Existence Results for Semilinear Functional Differential System with Nonlocal Conditions

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***Abstract**: In this paper, sufficient conditions are given for the existence of partial functional differential equations with nonlocal conditions in an abstract space with the help of the fixed point theorems.*

***Keywords:** Mild solutions, Nonlocal conditions, Fixed point theorems..*

I. INTRODUCTION

In this paper, we discuss the semilinear functional differential equation with nonlocal conditions of the form

$$
\begin{aligned}
x'(t) &= Ax(t) + f(t, x(t), x(\rho(t))) \\
x(0) &= \sum_{i=1}^{m} \nu_i \, x(t_i), \qquad t \in J = [0, T] \\
x(t) &= \varphi(t), \qquad t \in J_1 = [-r, 0]
\end{aligned}
\quad , \qquad (1.1)
$$

where $T > 0$; $0 < t_1 < t_2 < t_3 < \cdots < t_m < T$ and ν_i are real numbers. Let X be a Banach space with the norm $\|\cdot\|$ and the functional $f : J \times X^2 \to X$; $\rho : J \to [-r,T]$ are continuous functions. Let $E := C([-r,T];X)$ be the Banach space of continuous functions $x : [-r,T] \to X$, equipped with the norm,

$$\|x\|_E = \sup\{\|x(t)\| : t \in [-r,T]\}.$$

The notion of nonlocal conditions has been used to extend the study of the classical initial value evolution equation

$$u'(t) = Au(t) + f(t,u(t)), \qquad 0 \le t \le T, \qquad (1.2)$$

$$u(0) = u_0, \qquad (1.3)$$

to the following nonlocal evolution equation.

$$
\begin{aligned}
u'(t) &= Au(t) + f(t,u(t)), \qquad 0 \le t \le T, \\
u(0) + g(u) &= u_0,
\end{aligned}
$$

where $g : C([0,T];X) \to X$ is a continuous function. The equation (1.4)-(1.5) can be applied in physics with better effect than equation (1.2)-(1.3), see [1, 2] and the references therein related to this matter.

In [2], L.Byszewski studied the existence and uniqueness of mild, strong and classical solutions of the nonlocal Cauchy problem for a semilinear evolution equation of the form

$$
\begin{aligned}
&\frac{d}{dt}u(t) + Au(t) = f(t, u(t)), \quad t \in [t_0, t_0 + a] \\
&u(t_0) + g(t_1, \ldots\ldots, t_p, u(\cdot)) = u_0,
\end{aligned}
$$

,

where $0 \le t_0 < t_1 < \ldots.. < t_p \le t_0 + a, a > 0, -A$ is the infinitesimal generator of a C_0 semigroup on a Banach space $X, u_0 \in X$ and $f : [t_0,t_0 + a] \times X \to X, g : [t_0,t_0 + a]^p \times X \to X$ are continuous functions using the semigroup theory and the Banach fixed point theorem.In [3], L.Byszewski and H.Akca studied the existence of mild and classical solutions of a nonlocal Cauchy problem for a semilinear functional differential evolution equation

$$u'(t) + Au(t) = f(t,u(t),u(b_1(t)),\ldots..,u(b_m(t))),$$

$$u(t_0) + g(u) = u_0, \qquad t \in [t_0,t_0 + a],$$

where $t_0 > 0, a > 0, -A$ is the infinitesimal generator of a compact C_0-semigroup of operators on a Banach space using Schauder fixed point theorem.

In [4], H. Akca et al., proved the impulsive functional

$$
\begin{aligned}
u'(t) &= Au(t) + f(t,u_t), \qquad t \in (0,a], \quad t \ne \tau_k, \\
u(\tau_\kappa + 0) &= Q_k u(\tau_\kappa) = u(\tau_k) + I_\kappa, \qquad \kappa = 1,2,\ldots,k, \\
u(t) &+ (g(u_{t1},\ldots,u_{tp}))(t) = \varphi(t), \qquad t \in [-r,0],
\end{aligned}
$$

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differential equations with nonlocal conditions of the form where $0 < t_1 < \cdots < t_p \leq a$, $p \in \mathbb{N}$, A and $I_\kappa, \kappa = 1,2,...,k$ are linear operators acting in a Banach space E; f,g and φ are given functions satisfying some assumptions, $u_t(s) = u(t + s)$ for $t \in [0,a]$, $s \in [-r,0]$, $I_\kappa u(\tau_\kappa) = u(\tau_\kappa + 0) - u(\tau_\kappa - 0)$ and the impulsive moments τ_κ are such that $0 < \tau_1 < \cdots < \tau_\kappa < \cdots < a$, $\kappa \in \mathbb{N}$, by using the Banach contraction theorem.

Recently, Under sufficient conditions, Boucherif [5, 6] studied differential inclusions with nonlocal conditions through fixed point theory. The study of nonlocal problems in integro-differential equations have been treated in several works and we refer [7–10] and the references therein. Further, we utilize the technique developed in [11, 12].

Motivated from the above mentioned works. In this paper, we study the existence results for the system (1.1) by means of fixed point theory. The paper is organized as follows: some preliminaries are presented in the section 2. In section 3, we investigate the existence results of mild solutions for semilinear functional differential system using the Leray-Schauder alternative fixed point theorem and Banach fixed point theorem. Finally in section 4, we give an application for our abstract results.

II. PRELIMINARIES

Before proceeding to main result, we shall set forth some preliminaries that will be used in our subsequent discussion. We shall assume that A : D(A) → X is the infinitesimal generator of a compact analytic semigroup of uniformly bounded linear operators, (T(t))t≥0, and there exists M ≥ 1 such that kT(t)k ≤ M for all t ∈ J, (for more details we refer to [12]), and there exists a bounded operator B on D(B) = X given by the formula $B = (I - \sum_{i=1}^{m} \gamma_i T(t_i))^{-1}$. This is possible if, for instance $\sum_{i=1}^{m} |\gamma_i| < \frac{1}{M}$.

Definition 2.1. A map f : J × X × X → X is said to be L1-Carath`eodory if:$t \to f(t,x,y)$ *is strongly measurable for each* $x,y \in X$.

(i) $(x,y) \to f(t,x,y)$ *is continuous for almost all* $t \in J$.

(ii) for each positive integer $m > 0$, *there exists* $\alpha_m \in L^1(J : \Re^+)$ *such that*

$$\sup_{\|x\|\leq m;\ \|y\|\leq m} \|f(t,x,y)\| \leq \alpha_m(t), \text{ for } t \in J, \text{a.e.}$$

Definition 2.2. *$x \in E$ is a mild solution of equations (1.1) if*

$$x(t) = \begin{cases} \varphi(t), & t \in J_1, \\ \sum_{i=1}^{m} \gamma_i T(t) B \int_0^{t_i} T(t_i - s) f(s, x(s), x(\rho(s))) ds & \\ + \int_0^t T(t-s) f(s, x(s), x(\rho(s))) ds, & t \in J, \end{cases} \tag{2.1}$$

is satisfied.

Our existence theorem is based on the following theorem.

Theorem 2.1. *Let S be a convex subset of a Banach space E and assume that $0 \in S$. Let $F : S \to S$ be a completely continuous operator and let*

$$U(F) = \{x \in S : x = \lambda F x \quad \text{for some } 0 < \lambda < 1\}.$$

Then either is $U(F)$ unbounded or F has a fixed point.

III. EXISTENCE OF A SOLUTION :

In this section, we prove the existence theorem by using the following hypotheses:

(H_1) : There exists a continuous non-decreasing function for $\Omega : \Re^+ \to (0,\infty)$ and $p \in L^1(J;\Re^+)$ such that

k$f(t,x,y)$k ≤ $p(t)\Omega$(kxk + kyk), $t \in J; x,y \in X$.

(H_2) : The function $\rho : J \to [-r,T]$ is continuous and $t - r \leq \rho(t) \leq t$, for every $t \in J$.

Theorem 3.1. *If the hypotheses (H_1) −(H_2) be hold. Then the system (1.1) has a mild solution $x(t)$ on $[-r,T]$ provided that following inequality is satisfied:*

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$$\sup_{\varpi\in[0,\infty)} \frac{\varpi}{M\,\|p\|_{L^1}\,\Omega(2\varpi)\left[1+M\,\|B\|\sum_{i=1}^{m}|\gamma_i|\right]} > 1$$

. (3.1)

Proof. Let T be an arbitrary number $0 < T < +\infty$ satisfying (3.1). It follows from (3.1) that there exists $\beta > 0$ such that

$$\frac{\beta}{M\,\|p\|_{L^1}\,\Omega(2\beta)\left[1+M\,\|B\|\sum_{i=1}^{m}|\gamma_i|\right]} > 1 \quad (3.2)$$

Step-1:

For $\lambda \in (0,1)$, let consider the problems

$$\begin{aligned} x(t) &= \lambda\sum_{i=1}^{m}\gamma_i T(t)B\int_0^{t_i} T(t_i-s)f(s,x(s),x(\rho(s)))ds \\ &\quad +\lambda\int_0^t T(t-s)f(s,x(s),x(\rho(s)))ds,\ t\in J. \end{aligned} \quad (3.3)$$

Notice that if $x \in E$ is a solution of (3.3) for $\lambda = 1$, then x is a solution of (1.1).

Consider $U = \{x \in E; \|x\| < \beta\}$. Define $F : \bar{U} \to E$ by

$$\begin{aligned} Fx(t) &= \sum_{i=1}^{m}\gamma_i T(t)B\int_0^{t_i} T(t_i-s)f(s,x(s),x(\rho(s)))ds \\ &\quad + \int_0^t T(t-s)f(s,x(s),x(\rho(s)))ds \quad t\in J, \end{aligned}$$

we can easily show that F is continuous.

Step-2 : F maps bounded sets into bounded sets.

For, let $x \in B_\rho = \{v \in E : \|v\| \leq \}$, then $(H_1)-(H_2)$ implies that

$$\begin{aligned} \|Fx(t)\| &= \left\|\sum_{i=1}^{m}\gamma_i T(t)B\int_0^{t_i} T(t_i-s)f(s,x(s),x(\rho(s)))ds\right\| \\ &\quad +\left\|\int_0^t T(t-s)f(s,x(s),x(\rho(s)))ds\right\| \\ &\leq M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}\|f(s,x(s),x(\rho(s)))ds\| \\ &\quad +M\int_0^t\|f(s,x(s),x(\rho(s)))ds\| \\ &\quad +M\int_0^t p(s)\Omega(\|x(s)\|+\|x(\rho(s))\|)ds, \\ &\leq M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}p(s)\Omega(2\varpi)ds + M\int_0^t p(s)\Omega(2\varpi)ds, \end{aligned}$$

so that,

$$\|Fx\|_E = \sup_{t\in J}\|Fx(t)\| \leq M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\|p\|_{L^1}\Omega(2\varpi). \quad (3.4)$$

Step-3: $F(\bar{U})$ is a uniformly equicontinuous family of functions.

For, let $\tau_1 < \tau_2$ in J. Then

$$\begin{aligned} &\|Fx(\tau_1)-Fx(\tau_2)\| \\ &\leq \|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}\|T(t_i-s)\|\|f(s,x(s),x(\rho(s)))\|ds \times \|T(\tau_1)-T(\tau_2)\| \\ &+ \int_0^{\tau_1}\|T(\tau_1-s)-T(\tau_2-s)\|\|f(s,x(s),x(\rho(s)))\|ds \\ &+ \int_{\tau_1}^{\tau_2}\|T(\tau_2-s)\|\|f(s,x(s),x(\rho(s)))\|ds. \\ &\to 0\ \ as\ \ \tau_2\to\tau_1. \end{aligned}$$

Since, see [13, Proposition 1] and [6] that there exists $\eta > 0$ such that

$$\|T(\tau_1)-T(\tau_2)\| \leq \frac{\eta}{\sqrt{\tau_1}}\sqrt{\tau_2-\tau_1}$$

as $\tau_2 \to \tau_1$ we get $\|T(\tau_1)-T(\tau_2)\| \to 0$, $\max_{s\in J}\|T(\tau_1-s)-T(\tau_2-s)\| \to 0$ and also

$\int_{\tau_1}^{\tau_2}\alpha_\beta(s)ds \to 0$ as $\tau_2 \to \tau_1$. Because $\alpha_\beta \in L^1(J, \Re^+)$.

Step- 4: The set $\bar{U}(t) = \{Fx(t) : x \in \bar{U}\}$ is precompact in E.

For, let $t > 0$ and $0 < \varrho < t$. For $x \in \bar{U}$ define

$$\begin{aligned} F_\epsilon x(t) &= \sum_{i=1}^{m}\gamma_i T(t)B\int_0^{t_i} T(t_i-s)f(s,x(s),x(\rho(s)))ds \\ &\quad + \int_0^{t-\epsilon} T(t-s)f(s,x(s),x(\rho(s)))ds \end{aligned}$$

Since $T(t)$ is compact for every $t > 0$, the set Z $\bar{U}_\varrho =$ $\{F_\varrho x(t) : x \in \bar{U}\}$ is precompact in X. for every $\varrho \in (0,t)$. Moreover for $x \in \bar{U}$ we have t Z

≤

$F_\varrho x(t) - Fx(t)\| \leq \|T(t - s)f(s,x(s),x(\rho(s)))\|ds$

$t-\varrho$

$tM\alpha_\beta(s)ds.$

$t-\varrho \to 0$ as $\varrho \to 0.$

Since $\alpha_\beta(s) \in L^1$ and $meas([t-\varrho,t]) < \varrho$.

Step- 5:

Next, by (H_1) and (H_2) all solutions of (3.3) satisfy

$$\|x\| \leq M[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1]\|p\|_{L^1}\Omega(2\|x\|)$$

.

If $t \in J_1$, then $\|x(t)\| = \|\phi\|$ and the previous inequality holds. Consequently,

$$\|x\|_E \leq M[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1]\|p\|_{L^1}\Omega(2\|x\|_E)$$

.

Suppose, now that there exist $x \in \partial U$ and $\lambda \in (0,1)$ such that $x \in \lambda Fx$. Then x satisfies (3.3) and $\|x\|_E = \beta$. It follows from (3.4) that

$$\beta \leq M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\|p\|_{L^1}\Omega(2\beta)$$

.

This, obviously, contradicts the definition of β (see equation (3.2)). Moreover, the set U is bounded. Consequently, by Theorem 2.1, the operator F has a fixed point in E.

Therefore, the system (1.1) has a mild solution. Thus the proof is completed.

We now present another existence result for system (1.1). The Lipschitz condition on f is relaxed by using Wintner growth condition in the following Theorem.

Theorem 3.2. *Assume that* (H_2) *and the following condition holds*

(H_3) *: There exists* $\ell \in L^1([0,T],\Re^+)$ *such that* $\|f(t,x_1,y_1) - f(t,x_2,y_2)\| \leq \ell(t)\langle\|x_1 - x_2\| + \|y_1 - y_2\|\rangle$, $x_i, y_i \in X$ *and* $\|f(t,0,0)\| \leq \ell(t)$, *a.e.* $t \in J$,

then the system (1.1) has at least one mild solution on $[-r,T]$.Proof. The operator F defined in the proof of the previous theorem is completely continuous. Now, we prove that $U = \{x \in E: x \in \lambda F(x) \text{ for some } \lambda \in (0,1)\}$ is bounded. Let $x \in U$. Then for each $t \in J$

$$x(t) = \lambda\sum_{i=1}^{m}\gamma_i T(t)B\int_0^{t_i}T(t_i-s)f(s,x(s),x(\rho(s)))ds + \lambda\int_0^t T(t-s)f(s,x(s),x(\rho(s)))ds,$$

for some $\lambda \in (0,1)$. Then

$$\begin{aligned}\|x(t)\| &\leq \left\|\sum_{i=1}^{m}\gamma_i T(t)B\int_0^{t_i}T(t_i-s)f(s,x(s),x(\rho(s)))ds\right\| + \left\|\int_0^t T(t-s)f(s,x(s),x(\rho(s)))ds\right\| \\ &\leq M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}\|f(s,x(s),x(\rho(s)))ds\| + M\int_0^t\|f(s,x(s),x(\rho(s)))ds\| \\ &\leq M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\|\ell\|_{L^1} + M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}\ell(s)(\|x(s)\|+\|x(\rho(s))\|)ds \\ &\quad + M\int_0^t \ell(s)(\|x(s)\|+\|x(\rho(s))\|)ds, \\ &\leq M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\|\ell\|_{L^1} + 2M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\int_0^{t_i}\ell(s)\|x(s)\|ds \\ &\quad + 2M\int_0^t \ell(s)\|x(s)\|ds, \\ &\leq M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\|\ell\|_{L^1} + 2M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\int_0^t \ell(s)\|x(s)\|ds.\end{aligned}$$

$$\|x(t)\| \leq Q_1 + Q_2\int_0^t \ell(s)\|x(s)\|ds,$$

$$Q_1 = M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\|\ell\|_{L^1} \text{ and } Q_2 = 2M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]$$

Thuse.If $t \in J_1$, then $\|x(t)\| = \|\phi\|$ and the previous inequality holds.By applying Gronwall inequality, we getHence

$$\|x\|_E \leq Q_1\exp\left(Q_2\|\ell\|_{L^1}\right),\ t \in J,$$

$$\|x\|_E \leq \beta_1.$$

This shows that the set U is bounded.□As a consequence of Theorem 2.1, we deduce that F has a fixed point which is a mild solution of (1.1). This completes the proof. Concerning the existence and uniqueness of mild solution for the system (1.1), we establish in the following result.

Theorem 3.3. *Let assumption* (H_2) *be verified and the following condition holds*(H_4) *: There exists constants* $\ell_1 > 0$ *such that* $\|f(t,x_1,y_1) - f(t,x_2,y_2)\| \leq \ell_1(\|x_1 - x_2\| + \|y_1 - y_2\|)$, $x_i,y_i \in X$.*If*

$$\Lambda = 2M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\ell_1 < 1,$$

then there exists a unique mild solution for the system (1.1).*Proof.* The operator F defined as in the proof of the previous theorem. Now, we shall show that the operator F is a contraction. Let $x \in U$, then for each $t \in [-r,T]$ we have

$$\begin{aligned}\left\|Fx(t) - F\widetilde{x(t)}\right\| &\leq \left\|\sum_{i=1}^{m}\gamma_i T(t)B\int_0^{t_i}T(t_i-s)\left[f(s,x(s),x(\rho(s))) - f(s,\widetilde{x(s)},\widetilde{x(\rho(s))})\right]ds\right\| \\ &\quad + \left\|\int_0^t T(t-s)\left[f(s,x(s),x(\rho(s))) - f(s,\widetilde{x(s)},\widetilde{x(\rho(s))})\right]ds\right\| \\ &\leq M^2\|B\|\sum_{i=1}^{m}|\gamma_i|\ell_1\int_0^{t_i}\left[\|x(s)-\widetilde{x(s)}\| + \|x(\rho(s)) - \widetilde{x(\rho(s))}\|\right]ds \\ &\quad + M\ell_1\int_0^t\left[\|x(s)-\widetilde{x(s)}\| + \|x(\rho(s)) - \widetilde{x(\rho(s))}\|\right]ds \\ &\leq 2M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i|+1\right]\ell_1\int_0^t\|x(s)-\widetilde{x(s)}\|ds.\end{aligned}$$

Taking supremum over $t \in [-r,T]$, we get,

$$\|Fx - F\tilde{x}\|_E \leq 2M\left[M\|B\|\sum_{i=1}^{m}|\gamma_i| + 1\right]\ell_1 \|x - \tilde{x}\|_E.$$

Thus,

$\|Fx - F\tilde{x}\|_E \leq \Lambda\|x - \tilde{x}\|_E$, (3.5)

since $0 < \Lambda < 1$. This shows that operator F is a contraction. Uniqueness follows from (H_4). Consequently, by (3.5), the operator F satisfies all the assumptions of the Banach fixed point theorem. Therefore, in space U there is only one fixed point of F and this is the mild solution of the system (1.1). So, the proof of Theorem 3.3 is complete.

IV. CONCLUSION

In this section, we give an example of the partial differential equation to illustrate the application of our main theorem

$$\begin{aligned}
\frac{\partial v(t,u)}{\partial t} &= \frac{\partial^2 v(t,u)}{\partial u^2} + \mu(t,u,v(t,u),v(\rho(t),u)), \\
v(t,0) &= v(t,\pi) = 0, \quad t \in J = [0,1],\ u \in I = [0,\pi \\
v(0,u) &= \sum_{i=1}^{n}\alpha_i v(t_i,u), \quad u \in I \\
v(t,u) &= \varphi(t,u) \text{ for } -r \leq t \leq 0
\end{aligned}$$

] (4.1)

where $\mu : J \times I \times X \times X \to X$; $\rho : J \to [-r,1]$ are continuous and $t - r \leq \rho(t) \leq t$ for every $t \geq 0$ and $t_i \in J$; $\alpha_i \in \Re$ are prefixed numbers. Let $X = L^2[0,\pi]$. Define A an operator on X by $Av = \frac{\partial^2 v}{\partial u^2}$ with the domain

$$D(A) = \left\{v \in X \,\middle|\, v \text{ and } \frac{\partial v}{\partial u} \text{ are absolutely continuous}, \frac{\partial^2 v}{\partial u^2} \in X,\ v(0) = v(\pi) = 0.\right\}$$

It is well known that A generates a strongly continuous semigroup T(t) which is compact, analytic and self adjoint. Moreover, the operator A can be expressed as

$$Au = \sum_{n=1}^{\infty} n^2 < v, v_n > v_n, \quad v \in D(A)$$

,

where $v_n(\zeta) = (\frac{2}{\pi})^{\frac{1}{2}}\sin(n\zeta),\ n = 1,2,\ldots$, is the orthonormal set of eigenvectors of A.

Then the operator $(-A)^{\frac{1}{2}}$ is given by

$$(-A)^{\frac{1}{2}}v = \sum_{n=1}^{\infty} n < v, v_n > v_n$$

on the space

$$D[(-A)^{\frac{1}{2}}] = \{v \in X; \sum_{n=1}^{\infty} n < v, v_n > v_n \in X\}.$$

This satisfies $\|T(t)\| \leq 1$, $t \geq 0$, and hence is a contraction semigroup. In particular,

$$\|(-A)^{-\frac{1}{2}}\| = \frac{1}{\Gamma\frac{1}{2}}\int_0^\infty t^{\frac{1}{2}-1}\|T(t)\|dt \leq 1$$

.The problem (4.1) can be modeled as the abstract semilinear differential system (1.1).

By defining the operator f by $f(t,x,y)u = \mu(t,u,x(u),y(u))$. The next result a consequence of Theorem 3.1.

Proposition 4.1. Assume that the hypotheses (H1)−(H2) hold. Then there exists a mild solution v of the system (4.1) provided

$$\sup_{\varpi \in [0,\infty)} \frac{\varpi}{\|p\|_{L^1}\,\Omega(2\varpi)\left[1 + \|B\|\sum_{i=1}^{m}|\gamma_i|\right]} > 1 \tag{4.2}$$

is satisfied.

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