Fuzzy Almost Generalized E-Continuous Mappings in Smooth Topological Spaces

B. Vijayalakshmi, A. Vadivel

***Abstract*: We introduce and study several interesting properties of fuzzy almost generalized e-continuous mappings in smooth topological spaces with counter examples. We also introduce fuzzy r-$fT_{\frac{1}{2}}e$-space, r-fuzzy ge-space, r-fuzzy regular ge-space and r-fuzzy generalized e-compact space. It is seen that a fuzzy almost generalized e-continuous mapping, between a fuzzy r-$fT_{\frac{1}{2}}e$-space and a fuzzy topological space, becomes fuzzy almost continuous mapping.**

Index Terms: ***$fage$*-continuous, r-fge-space, r-fge-regular space, r-$fT_{\frac{1}{2}}e$-space.**

I. INTRODUCTION

See all undefined notions, of fuzzy sets in [28], of fuzzy topolological spaces in [[1]-[4]] [[10]-[15]], [[19]-[24]] and [26], of e-open sets in topology in [[5]-[9]].

Definition 1.1 *Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from a fts (X,τ_1) to another (Y,τ_2). Then f is called fuzzy e-irresolute [27] if $f^{-1}(\lambda)$ is a r-feo set in X for any r-feo set λ in Y.*.

II. FUZZY ALMOST GENERALIZED e-CONTINUOUS MAPPINGS

Definition 2.1 *A mapping $f:(X,\tau)\to(Y,\eta)$ is said to be fuzzy almost generalized e-continuous (in short, $fage$-Cts)mapping if $f^{-1}(\lambda)$ is r-fuzzy generalized e-open (in short, r-fgeo) set in X for every r-fuzzy regular open (in short, r-fro) set λ in Y.*

Theorem 2.1 *Let f be $fage$-Cts mapping iff $f^{-1}(\lambda)$ is r-fuzzy generalized e-closed (in short, r-fgec) set in X for every r-fuzzy regular closed (in short, r-frc) set λ in Y.*

Example 2.1 Consider the identity mapping $f:(X,\tau)\to(X,\eta)$, where $X=\{x,y,z\}$ and fuzzy topologies (in short, FT) τ, $\eta: I^X \to I$ on X, given by

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\overline{0} or \overline{1},\\ \frac{1}{2}, & \text{if } \lambda=\lambda_1,\lambda_2,\lambda_3,\lambda_4,\\ 0, & \text{otherwise.}\end{cases} \qquad \eta(\lambda)=\begin{cases}1, & \text{if} \lambda=\overline{0} \text{or} \overline{1},\\ \frac{1}{2}, & \text{if} \lambda=\lambda_4,\\ 0, & \text{otherwise.}\end{cases}$$

with
$\lambda_1(x)=0.4,\ \lambda_1(y)=0.6,\ \lambda_1(z)=0.5$;
$\lambda_2(x)=0.6,\ \lambda_2(y)=0.4,\ \lambda_2(z)=0.4$;
$\lambda_3(x)=0.6,\ \lambda_3(y)=0.6,\ \lambda_3(z)=0.5$;
$\lambda_4(x)=0.4,\ \lambda_4(y)=0.4,\ \lambda_4(z)=0.4$.
Then f is a fage-Cts mapping.

Remark 2.1 *It follows from Definitions, that every fuzzy generalized e continuous (in short, fge-Cts) mapping is an $fage$-Cts mapping.*

The forthcoming example shows that the converse of the Remark 2.1 is not true.

Example 2.2 Consider the identity mapping $f:(X,\tau)\to(X,\eta)$, where $X=\{x,y,z\}$ and FT τ, $\eta: I^X \to I$ on X given by

$$\tau(\lambda)=\begin{cases}1, & \text{if} \lambda=\overline{0} \text{or} \overline{1},\\ \frac{1}{2}, & \text{if} \lambda=\delta_1,\delta_2,\\ 0, & \text{otherwise,}\end{cases} \qquad \eta(\lambda)=\begin{cases}1, & \text{if} \lambda=\overline{0} \text{or} \overline{1},\\ \frac{1}{2}, & \text{if} \lambda=\delta_3,\delta_4,\\ 0, & \text{otherwise.}\end{cases}$$

with
$\delta_1(x)=0.3,\ \delta_1(y)=0.4,\ \delta_1(z)=0.5$;
$\delta_2(x)=0.6,\ \delta_2(y)=0.5,\ \delta_2(z)=0.5$;
$\delta_3(x)=0.4,\ \delta_3(y)=0.4,\ \delta_3(z)=0.4$;
$\delta_4(x)=0.5,\ \delta_4(y)=0.4,\ \delta_4(z)=0.5$.
Then f is fage-Cts mapping but f is not fge-Cts mapping. Since ω is $\frac{1}{2}$-fuzzy open set in Y,but $f^{-1}(\omega)$ is not $\frac{1}{2}$-fgeo in X.

Revised Manuscript Received on July 08, 2019.

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Theorem 2.2 *The statements (i) - (iv) are equivalent for every fuzzy mapping* $f:(X,\tau)\to(Y,\eta)$.

(1) f is faeg Cts.

(2) for each fuzzy point x_p in X and each r -fro q -nbd δ_2 of $f(x_p), \exists f(x_p)$ q -nbd λ of $x_p \ni f(\lambda,r)\le\delta_2$.

(3) $\forall\lambda$ in X, $f(geC_\tau(\lambda,r))\le eC_\tau(f(\lambda),r)$.

(4) $\forall\delta_2$ in Y, $geC_\eta(f^{-1}(\delta_2),r)\le f^{-1}(eC_\eta(\delta_2,r))$.

Proof. $(1)\Rightarrow(2)$: We know that $f(x_p)$ is a fuzzy point in $Y\ \forall\ x_p\in I^X$. Then Now, let $\delta_2\in I^Y$ be a r -fro $\ni f(x_p)q\delta_2$. For, $\lambda=f^{-1}(\delta_2)$ since f is fage continuous and δ_2 is r -fro set in X, we have λ is r -fgeo set of X and $x_p\in\lambda$. Therefore, $f(\lambda,r)=f(f^{-1}(\delta_2),r))\le\delta_2$.

$(2)\Rightarrow(3)$: Let $x_p\in geC_\tau(\lambda,r)$. Then $x_p q\lambda$ and $f(x_p)qf(\lambda)$ implies $f(x_p)\in eC_\tau(f(\lambda),r))$ and $x_p\in f^{-1}(eC_\tau(f(\lambda),r))$. Therefore, $geC_\tau(\lambda,r)\le f^{-1}(eC_\tau(f(\lambda),r)$.

$(3)\Rightarrow(4)$ and $(4)\Rightarrow(1)$ are clear.

Theorem 2.3 *A fage-Cts mapping* $f:(X,\tau_1)\to(Y,\tau_2)$ *with* (X,τ_1) *be an* r*-fuzzy* $T_{\frac{1}{2}}e$ *(in short,* $fT_{\frac{1}{2}}e$*)-space is fuzzy almost continuous (in short, fa-Cts).*

Proof. Let $\lambda\in I^Y$ be any r -frc. As f is fage-Cts, $f^{-1}(\lambda)\in I^X$ is r -fgec and since X is r - $fT_{\frac{1}{2}}e$ -space, $f^{-1}(\lambda)$ is r -fuzzy closed (in short, r -fc) set in X. Thus f is fa-Cts.

Example 2.3 Consider the identity mapping $f:(X,\tau)\to(X,\eta)$, where $X=\{x,y,z\}$ and FT τ, $\eta: I^X\to I$ on X, given by

$$\tau(\lambda)=\begin{cases}1, & \text{if}\lambda=\bar{0}\text{or}\,\bar{1},\\ \frac{1}{2}, & \text{if}\lambda=\delta_1,\delta_2,\delta_3,\delta_4,\\ 0, & \text{otherwise.}\end{cases}\qquad \eta(\lambda)=\begin{cases}1, & \text{if}\lambda=\bar{0}\text{or}\,\bar{1},\\ \frac{1}{2}, & \text{if}\lambda=\delta_5,\\ 0, & \text{otherwise.}\end{cases}$$

with

$\delta_1(x)=0.3,\ \delta_1(y)=0.4,\ \delta_1(z)=0.5$;

$\delta_2(x)=0.6,\ \delta_2(y)=0.5,\ \delta_2(z)=0.5$;

$\delta_3(x)=0.6,\ \delta_3(y)=0.5,\ \delta_3(z)=0.4$;

$\delta_4(x)=0.3,\ \delta_4(y)=0.4,\ \delta_4(z)=0.4$;

$\delta_5(x)=0.8,\ \delta_5(y)=0.4,\ \delta_5(z)=0.5$.

Then f is a fuzzy generalized e -irresolute (in short, fge -Irr.) and fuzzy e -irresolute (in short, fe -Irr.) mapping.

Theorem 2.4 *If* $f:X\to Y$ *is fge-Irr. and* $g:Y\to Z$ *is fage-Cts, then* $g\circ f:X\to Z$ *is fage-Cts.*

Proof. Let $\lambda\in I^Z$ be an r -fro. Since g is fage-Cts, $g^{-1}(\lambda)\in I^Y$ is r -fgeo. As f is fge-Irr., $f^{-1}(g^{-1}(\lambda))\in I^X$ is r -fgeo. This implies $(g\circ f)^{-1}(\lambda)\in I^X$ is r -fgeo. Thus, $g\circ f$ is fage-Cts.

Theorem 2.5 *If* $f:X\to Y$ *is an fuzzy continuous (In short, f-Cts) mapping and* $g:Y\to Z$ *is an fage-Cts mapping, where* Y *is* r *-* $fT_{\frac{1}{2}}e$ *-space, then* $g\circ f:X\to Z$ *is fa-Cts mapping.*

Proof. Let $\lambda\in I^Z$ be any r -frc. Since g is fage-Cts mapping, $g^{-1}(\lambda)\in I^Y$ is r -fgec. As Y is r - $fT_{\frac{1}{2}}e$ -space, $g^{-1}(\lambda)\in I^Y$ is r -fc. This implies $Y-g^{-1}(\lambda)$ is r -fo. As f is f-Cts, $f^{-1}(Y-g^{-1}(\lambda))\in I^X$ is r -fo $\Rightarrow$ $X-f^{-1}(g^{-1}(\lambda))$ is r -fo $\Rightarrow$ $f^{-1}(g^{-1}(\lambda)$ is r -fc $\Rightarrow$ $(g\circ f)^{-1}(\lambda)$ is r -fc Hence, $g\circ f$ is fa-Cts.

III. FUZZY GENERALIZED e -COMPACT SPACE AND FUZZY REGULAR ge -SPACE

Here we introduce the definition of fuzzy generalized e (in short, fge)-compact space and fuzzy regular ge (in short, frge)-space. Also some interesting theorems involving fge compact space and ffrge-space are established.

Definition 3.1 *An fts* (X,τ) *is said to be* r *-fge-compact space if every* r *-fuzzy generalized* e *open (in short,* r *-fgeo) cover of* X *has a finite subcover.*

Theorem 3.1 *An fage Cts surjection image of a fge compact space is fuzzy nearly (in short, fn) compact.*

Proof. Let $f:X\to Y$ be an fage-Cts surjection mapping and $\{\lambda_i\}_{i\in\Lambda}$ be an r -fro cover of Y. Since X is fge compact, $\{f^{-1}(\lambda_i)\}_{i\in\Lambda}$ is a family of r -fgeo cover of X. So, there exists a finite subset Λ_0 of Λ such that $\{f^{-1}(\lambda_i)\}_{i\in\Lambda_0}$ covers X. Since f is 1-1, onto, $\{\lambda_i\}_{i\in\Lambda_0}$ will be finite r -fro subcover of the r -fro cover $\{\lambda_i\}_{i\in\Lambda}$ of Y. So, Y is fn compact.

Definition 3.2 *An fts*

(X,τ) is said to be r-fuzzy regular generalised e (in short, frge)-space if every r-fgeo set in X is r-fro.

Remark 3.1 *There are spaces which are not r-frge-spaces.*

Example 3.1 Consider the fts, (X,τ) where $X=\{x,y,z\}$ and FT

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\bar{0} \text{ or } \bar{1},\\ \frac{1}{2}, & \text{if } \lambda=\delta_1,\delta_2,\\ 0, & \text{otherwise,}\end{cases}$$

with

$\delta_1(x)=0.3,\ \delta_1(y)=0.4,\ \delta_1(z)=0.5$;

$\delta_2(x)=0.7,\ \delta_2(y)=0.4,\ \delta_2(z)=0.5$;

$\delta_3(x)=0.8,\ \delta_3(y)=0.5,\ \delta_3(z)=0.5$;

$\delta_4(x)=0.8,\ \delta_4(y)=0.4,\ \delta_4(z)=0.5$.

Then the fts (X,τ) is not r-frge-space, since δ_3 and δ_4 are r-fgeo sets in $X,$ but not r-fro in (X,τ).

Definition 3.3 *An fts (X,τ) is said to be r-fge-space if every r-fgeo set in X is r-fo.*

Remark 3.2 *There are spaces which are not r-fge-spaces.*

Example 3.2 Consider the fts, (X,τ) where $X=\{x,y,z\}$ and FT

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\bar{0} \text{ or } \bar{1},\\ \frac{1}{2}, & \text{if } \lambda=\delta_1,\delta_2,\\ 0, & \text{otherwise,}\end{cases}$$

with

$\delta_1(x)=0.3,\ \delta_1(y)=0.5,\ \delta_1(z)=0.2$;

$\delta_2(x)=0.4,\ \delta_2(y)=0.5,\ \delta_2(z)=0.5$;

$\delta_3(x)=0.5,\ \delta_3(y)=0.7,\ \delta_3(z)=0.5$;

$\delta_4(x)=0.4,\ \delta_4(y)=0.6,\ \delta_4(z)=0.5$.

Then the fts (X,τ) is not r-fge-space, since δ_3 and δ_4 are r-fgeo but not r-fo in X.

Remark 3.3 *It is obvious from Definition 3.2 and 3.3 that every r-frge space is r-fge-space.*

Example 3.3 Consider the fts, (X,τ) where $X=\{x,y,z\}$ and FT

$$\tau(\lambda)=\begin{cases}1, & \text{if } \lambda=\bar{0} \text{ or } \bar{1},\\ \frac{1}{2}, & \text{if } \lambda=\delta_1,\delta_2,\\ 0, & \text{otherwise,}\end{cases}$$

with

$\delta_1(x)=0.3,\ \delta_1(y)=0.5,\ \delta_1(z)=0.2$;

$\delta_2(x)=0.5,\ \delta_2(y)=0.5,\ \delta_2(z)=0.5$.

Then the fts (X,τ) is r-fge-space but not r-frge-space since λ is r-fgeo and r-fo but not r-fro set in (X,τ).

Theorem 3.2 *Let $f:(X,\tau)\to(Y,\sigma)$ be fge-Cts surjection mapping. Then Y is r-fuzzy connected if X is r-fuzzy connected space and r-frge-space.*

Proof. Suppose (Y,σ) be not r-fuzzy connected, Then $\ni \lambda\in I^Y$ such that $\lambda\neq 0_Y,\ 1_Y$ and λ is both r-fo and r-fc in Y $\Rightarrow$ $f^{-1}(\lambda)$ is both r-fgeo and r-fgec in X $\Rightarrow$ $f^{-1}(\lambda)$ is both r-fro and r-frc in X $\Rightarrow$ $f^{-1}(\lambda)$ is both r-fo and r-fc in X. Also, $f^{-1}(\lambda)\neq 0_X, 1_X$. This contradicts that X is a r-fuzzy connected space, hence, Y is r-fuzzy connected.

Theorem 3.3 *Let $f:X\to Y$ be an fage Cts surjective mapping and X be r-frge-space as well as r-fn compact space. Then Y is r-fn compact.*

Proof. Let $\{\lambda_i\}_{i\in\Lambda}$ be any r-fro cover of Y. Since f is fage Cts, $f^{-1}(\lambda_i)_{i\in\Lambda}$ is a family of r-fgeo sets in X. X is r-frge-space. So, $f^{-1}(\lambda_i)_{i\in\Lambda}$ is a family of r-fro sets in X. X is r-fn compact. So, $\exists\Lambda_0$ of Λ $\ni$ $X\leq[f^{-1}(\lambda_i)_{i\in\Lambda_0}]$. Then $Y=f(X)\leq(\lambda_i)_{i\in\Lambda_0}$. So, $\{\lambda_i\}_{i\in\Lambda_0}$ is a finite subcover of r-fro sets of Y. So, Y is r-fn compact.

Theorem 3.4 *If $f:X\to Y$ is an fge Cts injective mapping and Y is a r-fuzzy Hausorff space, then X is r-fuzzy Hausdorff if it is r-frge-space.*

Proof. Let $x_p \neq y_q \in I^X$. Then $f(x_p) \neq f(y_q)$ in Y. Since Y is r-fuzzy Hausdorff, there exists r-fo nbds U and V of $f(x_p)$ and $f(y_q)$ respectively such that $U \wedge V = 0_X$. Since f is an fge Cts, $f^{-1}(U)$ and $f^{-1}(V)$ are r-fgeo in $X \Rightarrow f^{-1}(U)$ and $f^{-1}(V)$ are r-fro in X. [Since X is r-frge-space] $\Rightarrow f^{-1}(U)$ and $f^{-1}(V)$ are r-fuzzy open in X and contains respectively the fuzzy points x_p and y_q. Now, $x_p \in f^{-1}(U) = \lambda,$ say $y_q \in f^{-1}(V) = \delta_2,$ say. So, $\lambda \wedge \delta_2 = f^{-1}(U) \wedge f^{-1}(V) = f^{-1}(U \wedge V) = f^{-1}(0_X) = 0_X$. Thus, X is r-fuzzy Hausdorff.

IV. CONCLUSION

It is interesting to work under a new class of mappings viz. fage-Cts mapping, r-$fT_{\frac{1}{2}}e$-space, r-fge-compact space, r-fge-space, r-frge-space in $\check{S}$ostak's fuzzy topological spaces. It is seen that a fage-Cts surjection image of a r-fge-compact space is r-fn compact.

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