$. Hence the map $\psi : \mathcal{D}(A_\sim) \to \mathcal{D}(\mathrm{B}_\sim)$ is IFD continuous map.

Proposition 3.2

If $\psi_1 : \mathcal{D}(A_\sim) \to \mathcal{D}(\mathrm{B}_\sim)$ and $\psi_2 : \mathcal{D}(\mathrm{B}_\sim) \to \mathcal{D}(C_\sim)$ are IFD continuous maps, then $\psi_2 \circ \psi_1$ is also IFD continuous.

Proof: Since $\psi_1 : \mathcal{D}(A_\sim) \to \mathcal{D}(\mathrm{B}_\sim)$ is IFD continuous, for any two points *P* and *Q* which is IFD connected in $\mathcal{D}(A_\sim)$, their IFD images $\psi_1(P)$ and $\psi_1(Q)$ are IFD connected in $\mathcal{D}(\mathrm{B}_\sim)$. Also since $\psi_2 : \mathcal{D}(\mathrm{B}_\sim) \to \mathcal{D}(C_\sim)$ is IFD continuous, the map images $\psi_2(\psi_1(P))$ and $\psi_2(\psi_1(Q))$ of the IFD connected points $\psi_1(P)$ and $\psi_1(Q)$ are IFD connected in $\mathcal{D}(C_\sim)$. It follows that $\psi_2 \circ \psi_1 : \mathcal{D}(A_\sim) \to \mathcal{D}(C_\sim)$ is IFD continuous.

Definition 3.2

Let $\mathcal{D}(A_\sim)$ and $\mathcal{D}(\mathrm{B}_\sim)$ be IFD images in *E* and $P_1, Q_1 \in \mathcal{D}(A_\sim)$ and $P_2, Q_2 \in \mathcal{D}(\mathrm{B}_\sim)$. Then any two elements (P_1, P_2) and (Q_1, Q_2) of $\mathcal{D}(A_\sim) \times \mathcal{D}(\mathrm{B}_\sim)$ are said to be IFD connected if and only if they satisfy any one of the following conditions.

(i) P_2 and Q_2 are IFD connected in $\mathcal{D}(\mathrm{B}_\sim)$ and $P_1 = Q_1$.

(ii) P_1 and Q_1 are IFD connected in $\mathcal{D}(A_\sim)$ and $P_2 = Q_2$.

(iii) P_1 and Q_1 are IFD connected in $\mathcal{D}(A_\sim)$ and P_2 and Q_2 are IFD connected in $\mathcal{D}(\mathrm{B}_\sim)$.

Proposition 3.3

Let $\mathcal{D}(A_\sim)$ and $\mathcal{D}(\mathrm{B}_\sim)$ be digital images in *E*. Then the maps,

(i) $\Phi_1 : \mathcal{D}(A_\sim) \times \mathcal{D}(\mathrm{B}_\sim) \to \mathcal{D}(A_\sim)$ defined by $\Phi_1(P_1, P_2) = P_1$.

(ii) $\Phi_2 : \mathcal{D}(A_\sim) \times \mathcal{D}(\mathrm{B}_\sim) \to \mathcal{D}(\mathrm{B}_\sim)$ defined by $\Phi_2(Q_1, Q_2) = Q_2$ are IFD continuous.

Proof: Let (P_1, P_2) and (Q_1, Q_2) be IFD connected in $\mathcal{D}(A_\sim) \times \mathcal{D}(\mathrm{B}_\sim)$. Since $\Phi_1(P_1, P_2) = P_1$ and $\Phi_1(Q_1, Q_2) = Q_1$, by the Definition 3.2, P_1 and Q_1 are IFD connected in $\mathcal{D}(A_\sim)$ or $P_1 = Q_1$. Likewise, $\Phi_2(P_1, P_2) = P_2$ and

$\Phi_2(Q_1,Q_2)=Q_2$ are IFD connected in $\mathfrak{D}(B_{\sim})$ or $P_2=Q_2$. Thus the given maps are IFD continuous.

Proposition 3.4

The maps $\psi_1 : \mathfrak{D}(A_{\sim}) \to \mathfrak{D}(C_{\sim})$ and $\psi_2 : \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(D_{\sim})$ are IFD continuous surjections if and only if the map $\psi_1 \times \psi_2 : \mathfrak{D}(A_{\sim}) \times \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(C_{\sim}) \times \mathfrak{D}(D_{\sim})$ defined by is an IFD continuous surjection.

Proof: First assume that $\psi_1 : \mathfrak{D}(A_{\sim}) \to \mathfrak{D}(C_{\sim})$ and $\psi_2 : \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(D_{\sim})$ are IFD continuous surjections. Let (P_1,P_2) and (Q_1,Q_2) be IFD connected in $\mathfrak{D}(A_{\sim}) \times \mathfrak{D}(B_{\sim})$. Then from the Definition 3.2, P_1 and Q_1 are either equal or IFD connected in $\mathfrak{D}(A_{\sim})$. Hence $\psi_1(P_1)$ and $\psi_1(Q_1)$ are either IFD connected in $\mathfrak{D}(C_{\sim})$ or equal.

Likewise, from the Definition 3.2, P_2 and Q_2 are either equal or IFD connected in $\mathfrak{D}(B_{\sim})$. Hence $\psi_2(P_2)$ and $\psi_2(Q_2)$ are either IFD connected in $\mathfrak{D}(D_{\sim})$ or equal. Hence from the Definition 3.2, $(\psi_1 \times \psi_2)(P_1,P_2)$ and $(\psi_1 \times \psi_2)(Q_1,Q_2)$ are IFD connected in $\mathfrak{D}(C_{\sim}) \times \mathfrak{D}(D_{\sim})$. Thus $\psi_1 \times \psi_2 : \mathfrak{D}(A_{\sim}) \times \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(C_{\sim}) \times \mathfrak{D}(D_{\sim})$ is IFD continuous.

Now let $(R_1,R_2) \in \mathfrak{D}(C_{\sim}) \times \mathfrak{D}(D_{\sim})$. Since $\psi_1 : \mathfrak{D}(A_{\sim}) \to \mathfrak{D}(C_{\sim})$ and $\psi_2 : \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(D_{\sim})$ are IFD continuous surjections, there exists $S_1 \in \mathfrak{D}(A_{\sim})$ and $S_2 \in \mathfrak{D}(B_{\sim})$ such that $\psi_1(S_1)=R_1$ and $\psi_2(S_2)=R_2$. Thus, $(\psi_1 \times \psi_2)(S_1,S_2)=(\psi_1(R_1),\psi_2(R_2))$ and so $\psi_1 \times \psi_2$ is an IFD continuous surjection.

Conversely, assume that $\psi_1 \times \psi_2 : \mathfrak{D}(A_{\sim}) \times \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(C_{\sim}) \times \mathfrak{D}(D_{\sim})$ is IFD continuous. Now from the Proposition 3.3, the maps $\Phi_1 : \mathfrak{D}(A_{\sim}) \times \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(A_{\sim})$ and $\Phi_2 : \mathfrak{D}(A_{\sim}) \times \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(B_{\sim})$ defined by $\Phi_1(P_1,P_2)=P_1$ and $\Phi_2(Q_1,Q_2)=Q_2$ are IFD continuous. From the Proposition 3.2, it follows that the maps $\psi_1 = \Phi_1 \circ (\psi_1 \times \psi_2)$ and $\psi_2 = \Phi_2 \circ (\psi_1 \times \psi_2)$ are IFD continuous. Also, since $\psi_1 \times \psi_2$ is IFD continuous surjection, it is obvious that ψ_1 and ψ_2 are also IFD continuous surjections.

IV. INTUITIONISTIC FUZZY DIGITAL CONNECTED MAPS

Here we presented the concept of intuitionistic fuzzy digital connected maps and proved some of its properties.

Definition 4.1

Let $\mathfrak{D}(A_{\sim})$ and $\mathfrak{D}(B_{\sim})$ be IFD images in *E*. Let $\psi : \mathfrak{D}(A_{\sim}) \to \mathfrak{D}(B_{\sim})$ be an IFD continuous surjection. The map ψ is said to be an intuitionistic fuzzy digital connected map (or IFD connected map) iff for every IFD connected subset $B_{1\sim}$ of $\mathfrak{D}(B_{\sim})$ its inverse image $\psi^{-1}(B_{1\sim})$ is IFD connected in $\mathfrak{D}(A_{\sim})$.

Proposition 4.1

Let $\mathfrak{D}(A_{\sim})$ and $\mathfrak{D}(B_{\sim})$ be IFD images which are IFD connected and let $P \in \mathfrak{D}(B_{\sim})$. Then the map $\psi : \mathfrak{D}(A_{\sim}) \to \{P\}$ is an IFD connected map.

Proof: From the Definition 4.1, the proof follows.

Definition 4.2

Let $\psi : \mathfrak{D}(A_{\sim}) \to \mathfrak{D}(B_{\sim})$ be an IFD continuous surjection map between the IFD images $\mathfrak{D}(A_{\sim})$ and $\mathfrak{D}(B_{\sim})$. If the inverse $\psi^{-1} : \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(A_{\sim})$ is also IFD continuous then ψ is said to be an intuitionistic fuzzy digital homeomorphism (or IFD homeomorphism) between $\mathfrak{D}(A_{\sim})$ and $\mathfrak{D}(B_{\sim})$.

Proposition 4.2

If $\psi : \mathfrak{D}(A_{\sim}) \to \mathfrak{D}(B_{\sim})$ is an IFD homeomorphism between the digital images $\mathfrak{D}(A_{\sim})$ and $\mathfrak{D}(B_{\sim})$ then ψ is an IFD connected map.

Proof: The proof follows from the Definitions 4.1and 4.2.

Proposition 4.3

If $\mathfrak{D}(A_{\sim})$ is IFD homeomorphic to $\mathfrak{D}(B_{\sim})$, then the number of IFD connected subsets of $\mathfrak{D}(A_{\sim})$, is same as the number of IFD connected subsets of $\mathfrak{D}(B_{\sim})$.

Proof: Assume that $\mathfrak{D}(A_{\sim})$ have n_1 number of IFD connected subsets, say $A_{1\sim}, A_{2\sim}, \ldots, A_{n_1\sim}$ and $\mathfrak{D}(B_{\sim})$ have n_2 number of IFD connected subsets, say $B_{1\sim}, B_{2\sim}, \ldots, B_{n_2\sim}$. If $n_1 \neq n_2$, then $n_1 > n_2$ or $n_2 > n_1$. Suppose $n_1 > n_2$, consider, $\psi : \mathfrak{D}(A_{\sim}) \to \mathfrak{D}(B_{\sim})$ be an IFD homeomorphism. Then $\psi^{-1} : \mathfrak{D}(B_{\sim}) \to \mathfrak{D}(A_{\sim})$ is also an IFD homeomorphism. Since ψ^{-1} is IFD continuous, $\psi^{-1}(B_{1\sim})$ is an IFD connected subset of $\mathfrak{D}(A_{\sim})$. Hence $\psi^{-1}(B_{1\sim}) \subseteq A_{k\sim}$, $1 \le k \le n_1$, for some *k*. By reordering, assume $\psi^{-1}(B_{1\sim}) \subseteq A_{1\sim}$. Continuing this way, we have $\psi^{-1}(B_{k\sim}) \subseteq A_{k\sim}$, , where $k = 1,2,\ldots,n_2$. But this process left out $A_{(k+1)\sim},\ldots,A_{n_1\sim}$.. This contradicts the fact that ψ^{-1} is subjective and so we cannot have $n_1 > n_2$. In the same way, by considering $\psi : \mathcal{D}(A_{\sim}) \to \mathcal{D}(B_{\sim})$ we can prove $n_2 > n_1$ is not true. Thus $n_1 = n_2$ and this completes the proof.

Proposition 4.4

If $\psi_1 : \mathcal{D}(A_{\sim}) \to \mathcal{D}(B_{\sim})$ and $\psi_2 : \mathcal{D}(B_{\sim}) \to \mathcal{D}(C_{\sim})$ are IFD connected maps, then $\psi_2 \circ \psi_1$ is also an IFD connected map.

Proof: Since $\psi_2 : \mathcal{D}(B_{\sim}) \to \mathcal{D}(C_{\sim})$ is an IFD connected map, by the Definition 4.1, if $C_{1\sim}$ is an IFD connected subset of $\mathcal{D}(C_{\sim})$, then $\psi_2^{-1}(A_{\sim})$ is IFD connected in $\mathcal{D}(B_{\sim})$. Also Since $\psi_1 : \mathcal{D}(A_{\sim}) \to \mathcal{D}(B_{\sim})$ is an IFD connected map, by the Definition 4.1, $(\psi_1 \circ \psi_2)^{-1}(C_{1\sim}) = \psi_1^{-1}(\psi_2^{-1}(C_{1\sim}))$ is IFD connected in $\mathcal{D}(A_{\sim})$. Thus, the map $\psi_1 \circ \psi_2 : \mathcal{D}(A_{\sim}) \to \mathcal{D}(C_{\sim})$ is an IFD connected map.

Proposition 4.5

Let $\psi_1 : \mathcal{D}(A_{\sim}) \to \mathcal{D}(C_{\sim})$ and $\psi_2 : \mathcal{D}(B_{\sim}) \to \mathcal{D}(D_{\sim})$ be IFD continuous surjections. Then $\psi_1 \times \psi_2 : \mathcal{D}(A_{\sim}) \times \mathcal{D}(B_{\sim}) \to \mathcal{D}(C_{\sim}) \times \mathcal{D}(D_{\sim})$ is an IFD connected map if and only if ψ_1 and ψ_2 are IFD connected maps.

Proof: If ψ_1 and ψ_2 are IFD connected maps, then it is clear that they are IFD continuous surjections. Hence from the Propositions 3.4, $\mathcal{D}(A_{\sim}) \times \mathcal{D}(B_{\sim}) \to \mathcal{D}(C_{\sim}) \times \mathcal{D}(D_{\sim})$ is IFD continuous surjection.

To prove $\psi_1 \times \psi_2$ is an IFD connected map, let $C_{1\sim} \times D_{1\sim}$ be an IFD connected set in $\mathcal{D}(C_{\sim}) \times \mathcal{D}(D_{\sim})$ where $C_{1\sim} \subseteq \mathcal{D}(C_{\sim})$ and $D_{1\sim} \subseteq \mathcal{D}(D_{\sim})$. Also let $A_{1\sim} \times B_{1\sim} = (\psi_1 \times \psi_2)^{-1}(C_{1\sim} \times D_{1\sim})$, where $A_{1\sim} \subseteq \mathcal{D}(A_{\sim})$; $B_{1\sim} \subseteq \mathcal{D}(B_{\sim})$ such that $A_{1\sim} = (\psi_1)^{-1}(C_{1\sim})$; $B_{1\sim} = (\psi_2)^{-1}(D_{1\sim})$. Now consider any two points (P_1, P_2) and (Q_1, Q_2) in $A_{1\sim} \times B_{1\sim}$. Since $\psi_1^{-1}(C_{1\sim}) = A_{1\sim}$ is an IFD connected set, P_1 and Q_1 are IFD connected in $A_{1\sim}$. Let ρ_1 be a path between P_1 and Q_1. Hence $\rho_1 \times \{P_2\}$ is a path in $\psi_1^{-1}(C_{1\sim}) \times \{P_2\}$ between (P_1, P_2) and (Q_1, P_2). Since $\psi_2^{-1}(D_{1\sim}) = B_{1\sim}$ is an IFD connected set, P_2 and Q_2 are IFD connected in $B_{1\sim}$ Let ρ_2 be a path between P_2 and Q_2. Hence $\{Q_1\} \times \rho_2$ is a path in $\psi_2^{-1}(D_{1\sim}) \times \{P_2\}$ between (P_1, P_2) and (Q_1, P_2). Therefore $\rho_1 \times \rho_2$ is a path in $A_{1\sim} \times B_{1\sim}$. Hence $A_{1\sim} \times B_{1\sim}$ is IFD connected and so $\psi_1 \times \psi_2$ is an IFD connected map.

Conversely, assume that $\psi_1 \times \psi_2$ is IFD connected. by Proposition 3.4, ψ_1 and ψ_2 are IFD continuous surjections. Let $C_{1\sim}$ be an IFD connected subset of $C_{\sim}$ and let $P \in D_{\sim}$. Then $C_{1\sim} \times \{P\}$ is an IFD connected subset of $C_{\sim} \times D_{\sim}$. Since $\psi_1 \times \psi_2$ is IFD connected, $\psi_1^{-1}(A_{1\sim}) \times \psi_2^{-1}(P) = (\psi_1 \times \psi_2)^{-1}(A_{1\sim} \times \{P\})$ is IFD connected in $A_{\sim} \times B_{\sim}$. Now $\Phi_1(\psi_1^{-1}(A_{1\sim}) \times \psi_2^{-1}(\{P\}) = \psi_1^{-1}(A_{1\sim})$. By Proposition 3.3, Φ_1 is IFD continuous and so $\psi_1^{-1}(A_{1\sim})$ is IFD connected in $\mathcal{D}(A_{\sim})$. Hence ψ_1 is an IFD connected map. Similarly, it can be proved for ψ_2 also.

Proposition 4.6

Let $\mathcal{D}(A_{\sim})$, $\mathcal{D}(C_{\sim})$, $\mathcal{D}(B_{\sim})$ and $\mathcal{D}(D_{\sim})$ be IFD images in E such that $\mathcal{D}(A_{\sim}) \cap \mathcal{D}(B_{\sim}) = \{P\}$ and $\mathcal{D}(C_{\sim}) \cap \mathcal{D}(D_{\sim}) = \{Q\}$. Then the following statements are equivalent.

(i) The maps $\psi_1 : \mathcal{D}(A_{\sim}) \to \mathcal{D}(C_{\sim})$ and $\psi_2 : \mathcal{D}(B_{\sim}) \to \mathcal{D}(D_{\sim})$ such that $\psi_1(P) = \psi_2(P) = Q$ are IFD connected maps.

(ii) The map $\psi_1 * \psi_2 : A_{\sim} \cup B_{\sim} \to C_{\sim} \cup D_{\sim}$ defined by

$$(\psi_1 * \psi_2)(P) = \begin{cases} \psi_1(P) \text{ if } P \in \mathcal{D}(A_{\sim}) \\ \psi_2(P) \text{ if } P \in \mathcal{D}(B_{\sim}) \end{cases}$$ is an IFD connected map.

Proof: (i) $\Rightarrow$ (ii) If ψ_1 and ψ_2 are IFD connected maps, then it is clear that $\psi_1 * \psi_2$ is IFD continuous surjection. Let $C_{1\sim}$ be an IFD connected subset of $C_{\sim} \cup D_{\sim}$ and let $Q_0, Q_1 \in \psi_1^{-1}(C_{1\sim})$. Since $C_{1\sim}$ be an IFD connected set there exists a path ρ between $\psi_1(Q_0)$ to $\psi_1(Q_1)$. Let $\rho : \psi_1(Q_0) = P_1, P_2, \ldots, \psi_1(Q_1) = P_i$. Then there arises the following three cases.

Case (i): Suppose $C_{1\sim} \subset C_{\sim}$, Since ψ_1 is an IFD connected map $\psi_1^{-1}(P)$ is IFD connected in $A_{\sim}$.

Case (ii): Suppose $C_{1\sim} \subset D_{\sim}$, $(\psi_1 * \psi_2)^{-1}(P) = (\psi_2)^{-1}(P)$. Since ψ_2 is an IFD connected map $\psi_2^{-1}(P)$ is IFD connected in $B_{\sim}$.

Case (iii): If $C_{1\sim}$ is partly contained in $C_{\sim}$ and partly contained in $D_{\sim}$, then Q_0 must belong to $C_{1\sim}$. Now assume the path $\rho \subset C_{1\sim}$ where $P_1, P_3, P_5 \ldots, \in C_{\sim} \cap C_{1\sim}$

and $P_2, P_4, P_6 \ldots, \in D_{\sim} \cap C_{1\sim}$.

Let $A_{1\sim} = \bigcup_{i=1,3,\ldots} \psi_1^{-1}(P_i)$ and $A_{2\sim} = \bigcup_{i=2,4,\ldots} \psi_2^{-1}(P_i)$. Then $A_{1\sim}$ and $A_{2\sim}$ are IFD connected sets containing P. Hence $(\psi_1 * \psi_2)^{-1}(P) = A_{1\sim} \cup A_{2\sim}$ is an IFD connected subset of $(\psi_1 * \psi_2)^{-1}(C_{1\sim})$. Thus $(\psi_1 * \psi_2)^{-1}(C_{1\sim})$ is IFD connected.

Conversely assume $\psi_1 * \psi_2$ is IFD connected map. Then ψ_1 and ψ_2 are IFD continuous surjections. Now let $C_{1\sim}$ be an IFD connected subset of $\mathscr{D}(C_{\sim})$ and let $C_{2\sim}$ be an IFD connected subset of $\mathscr{D}(B_{\sim})$. Since $\psi_1 * \psi_2$ is an IFD connected map, $(\psi_1 * \psi_2)^{-1}(C_{1\sim})$ and $(\psi_1 * \psi_2)^{-1}(C_{2\sim})$ are IFD connected sets in $A_{\sim} \cup B_{\sim}$. But $(\psi_1 * \psi_2)^{-1}(C_{1\sim}) = (\psi_1)^{-1}(C_{1\sim})$ is an IFD connected subset of $\mathscr{D}(A_{\sim})$ and $(\psi_1 * \psi_2)^{-1}(C_{2\sim}) = (\psi_2)^{-1}(C_{2\sim})$ is an IFD connected subset of $\mathscr{D}(B_{\sim})$. Hence ψ_1 and ψ_2 are IFD connected maps.

V. CONCLUSION

Intuitionistic fuzzy logic has massive applications in image processing. Being continuity and connectivity the topological properties, they are very useful in analyzing the properties of the parts of the digital images. Thus we have developed the concepts of intuitionistic fuzzy digital continuous maps and intuitionistic fuzzy digital connected maps between any two intuitionistic fuzzy digital images and their properties. Also we defined intuitionistic fuzzy digital homeomorphisms and analyzed its properties.

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