Oscillatory Behavior of Solutions of Fourth-order Mixed Neutral Difference Equations with Asynchronous Non Linearities

Shirmila Premkumari. K

Abstract*: *In this article, oscillation criteria for solutions of fourth order mixed type neutral difference equation with asynchronous non linearities of the form

$$\Delta^2(a_n\Delta^2(x_n + b_n x_{n-\tau_1} + c_n x_{n+\tau_2})) + q_n x^{\alpha}_{n+1-\sigma_1} + p_n x^{\beta}_{n+1+\sigma_2} = 0$$

where{an}, {bn}, {cn}, {qn} and {pn} are established. Examples are provided to illustrate the results.

Keywords*: *Oscillation, Neutral difference equation, asynchronous.

I. INTRODUCTION

Neutral difference equations exist in stability theory, exist theory, network systems and so on. It has applications in problems dealing with vibrating masses to elastic bar and variational problems. Consider the fourth-order mixed type neutral difference equation with asynchronous non linearities of the form

$$\Delta^2(a_n\Delta^2(x_n + b_n x_{n-\tau_1} + c_n x_{n+\tau_2})) + q_n x^{\alpha}_{n+1-\sigma_1} + p_n x^{\beta}_{n+1+\sigma_2} = 0 \qquad (1)$$

where $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, $\{p_n\}$ and $\{q_n\}$ are positive real sequences, α and β are ratios of positive odd integers τ_1, τ_2, σ_1 and σ_2 are positive integers and $n \in N$ where $N = \{n_0, n_0 + 1, n_0 + 2, \ldots\}$, n_0 is a non negative integers. The forward difference operator is defined by $\Delta x_n = x_{n+1} - x_n$.

Let $\theta = \max\{\tau, \sigma_1\}$. By a solution of (1) a real sequence $\{x_n\}$ which is defined for all $n \geq n_0 - \theta$ and satisfies equation (1) for all $n \in N$. A non trivial solution$\{x_n\}$ is said to be non oscillatory if it is either eventually positive or eventually negative and it is oscillatory otherwise .

In the past two years there has been an increasing interest in the study of oscillatory behavior of solution of difference equations. See [1–10] and reference cited therein. If $\alpha = \beta$ in (1) then it is a synchronous case. If $\alpha = \beta$, then (1) is an equation with asynchronous non linearities.

In this paper we discuss the oscillatory and asymptotic behavior of solutions of equations with asynchronous non linearities.

I. SOME PRELIMINARY LEMMAS

In this section, we present some oscillation criteria for equation (1). For all suf- ficiently large n, consider a functional inequality holds and assume the following conditions.

(H1) $\{a_n\}$ is a positive non-decreasing sequence such that$\sum_{n=n_0}^{\infty} \frac{1}{a_n} = \infty$;

(H2) $\{b_n\}$ and $\{c_n\}$ are real sequences such that $0 \leq b_n \leq b$ and $0 \leq c_n \leq c$ with
$b + c < 1$;

(H3) $\{P_n\}$ and $\{Q_n\}$ are positive real sequences;

(H4) α and β are both rations of odd positive integer τ_1, τ_2, σ_1 and σ_2 are nonnegative integers.

Result:

Here we adopt the following notations:

$y_n = x_n + b_n x_{n-\tau_1} + c_n x_{n+\tau_2}$

$Q_n = \min\{q_n, q_{n-\tau_1}, q_{n+\tau_2}\}$

$P_n = \min\{P_n, P_{n-\tau_1}, P_{n+\tau_2}\}$

$W_s = Q_s + M^{\beta-\alpha} P_s$

Lemma 2.1. Assume $A \geq 0$, $B \geq 0$, $\alpha \geq 1$ $(A + B)^{\alpha} \leq 2^{\alpha-1}(A^{\alpha} + \beta^{\alpha})$ (see [11]).

Lemma 2.2. Let $\{x_n\}$ be a positive solution of equation (1). Then there are two cases hold for $n \geq n_1 \in N$ sufficiently large:

(1) $y_n > 0$, $\Delta y_n > 0$, $\Delta^2 y_n > 0$, $\Delta(a_n\Delta^2 y_n) \leq 0$, $\Delta^2(a_n\Delta^2 y_n) \leq 0$;

(2) $y_n > 0$, $\Delta y_n < 0$, $\Delta^2 y_n > 0$, $\Delta(a_n\Delta^2 y_n) \leq 0$, $\Delta^2(a_n\Delta^2 y_n) \leq 0$.

(see [11])

Lemma 2.3. Let $y_n > 0$, $\Delta y_n > 0$, $\Delta^2 y_n > 0$, $\Delta^3 y_n \leq 0$, $\Delta^4 y_n \leq 0$ for all $n \geq N \in N$. Then for any $\xi \in (0, 1)$ and some integer N_1, the following inequalities

$$\frac{y_{n+1}}{\Delta y_n} \geq \frac{n-N}{2} \geq \frac{\xi_n}{2} \qquad (2)$$

for $n \geq N_1 > N$ hold.

Lemma 2.4. Suppose that$\{x_n\}$ be a positive solution of equation (1) with the upper bound M , and the corresponding yn satisfies (2) of lemma 2.2. Also if

$$\sum_{n=n_0}^{\infty} \sum_{s=n}^{\infty} \left(\frac{1}{a_s} \sum_{t=s}^{\infty} \left(q_t + M^{\beta-\alpha} P_t \right) \right) = \infty \qquad (3)$$

holds, then $\lim_{n\to\infty} x_n = 0$.

Proof.

Let $\{x_n\}$ be a positive solution of equation (1) satisfying $x_n \leq M$.

$\lim y_n = \lambda \geq 0$ exists. It can be proved that $\lambda = 0$. If not, $\lambda > 0$,

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and for any $n\to\infty$ > 0, we have $\lambda+\epsilon >$ eventually. Choose

$$0<\epsilon<\frac{\lambda(1-b-c)}{b+c}$$

then we have $x_n = y_n - b_n x_{n-\tau_1} - c_n x_{n+\tau_2}$
$> \lambda-(b+c)y_{n-\tau_1}$
$> \lambda-(b+c)(\lambda+\epsilon)$

$= g(\lambda+\epsilon)$
$> g y_n$,
where

$$g = \frac{\lambda-(b+c)(\lambda+\epsilon)}{\lambda+\epsilon} > 0.$$

Further,

$$\Delta(a_n\Delta y_n) \le -q_n g^\alpha y^\alpha_{n+1-\sigma_1} - P_n g y^\beta_{n+1-\sigma_2} \le -g^\alpha(q_n + M^{\beta-\alpha}P_n)y^\alpha_{n+1-\tau_1}$$

Summing the above inequality from n to ∞, and using the relation $y_n \ge \lambda$, we have

$$\Delta^2 y_n \ge (g\lambda)^\alpha\left(\frac{1}{a_n}\sum_{r=n}^{\infty}(q_r + M^{\beta-\alpha}p_r)\right) \quad (4)$$

Summing the two sides of (4) from n to ∞

$$-\Delta y_n \ge (g\lambda)^\alpha\left(\sum_{s=n}^{\infty}\frac{1}{a_s}\sum_{t=s}^{\infty}(q_t + M^{\beta-\alpha}p_t)\right)$$

Since, $y_n > 0$, $\Delta y_n < 0$.
Summing from n_1 to ∞ leads to

$$y_{n_1} \ge (g\lambda)^\alpha\sum_{n=n_1}^{\infty}\sum_{s=n}^{\infty}\frac{1}{a_s}\sum_{t=s}^{\infty}(q_t + M^{\beta-\alpha}p_t)$$

which is a contradiction to (3). Then $\lambda = 0$, which together with the inequality $0 < x_n < y_n$ implies that $\lim_{n\to\infty} x_n = 0$. The proof is complete.

Let $Q_n = \min\{q_n, q_{n-\tau_1}, q_{n+\tau_2}\}$, $P_n = \min\{P_n, P_{n-\tau_1}, P_{n+\tau_2}\}$

Theorem 2.1. Suppose that $\{x_n\}$ be a bounded positive solution of equation (1) with the upper bound M , and the condition (3) holds, $\sigma_1 \ge \tau_1$ and $\alpha_1 \ge \beta \ge 1$. If there exists a positive real sequence $\{\eta_n\}$ and an integer $N_1 \in$ such that for some $\xi \in (0, 1)$ and $\delta > 0$.

$$\limsup_{n\to\infty}\sum_{s=N_1}^{n-1}\left(\eta_s\left(\left(\frac{\delta}{4}\right)^{\alpha-1}\frac{\xi(n-\sigma_1)^\alpha}{2^\alpha}\omega_s\right) - \left(\frac{1+b^\alpha+\frac{c^\beta}{2^{\beta-1}}}{4}\right)\times\left(\frac{a_{s-\sigma_1}(\Delta\eta_s)^2}{\eta_s}\right)\right) \quad (5)$$

holds, then every such solution $\{x_n\}$ of equation (1) oscillates or $\lim_{n\to\infty} x_n = 0$.

Proof.

Let $\{x_n\}$ be a non oscillatory solution of equation (1), and $x_n \le M$. Let that there exists an integer $N \ge n_0$ such that x_n, $x_{n-\sigma_1}$, $x_{n+\sigma_2}$, $x_{n-\tau_1}$, $x_{n+\tau_2} \in (0, M]$ for all $n > N$, we have,

$$\Delta^2(a_n\Delta^2 y_n) + q_n x^\alpha_{n+1-\sigma_1} + P_n x^\beta_{n+1+\sigma_2} + b^\alpha\Delta^2(a_{n-\tau}\Delta^2 y_{n-\tau_1}) + b^\alpha q_{n-\tau} x_{n+1-\tau_1-\sigma_1} + b^\alpha p_{n-\tau_1} x^\beta_{n+1-\tau_1-\sigma_2} + \frac{c^\beta}{2^{\beta-1}}\Delta^2(a_{n+\tau_2}\Delta^2 y_{n+\tau_2}) + \frac{c^\beta}{2^{\beta-1}} q_{n+\tau_2} x^\alpha_{n+1+\tau_2-\sigma_1} + \frac{c^\beta}{2^{\beta-1}} p_{n+\tau_2} x^\beta_{n+1+\tau_2-\sigma_2} = 0 \quad (6)$$

By lemma (2.1) and $\beta \le \alpha$ in (6), we have

$$\Delta^2(a_n\Delta^2 y_n) + b^\alpha\Delta^2(a_{n-\tau_1}\Delta^2 y_{n-\tau_1}) + \frac{c^\beta}{2^{\beta-1}}\Delta^2(a_{n+\tau_2}\Delta^2 y_{n+\tau_2}) + \frac{c^\beta Q_n M^{\beta-\alpha}}{2^{\beta-1}4^{\alpha-1}}y^\alpha_{n+1-\sigma_1} + y^\alpha_{n+1+\sigma_2} \le 0 \quad (7)$$

By lemma 2.2, there are two cases for y_n to be considered. Assume that (1) holds for all $n \ge N_1 \ge N$.

It follows from $\Delta y_n > 0$, that $y_{n+\sigma_2} > y_{n-\sigma_1}$.

$$\Delta^2(a_n\Delta^2 y_n) + b^\alpha\Delta^2(a_{n-\tau_1}\Delta^2 y_{n-\tau_1}) + \frac{c^\beta}{2^{\beta-1}}\Delta^2(a_{n+\tau_2}\Delta^2 y_{n+\tau_2}) + \frac{Q_n}{4^{\alpha-1}}y^\alpha_{n+1-\sigma_1} \le 0 \quad (8)$$

Define

$$v_1(n) = \eta_n\frac{a_n\Delta^2 y_n}{\Delta y}, \ n \ge N_1. \quad (9)$$

Then $v_1(n) > 0$ for $n \ge N_1$.

$$\Delta v_1(n) = \frac{\Delta\eta_s}{\eta_{s+1}}v_1(n+1) + \eta_s\frac{\Delta(a_n\Delta^2 y_n)}{\Delta y_{n-\sigma_1}} - v_1(n+1)\frac{\Delta^2 y_{n-\sigma_1}}{\Delta y_{n-\sigma_1}}$$

$$\Delta^2(a_n\Delta^2 Z_n) = -q_n x^\alpha_{n+1-\sigma_1} - p_n x^\beta_{n+1+\sigma_2} < 0 \quad (10)$$

and

$$a_{n-\sigma_1}\Delta y_{n-\sigma_1} \ge a_{n+1}\Delta y_{n+1}.$$

From (9), we have

$$\Delta v(n) \le \frac{\Delta\eta_n}{\eta_{n+1}}v_1(n+1) + \eta_n\frac{\Delta(a_n\Delta^3 y_n)}{\Delta y_{n-\sigma_1}} - \frac{\eta_n}{\eta_{n+1}^2 a_{n-\sigma_1}}v^2(n+1) \quad (11)$$

$$v_2(n) = \eta_{n-1}\frac{a_n\Delta^2 y_{n-\tau}}{\Delta y_{n-\sigma_1}}, \ n \ge N_1 \quad (12)$$

Then $v_2(n) > 0$ for $n > N_1$, which together with (12) yields

$$\Delta v_2(n) \le \frac{\Delta\eta_n}{\eta_{n+1}}v_2(n+1) + \eta_n\frac{\Delta(a_{n-\tau_1}\Delta^2 y_{n-\tau_1})}{\Delta y_{n-\sigma_1}} - \frac{\eta_n}{\eta^2_{n+1}a_{n-\sigma_1}}v_2^2(n+1). \quad (13)$$

Similarly define

$$v_3(n) = \eta_n\frac{a_{n+\tau_2}\Delta y^2_{n+\tau_2}}{\Delta y_{n-\sigma_1}}, \ n \ge N_1. \quad (14)$$

Then we have,

$$\Delta v(n) \le \frac{\Delta\eta_n}{\eta_{n+1}}v_3(n+1) + \eta_n\frac{\Delta(a_{n+\tau_2}\Delta^3 y_{n+\tau_2})}{\Delta y_{n-\sigma_1}} - \frac{\eta_n}{\eta^2_{n+1}a_{n-\sigma_1}}v_3^2(n+1) \quad (15)$$

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$$\Delta v_1(n) + b^\alpha \Delta v_2(n) + \frac{c^\beta}{2^{\beta-1}} \Delta v_3(n)$$

$$\leq -\eta_n \frac{Q_n y^\alpha_{n+1-\sigma_1}}{4^{\alpha-1} \Delta y_{n-\sigma_1}} + \frac{\Delta\eta_n}{\eta_{n+1}} - \eta_n \frac{v_1^2(n+1)}{\eta^2_{n+1} a_{n-\sigma_1}}$$

$$+ b^\alpha \frac{\Delta\eta_n v_2(n+1)}{\eta_{n+1}} - \eta_n \frac{v_2^2(n+1)}{\eta^2_{n+1} a_{n-\sigma_1}}$$

$$+ \frac{c^\beta}{2^{\beta-1}} \frac{\Delta\eta_n v_3(n+1)}{\eta_{n+1}} - \eta_n \frac{v_3^2(n+1)}{\eta^2_{n+1} a_{n-\sigma_1}} \quad (16)$$

Since $\{a_n\}$ is nondecreasing and $\Delta^2 y_n > 0$, $\Delta^3 y_n < 0$, $\Delta^4 y_n < 0$ for $n \geq N_1$. By lemma (2.3) for any $\xi \in (0, 1)$,

$$\frac{y_{n+1-\sigma_1}}{\Delta y_{n-\sigma_1}} \geq \frac{\xi(n-\sigma_1)}{2} \quad (17)$$

$$\Delta v_1(n) + b^\alpha \Delta v_2(n) + \frac{c^\beta}{2^{\beta-1}} \Delta v_3(n) \leq -\eta_n \frac{\delta^{\alpha-1}}{4} \frac{\xi(n-\sigma_1)^\alpha}{2^\alpha} [Q_s + M^{\beta-\alpha} P_n] + \frac{\left(1 + b^\alpha + \frac{c^\beta}{2^{\beta-1}}\right) a_{n-\sigma_1} (\Delta\eta_n)^2}{4\eta_n}$$

Summing the inequality from $N_2 \geq N_1$ to $n-1$.

$$\sum_{s=N_2} \eta_s \left(\left(\frac{\delta}{4}\right)^{\alpha-1} \frac{\xi(n-\sigma_1)^\alpha}{2^\alpha} [Q_s + M^{\beta-\alpha} P_s] \right) + \frac{\left(1 + b^\alpha + \frac{c^\beta}{2^{\beta-1}}\right) a_{s-\sigma_1} (\Delta\eta_s)^2}{4\gamma_s}$$

$$\leq v_1(N_2) + b^\alpha v_2(N_2) + \frac{c^\beta}{2^{\beta-1}} v_3(N_2) \quad (18)$$

Let $\eta_n = N$, $\alpha = \beta = 1$. Then the following corollary is easily obtained.

Corollary 2.1. Suppose that $\{x_n\}$ be a bounded positive solution of equation (1) with the upper bound M , and $\alpha = 1$. Assume that (5) holds and $\sigma_1 > \tau_1$. If there is an integer $N_1 \in N$ such that for some $\xi \in (0, 1)$ and $\delta > 0$.

$$\limsup_{n\to\infty} \sum_{s=N_1}^{n-1} \left(\eta_s \left(\frac{\delta}{4}\right)^{\alpha-1} \frac{\xi(n-\sigma_1)^\wedge\alpha}{2^\alpha} W_s - \frac{1+b+c}{4s} a_{s-\sigma_1} \right)$$

holds, then every such solution $\{x_n\}$ of (1) oscillate (or) $\lim_{n\to\infty} x_n = 0$, where

$$W_s = Q_s + M^{\beta-\alpha} P_s.$$

Theorem 2.2. Suppose that $\{x_n\}$ be a bounded positive solution of equation (1) with the upper bound M . Assume that the condition (7) holds, $\sigma_1 < T_1$, and $\alpha \geq \beta \geq 1$. If there exists a positive real sequence η_n, and an integer $N_1 \in$ such that for some $\xi \in (0, 1)$ and $\delta > 0$

$$\limsup_{n\to\infty} \sum_{s=N}^{n-1} \eta_s \left(\frac{\delta}{4}\right)^{\alpha-1} \frac{\xi(n-\sigma_1)^\wedge\alpha}{2^\alpha} \omega_s - \frac{\left(1 - b^\alpha + \frac{c^\beta}{2^{\beta-1}}\right) a_{s-\tau_1} (\Delta\tau_s)^2}{4\tau_s} \quad (19)$$

holds, then every solution $\{x_n\}$ of equation (1) oscillates or $\lim_{n\to\infty} x_n =$.

Proof.

Assume that case 1 of lemma 2.2 holds for all $n \geq N_1 \geq \mathbb{N}$. Then define,

$$v_1(n) = \eta_n \frac{a_n \Delta^2 y_n}{\Delta y_{n-\tau_1}}, \quad n \geq N_1,$$

$$v_2(n) = \eta_n \frac{a_{n-\tau_1} \Delta^2 y_{n-\tau_1}}{\Delta y_{n-\tau_1}}, \quad n \geq N_1,$$

$$v_3(n) = \eta_n \frac{a_{n+\tau_2} \Delta^2 y_{n+\tau_2}}{\Delta y_{n+\tau_2}}, \quad n \geq N_1,$$

We have,

$$\Delta v_1(n) + b^\alpha v_2(n) + \frac{c^\beta}{2^{\beta-1}} \Delta v_3(n) \leq -\eta_n \frac{Q_n}{4^{\alpha-1}} \frac{Z^\alpha_{n+1-\tau_1}}{\Delta y_{n-\tau_1}} + \frac{\Delta\eta_n}{\eta_{n+1}}$$

$$-\eta_n \frac{\omega_1^2(n+1)}{\eta^2_{n+1} a_{n-\tau_1}} + b^\alpha \left(\frac{\Delta\eta_n v_2(n+1)}{\eta_{n+1}} - \eta_n \frac{\omega}{\eta} \right.$$

$$+ \frac{c^\beta}{2^{\beta-1}} \left(\frac{\Delta\eta_n v_3(n+1)}{\eta_{n+1}} - \eta_n \frac{\omega_3^2(n+1)}{\eta^2_{n+1} a_{n-\tau_1}} \right) \quad (20)$$

On the other hand, we have for any $\xi \in (0, 1)$

$$\frac{y_{n+1-\sigma_1}}{\Delta y_{n-\tau_1}} = \frac{y_{n+1-\sigma_1}}{\Delta y_{n-\sigma_1}} \frac{y_{n+1-\sigma_1}}{\Delta y_{n-\tau_1}} \geq \frac{\xi(n-\sigma_1)}{2}$$

for all $n \geq N_2$, we obtain

$$\Delta v_1(n) + b^\alpha v_2(n) + \frac{c^\beta}{2^{\beta-1}} \Delta v_3(n) \leq -\eta_n \omega_n + \frac{\left(1 + b^\alpha + \frac{c^\beta}{2^{\beta-1}}\right) a_n}{4\gamma_n}$$

The proof is similar to theorem 2.1 Summing the inequality from N_2 to $n-1$, we obtain

$$\sum_{s=N_2}^{n-1} \eta_s \left(\left(\frac{\delta}{4}\right)^{\alpha-1} \frac{\xi(n-\sigma_1)^\alpha}{2^\alpha} [Q_s + M^{\beta-\alpha} P_s] \right) + \left(1 + b^\alpha + \frac{c^\beta}{2^{\beta-1}}\right) a_s$$

$$\leq v_1(N_2) + b^\alpha v_2(N_2) + \frac{c^\beta}{2^{\beta-1}} v_3(N_2)$$

takinglim sup on both sides yields a contradiction to (19). The case (2) can be proved similarly. The proof is completed.

Let $\eta_n = n$, $\alpha = \beta = 1$. Then we obtain the following result.

Corollary 2.2. Suppose that $\{x_n\}$ be a bounded possible solution of equation (1)with the upper bound M . Assume that condition (3) holds and $\tau_1 \geq \sigma_1$. If

$$\limsup_{n\to\infty} \sum_{s=N_1}^{n-1} \left(\eta_s \left(\frac{\delta}{4}\right)^{\alpha-1} \frac{\xi(n-\sigma_1)^\alpha}{2^\alpha} W_s - \frac{1+b+c}{4\eta_s} q_{s-\tau_1} \right) = \infty$$

holds for all sufficiently large N , then every such solution $\{x_n\}$ of equation (1)oscillates or $\lim_{n\to\infty} x_n = 0$ where $W_s = Q_s + M^{\beta-\alpha} P_s$.

Theorem 2.3. Assume that condition (3) holds, $\sigma_1 \geq \tau_1$ and $1 \leq \alpha \leq \beta$. If there exist a positive real sequence $\{\eta_n\}$ and an integer $N_1 \in$ with

$$\lim_{n\to\infty} \sup \sum_{s=N_2}^{n-1} \left(\eta_s \left(\frac{\delta}{4}\right)^{\alpha-1} \frac{\xi(n-\sigma_1)^\alpha}{2^\alpha} W_s - \frac{\left(1 + b^\alpha + \frac{c^\beta}{2^{\beta-1}}\right) a_{s-\tau_1} (\Delta\tau_s)^2}{4\tau_s} \right) = \infty \quad (23)$$

holds, then every solution $\{x_n\}$ of equation (1) oscillates (or) $\lim_{n\to\infty} x_n = 0$.

Theorem 2.4. Assume that condition (3) holds, $\sigma_1 \leq \tau_1$ and $1 \leq \alpha \leq \beta$. If there is a positive real sequence $\{\eta_s\}$ and an integer $N_1 \in \mathbb{N}$ with

$$\limsup_{n\to\infty} \sum_{s=N_2}^{n-1} \left(\eta_s \left(\frac{\delta}{4}\right)^{\alpha-1} \frac{\xi(n-\sigma_1)^\alpha}{2^\alpha} W_s - \frac{1 + b^\alpha + \frac{c^\beta}{2^{\beta-1}} a_{s-\tau_1} (\Delta\tau_s)^2}{4\tau_s} \right) = \infty \quad (24)$$

holds, then every solution $\{x_n\}$ of equation (1) oscillates (or) $\lim_{n\to\infty} x_n = 0$.

I. EXAMPLE

We present two examples to illustrate the main results.

Example 3.1. Consider the fourth order difference equation

$$\Delta^4 \left(x_n + \frac{1}{3} x_{n-1} + \frac{1}{3} x_{n+1} \right) + \frac{2^n}{320} x^2_{n-2} + \frac{31}{30} 4^n x^3_{n+1} = 0, n \geq 1$$

$a_n = 1$, $b_n = c_n = \frac{1}{3}$, $q_n = \frac{2^n}{320}$,
$p_n = \frac{31}{30} 4^n$, $\tau_1 = 1$, $\tau_2 = 1$,

$\sigma_1 = 3, \sigma_2 = 0, \alpha = 2 \leq \beta = 3.$

Take $\eta_n = 1$. It is easy to verity that (3) hold. On the otherhand, condition (5) is also true.

Therefore by theorem (6), every Solution $\{x_n\}$ of (25)

$$\lim_{n\to\infty} x_n = 0$$

Example 3.2. Consider the fourth order difference equation

$$\Delta^2\left(a_n\Delta^2\left(x_n + b_n x_{n-\tau_1} + c_n x_{n+\tau_2}\right)\right) + \frac{c}{n}x^2_{n+1-\sigma_1} + \frac{d}{n}x_{n+1+\sigma_2} = 0 \qquad (26)$$

wherec,d are positive constant, $0 \leq b_n \leq b$, $0 \leq c_n \leq c$ and $b + c < 1$. Let $a_n = n$, $b_n = c_n = \frac{1}{3}$, $q_n = \frac{c}{n}$, $p_n = \frac{d}{n}$, $\sigma = 2 > \beta = 1$. Take η_n. It is easy to see that theorem (6), (or) theorem (8) is well satisfied.

Therefore every solution $\{x_n\}$ of equation (26) oscillates (or) $\lim_{n\to\infty} x_n =$.

II. CONCLUSION

In this paper we solve non linear differential equation and inequality of differential equations in our real life problems. We find new theorem, definition and lemmas of those inequalities and those non linear differential equations.

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