

A Special Research on Homo Cordial Labeling of Spider Graph

S.Sriram, R.Govindarajan

Abstract: Let $G=(V, E)$ be a graph with p vertices and q edges. A Homo Cordial Labeling of a graph G with vertex set V is a bijection from V to $\{0, 1\}$ such that each edge uv is assigned the label 1 if $f(u)=f(v)$ or 0 if $f(u) \neq f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. The graph that admits a Homo- Cordial labelling is called Homo Cordial graph. In this paper we prove that the

spider graph $SP(1^m, 2^t)$ is Homo-Cordial labelling graph and further study on the generalisation of labelling spider graph $SP(1^m, 2^t)$

Keywords : Homo Cordial graphs, Homo Cordial labelling.

I. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. Gallian[1] has given a dynamic survey of graph labelling. The origin of graph labelings can be attributed to Rosa. A Path related Homo Cordial graph was introduced by Dr.A.Nellai Murugan and A.Mathubala[2,3,4]. Motivated towards the labelling of homo cordial labelling of graphs In this paper we prove that spider graph $SP(1^m, 2^t)$ is Homo Cordial labelling graph. Further to generalise the concept of homo cordial labelling of spider graph $SP(1^m, 2^t)$ we have ascertained the ways in which the number of labels assigned with 0 and number of labels assigned with 1 so as to identify the phenomena of spider graph $SP(1^m, 2^t)$ to be called a homo cordial labelling graph.

II. PRELIMINARIES

Definition 2.1: A tree is called a spider if it has a centre vertex C of degree $R>1$ and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has X_1 's of length a_1 ,

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X_2 's paths of length a_2 etc. We shall denote the spider by $SP(a_1^{x_1}, a_2^{x_2}, a_3^{x_3} \dots a_m^{x_m})$ where $a_1 \geq a_2 \geq a_3 \geq \dots \geq a_m$ and $x_1 + x_2 + \dots + x_m = R$

III. MAIN RESULTS

Theorem .3.1 : The Spider graph $SP(1^m, 2^t)$ is a homo cordial labelling graph

Proof: Let $G = SP(1^m, 2^t)$ be a Spider graph.

We know that a tree is called a spider if it has a centre vertex C of degree $R>1$ and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has X_1 's of length a_1 , X_2 's paths of length a_2 etc. We shall denote the spider by $SP(a_1^{x_1}, a_2^{x_2}, a_3^{x_3} \dots a_m^{x_m})$ where $a_1 \geq a_2 \geq a_3 \geq \dots \geq a_m$ and $x_1 + x_2 + \dots + x_m = R$

Define the Vertex set $V(SP(1^m, 2^t)) = \{u, v_i, u_j : 1 \leq i \leq m, 1 \leq j \leq 2t\}$ and edge set $E(SP(1^m, 2^t)) = \{e_i = uv_i : 1 \leq i \leq m, e'_i = u u_j, 1 \leq j \leq t, e''_i = u_j u_{j+1} : 1 \leq j \leq t\}$

Now to label the vertices let us consider the bijective function $f: V \rightarrow \{0, 1\}$ such that such that each edge uv is assigned the label 1 if $f(u)=f(v)$ or 0 if $f(u) \neq f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. We define the labelling of vertices as follows

$$\begin{aligned} f(u) &= 0 \quad f(v_i) = 1 \quad \text{for } i \equiv 1 \pmod{m} \\ f(v_i) &= 0 \quad \text{for } i \equiv 0 \pmod{m} \quad \text{where } 1 \leq i \leq m \\ f(u_j) &= 1 \quad \text{for } j \equiv 1 \pmod{t} \\ f(u_j) &= 0 \quad \text{for } j \equiv 0 \pmod{t} \quad \text{where } 1 \leq j \leq 2t \end{aligned}$$

Then the induced edge labelling for the graph



$$G = SP(1^m, 2^t) \text{ are}$$

$$f^*(uv_i) = 0 \text{ for } i \equiv 1(\text{mod } m)$$

$$f^*(uv_i) = 1 \text{ for } i \equiv 0(\text{mod } m)$$

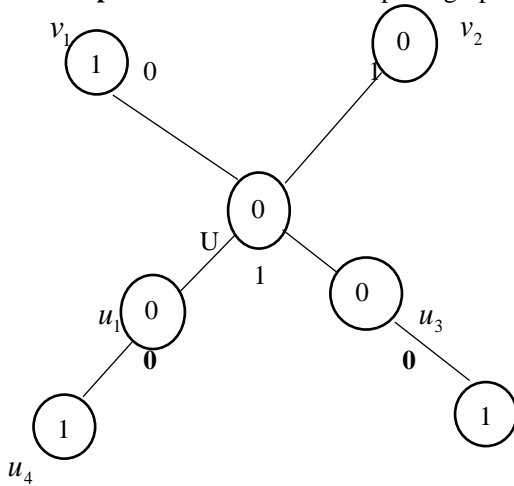
$$f^*(uu_j) = 1$$

$$f^*(u_j u_{t+j}) = 0 \text{ where } 1 \leq j \leq 2t$$

Noticing the induced edge labelling we find that the number of vertices labelled with 0 is n and the number of vertices labelled with 1 is n and that the number of edges labelled with 0 is n+1 and the number of edges labelled with 1 is n. Hence $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Therefore the Spider graph $G = SP(1^m, 2^t)$ is a homo cordial labelling graph.

Example 3.2 : Consider the Spider graph $SP(1^2, 2^2)$



In order to generalise the Spider graph $SP(1^m, 2^t)$ for the various types we classify according to the values of m and t as follows

Case.(i) $m \neq t$

Case.(ii) $m = t$

Case.(iii) $m \neq t$

Where we define a relation between the possibilities of 1^m with that of 2^t as

$$A_1 = \{1, 2, 3, \dots, m\}, \text{ where } A_1 \text{ is the possibilities of } 1^m$$

$$\text{and } A_2 = \{1, 2, \dots, t\}, \text{ where } A_2 \text{ is the possibilities of } 2^t$$

then the relation $R = A_1 \times A_2$ is defined as either p, =, f as in Case.(i), Case.(ii) and Case.(iii) respectively.

Suppose for $m=2$ and $t=3$ we have $A_1 = \{1, 2\}$ and $A_2 = \{1, 2, 3\}$ therefore we have

$$R = A_1 \times A_2 = \{(1,1)(1,2)(1,3)(2,2)(2,3)(2,1)\}$$

we identify that $(1,2)(1,3)(2,3)$ belongs to Case.(i), $(1,1)(2,2)$ belongs to Case. (ii) and $(2,1)$ belongs to Case.(iii) giving rise to the number of elements in the relation R and for $m=2$ and $t=3$ we have total number of elements = $2 \times 3 = 6$ and hence in general $m \times n$ elements for a spider graph $SP(1^m, 2^t)$

Definition 3.3 : For a Spider graph $SP(1^m, 2^t)$ let us define a relation R from A_1 to A_2 defined as a subset of the Cartesian product $A_1 \times A_2$ such that $A_1 = \{x : 1 \leq x \leq m\}$ and $A_2 = \{y : 1 \leq y \leq t\}$.

The number of vertices among each of the possibilities of 1^m is 1 excluding the centre vertex and the number of edges contributed for each of the possibilities of 1^m is 1. Also the number of vertices among each of the possibilities of 2^t is 2 excluding the centre vertex and the number of edges contributed for each of the possibilities of 2^t is 2.

Definition 3.4: Basic Spider graph $SP(1^m, 2^t)$ for $m=1$ and $t=1$. The Spider graph $SP(1^1, 2^1)$ is defined to have the vertex set $V(SP(1^1, 2^1)) = \{u, u_1, v_1, v_2\}$ and $E(SP(1^1, 2^1)) = \{uu_1, uv_1, v_1v_2\}$ with number of vertices of $SP(1^m, 2^t) = 4$ and number of edges $SP(1^m, 2^t) = 3$.

Note.1 : For each possibility of 1^m the number of vertices increases by 1 and edges increase by 1 and for each possibility of 2^t the number of vertices increases by 2 and edges increases by 2.

Note.2 : By Labelling procedure adopted in Theorem.3.1 to prove Spider graph $SP(1^m, 2^t)$ is homo cordial graph we

notice that the basic Spider graph $SP(1^m, 2^t)$ for $m=1$ and $t=1$ has 4 labels (both vertices and edges) assigned with 0 and 3 labels(both vertices and edges) assigned with 1.

Definition 3.5: One part of Spider graph $SP(1^m, 2^t)$:

For a Spider graph $SP(1^m, 2^t)$ from the basic Spider graph $SP(1^1, 2^1)$ we add one part denoted by $T(F_1)$ and the resulting Spider graph is



$SP(1^2, 2^1)$. The resulting graph $SP(1^2, 2^1)$ is said to increase the number of labels assigned as 0 by 1 and the number of labels assigned as 1 by 1 to the existing basic Spider graph $SP(1^1, 2^1)$

Definition 3.6: One part of Spider graph $SP(1^m, 2^t)$:
For a Spider graph $SP(1^m, 2^t)$ from the basic Spider graph $SP(1^1, 2^1)$ we add one part denoted by $T(F_2)$ and the resulting Spider graph is $SP(1^1, 2^2)$. The resulting graph $SP(1^1, 2^2)$ is said to increase the number of labels assigned as 0 by 2 and the number of labels assigned as 1 by 2 to the existing basic Spider graph $SP(1^1, 2^1)$

Note.1: We denote the number of labels assigned with 0 (both vertices and edges) to the Spider graph $SP(1^m, 2^t)$ by $T_0(SP(1^m, 2^t))$ and the number of labels assigned with 1 (both vertices and edges) by $T_1(SP(1^m, 2^t))$

Theorem:3.7: If Spider graph $SP(1^m, 2^t)$ is homo cordial graph then

$$T_0(SP(1^m, 2^t)) = T_0(SP(1^1, 2^1)) + (m-1)T_0(F_1) \quad \text{and}$$

$$T_1(SP(1^m, 2^t)) = T_1(SP(1^1, 2^1)) + (m-1)T_1(F_1)$$

Proof: Consider the Spider graph $SP(1^m, 2^t)$ which is proved as homo cordial graph as in Theorem.3.1. Let us prove the theorem by method of Mathematical induction on m, where m is the number of paths from the centre vertex.

Let us prove for the 1st positive integer m i.e m=2

We have $SP(1^1, 2^1)$ which is the basic Spider graph and contains labelling of 4 0's (both vertices and edges) and 3 1's (both vertices and edges). On adding one part $T(F_1)$ we find that there is an increase in label assigned with 0 by 1 and label assigned with 1 by 1. Hence $SP(1^2, 2^1)$ has 5 0's and 4 1's. Hence

$$T_0(SP(1^2, 2^1)) = T_0(SP(1^1, 2^1)) + (2-1)T_0(F_1)$$

$$T_0(SP(1^2, 2^1)) = T_0(SP(1^1, 2^1)) + T_0(F_1)$$

$$T_0(SP(1^2, 2^1)) = 4 + 1 = 5$$

Similarly

$$T_1(SP(1^m, 2^1)) = T_1(SP(1^1, 2^1)) + (m-1)T_1(F_1)$$

$$T_1(SP(1^2, 2^1)) = T_1(SP(1^1, 2^1)) + (2-1)T_1(F_1)$$

$$T_1(SP(1^2, 2^1)) = T_1(SP(1^1, 2^1)) + T_1(F_1)$$

$$T_1(SP(1^2, 2^1)) = 3 + 1 = 4$$

Hence it is true for m=2.

Now let us assume that the result is true for m=k

$$T_0(SP(1^k, 2^1)) = T_0(SP(1^1, 2^1)) + (k-1)T_0(F_1)$$

$$T_1(SP(1^k, 2^1)) = T_1(SP(1^1, 2^1)) + (k-1)T_1(F_1)$$

Now let us prove for m=k+1

i.e to Prove

$$T_0(SP(1^{k+1}, 2^1)) = T_0(SP(1^1, 2^1)) + (k)T_0(F_1)$$

$$T_1(SP(1^{k+1}, 2^1)) = T_1(SP(1^1, 2^1)) + (k)T_1(F_1)$$

$$\text{Consider } T_0(SP(1^{k+1}, 2^1)) = T_0(SP(1^k, 2^1)) + T_0(F_1)$$

$$T_0(SP(1^{k+1}, 2^1)) = T_0(SP(1^1, 2^1)) + (k-1)T_0(F_1) + T_0(F_1)$$

Hence

$$T_0(SP(1^{k+1}, 2^1)) = T_0(SP(1^1, 2^1)) + (k)T_0(F_1)$$

Similarly we can prove for

$$T_1(SP(1^{k+1}, 2^1)) = T_1(SP(1^1, 2^1)) + (k)T_1(F_1)$$

Hence the theorem by induction.

Corollary 3.8 : If for Spider graph $SP(1^m, 2^1)$ which is homo cordial labelling graph removal of each part $T(F_1)$ reduces the total number of vertices and edges labelled with 0 by 1 and total number of vertices and edges labelled with 1 by 1 and reduces to the basic Spider graph $SP(1^1, 2^1)$

Proof: Consider the Spider graph $SP(1^m, 2^1)$ which is homo cordial labelling graph. Now we remove one part $T(F_1)$ and one part of $T(F_1)$ consists of total number of vertices and edges labelled with 0 as 1 and total number of vertices and edges labelled with 1 as 1. Continuing this procedure m-1 times we can reduce the Spider graph $SP(1^m, 2^1)$ to the basic Spider graph $SP(1^1, 2^1)$. Hence the proof.

Theorem:3.9: If Spider graph $SP(1^m, 2^t)$ is homo cordial graph then

$$T_0(SP(1^1, 2^t)) = T_0(SP(1^1, 2^1)) + (t-1)T_0(F_2)$$

$$T_1(SP(1^1, 2^t)) = T_0(SP(1^1, 2^1)) + (t-1)T_0(F_2)$$

Proof: Consider the Spider graph $SP(1^m, 2^t)$ which is proved as homo cordial graph as in Theorem.3.1. Let us prove the theorem by method of Mathematical induction on t, where t is the number of paths from the centre vertex.

Let us prove for the 1st positive integer t i.e t=2

We have $SP(1^1, 2^1)$ which is the basic Spider graph and contains labelling of 4 0's (both vertices and edges) and 3 1's (both vertices and edges). On adding one part $T(F_2)$ we find that there is an increase in label assigned with 0 by 2 and label assigned with 1 by 2. Hence $SP(1^2, 2^2)$ has 6 0's and 5 1's. Hence

$$T_0(SP(1^1, 2^2)) = T_0(SP(1^1, 2^1)) + (2-1)T_0(F_2)$$

$$T_0(SP(1^1, 2^2)) = T_0(SP(1^1, 2^1)) + T_0(F_2)$$

$$T_0(SP(1^1, 2^2)) = 4 + 2 = 6$$

Similarly

$$T_1(SP(1^1, 2^t)) = T_1(SP(1^1, 2^1)) + (t-1)T_1(F_2)$$

$$T_1(SP(1^1, 2^2)) = T_1(SP(1^1, 2^1)) + (2-1)T_1(F_2)$$

$$T_1(SP(1^1, 2^2)) = T_1(SP(1^1, 2^1)) + T_1(F_2)$$

$$T_1(SP(1^1, 2^2)) = 3 + 2 = 5$$

Hence it is true for m=2.

Now let us assume that the result is true for t=k

$$T_0(SP(1^1, 2^k)) = T_0(SP(1^1, 2^1)) + (k-1)T_0(F_2)$$

$$T_1(SP(1^1, 2^k)) = T_1(SP(1^1, 2^1)) + (k-1)T_1(F_2)$$

Now let us prove for m=k+1

i.e to Prove

$$T_0(SP(1^1, 2^{k+1})) = T_0(SP(1^1, 2^1)) + (k)T_0(F_2)$$

$$T_1(SP(1^1, 2^{k+1})) = T_1(SP(1^1, 2^1)) + (k)T_1(F_2)$$

Consider

$$T_0(SP(1^1, 2^{k+1})) = T_0(SP(1^1, 2^1)) + (k-1)T_0(F_2) + T_0(F_2)$$

$$T_0(SP(1^1, 2^{k+1})) = T_0(SP(1^1, 2^1)) + kT_0(F_2)$$

Hence

$$T_0(SP(1^{k+1}, 2^1)) = T_0(SP(1^1, 2^1)) + (k)T_0(F_1)$$

Similarly we can prove for

$$T_1(SP(1^1, 2^{k+1})) = T_1(SP(1^1, 2^1)) + kT_1(F_2)$$

Hence the theorem by induction.

Corollary 3.10 : If for Spider graph $SP(1^1, 2^t)$ which is homo cordial labelling graph removal of each part $T(F_2)$ reduces the total number of vertices and edges labelled with 0 by 2 and total number of vertices and edges labelled with 1 by 2 and reduces to the basic Spider graph $SP(1^1, 2^1)$

Proof: Consider the Spider graph $SP(1^1, 2^t)$ which is homo cordial labelling graph. Now we remove one part $T(F_2)$ and one part of $T(F_2)$ consists of total number of vertices and edges labelled with 0 as 2 and total number of vertices and edges labelled with 1 as 2. Continuing this procedure t-1 times we can reduce the Spider graph $SP(1^1, 2^t)$ to the basic Spider graph $SP(1^1, 2^1)$. Hence the proof.

Theorem:3.11: If Spider graph $SP(1^m, 2^t)$ is homo cordial graph then

$$T_0(SP(1^m, 2^t)) = 2T_0(SP(1^1, 2^1)) + (m-1)T_0(F_1) + (t-1)T_0(F_2)$$

$$T_1(SP(1^m, 2^t)) = 2T_1(SP(1^1, 2^1)) + (m-1)T_1(F_1) + (t-1)T_1(F_2)$$

Proof: Given the Spider graph $SP(1^m, 2^t)$ is a homo cordial graph. Now from theorem 3.7 & theorem .3.8 we find the following results

$$T_0(SP(1^m, 2^1)) = T_0(SP(1^1, 2^1)) + (m-1)T_0(F_1)$$

$$T_1(SP(1^m, 2^1)) = T_1(SP(1^1, 2^1)) + (m-1)T_1(F_1)$$

$$T_0(SP(1^1, 2^t)) = T_0(SP(1^1, 2^1)) + (t-1)T_0(F_2)$$

$$T_1(SP(1^1, 2^t)) = T_0(SP(1^1, 2^1)) + (t-1)T_0(F_2)$$

Now Consider

$$T_0(SP(1^m, 2^t)) = T_0(SP(1^m, 2^1)) + T_0(SP(1^1, 2^t))$$

Which on substituting from the result

$$T_0(SP(1^m, 2^t)) = T_0(SP(1^1, 2^1)) + (m-1)T_0(F_1) + T_0(SP(1^1, 2^1)) + (t-1)T_0(F_2)$$

Simplifying we have

$$T_0(SP(1^m, 2^t)) = 2T_0(SP(1^1, 2^1)) + (m-1)T_0(F_1) + (t-1)T_0(F_2)$$

Similarly consider

$$T_1(SP(1^m, 2^t)) = T_1(SP(1^m, 2^1)) + T_1(SP(1^1, 2^t))$$

Which on substituting from the result

$$T_1(SP(1^m, 2^t)) = T_1(SP(1^1, 2^1)) + (m-1)T_1(F_1) + T_1(SP(1^1, 2^t))$$

Simplifying we have

$T_1(SP(1^m, 2^t)) = 2T_1(SP(1^1, 2^1)) + (m-1)T_1(F_1) + (t-1)T_0(F_2)$ (m-1)(n-1) times we get the Spider graph $SP(1^m, 2^t)$ with the labelling as specified by Theorem.3.1. Hence the proof..

Hence the proof.

Corollary 3.12 : If for Spider graph $SP(1^m, 2^t)$ which is homo cordial labelling graph removal of each part $T(F) = T(F_1) + T(F_2)$ reduces the total number of vertices and edges labelled with 0 by 3 and total number of vertices and edges labelled with 1 by 3 and reduces to the basic Spider graph $SP(1^1, 2^1)$

Proof: Consider the Spider graph $SP(1^m, 2^t)$ which is homo cordial labelling graph . Now we remove one part $T(F) = T(F_1) + T(F_2)$ which consists of total number of vertices and edges labelled with 0 as 3 and total number of vertices and edges labelled with 1 as 3. Continuing this procedure (m-1) (t-1) times we can reduce the Spider graph $SP(1^m, 2^t)$ to the basic Spider graph $SP(1^1, 2^1)$. Hence the proof.

Theorem:3.13: If G is a Spider graph $SP(1^m, 2^t)$ then the following are equivalent

- (a) Spider graph $SP(1^m, 2^t)$ is homo cordial graph
- (b) $T_0(SP(1^m, 2^t)) = 2T_0(SP(1^1, 2^1)) + (m-1)T_0(F_1) + (t-1)T_0(F_2)$ and $T_1(SP(1^m, 2^t)) = 2T_1(SP(1^1, 2^1)) + (m-1)T_1(F_1) + (t-1)T_0(F_2)$
- (c) Each part $T(F)$ of Spider graph $SP(1^m, 2^t)$ has 3 0's and 3 1's

Proof: Consider the graph G, the Spider graph $SP(1^m, 2^t)$. To prove that the following are equivalent we claim

- (a) Implies (b)
- (b) Implies (c)
- (c) Implies (a)

For (a) implies (b)

By considering the Theorem.3.1 and the labelling procedure we can claim the results in Theorem.3.11 and hence (b) is true

For (b) implies (c)

By considering the result obtained in theorem 3.11 and from the corollary 3.12 we are done.

For (c) implies (a)

By considering (c) we can obtain by adding each part $T(F)$ to the basic Spider graph $SP(1^1, 2^1)$ consecutively

IV. CONCLUSION

In this paper we have considered Spider Graph $SP(1^m, 2^t)$ and proved that it is homo cordial labelling graph and have identified a generalisation method for Spider graph. We are investigating on the other types of Spider graphs which can be also labelled so as to prove that they are homo cordial labelling graph.

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