Solving Assignment Problems using Consecutive Integers with Residue Partitioning

P. Geetha, A.Gnanam

***Abstract*: In this paper we introduce a new approach to solve the assignment problems using consecutive integers with residue partitioning by assigning ones in the Problems. We also give an illustration, using consecutive intergers with residue partitioning by the assingnment of ones and depending on the problems two procedures have been adopted.**

***Keywords* : Partitions, Consecutive integers with residue partitioning.**

I. INTRODUCTION

The theory of partitions [1], [2] is one of very few branches of mathematics that can be appreciated by anyone who is endowed with little more than a lively interest in the subject. Its applications are found wherever discrete objects are to be counted or classified, whether in the molecular and the atomic studies of matter, in the theory of numbers [3], [4] or combinatorial problems from all sources. The theory of partitions has an interesting history. Certain special problems in partitions certainly date back to the middle ages. Euler indeed laid the foundations of the theory of partitions. Many of the other great mathematicians – Cayley, Gauss, Hardy, Jacobi, Lagrange, Legendre, Littlewood, Rademacher, Ramanujan, Schur and Sylvester have contributed to the development of number theory. The representation of positive integers by sums of other positive integers is known as the fundamental additive decomposition process. For studying partitions, the graphical representation is an effective elementary device. Representation of a positive integer as a sum of two or more squares is also a partition, where each part is a square or a square number. There are many kinds of partitions [5], [6], [7] depending on the parts represented. The partition of a positive integer n can be defined as : A finite non – decreasing sequence $\lambda = (\lambda_1, \lambda_2, ...\lambda_k)$ such that $\sum_{i=1}^{k} \lambda_i = n$ and $\lambda_i > 0$ for all $i = 1, 2, ...k$. The λ_i are called the parts of the partition and k is called the length of the partition.

Operations Research [8], [9], [10] is the study of optimization techniques. It is applied decision theory. The existence of optimization techniques can be traced at least to the days of Newton and Lagrange. Rapid development and invention on new techniques occurred since the World War II essentially, because of the necessity to win the war with the limited resources available. Different teams had to do research on military operations in order to invent techniques to manage with available resources so as to obtain the desired objective. Number theory, an interesting branch of mathematics that deals with integers and their properties plays an important role in operations research [11], [12]. The assignment problem is a particular case of the transportation problem in which the objective is to assign a number of tasks to an equal number of facilities at a minimum cost or maximum profit. Suppose that we have 'n' jobs to be performed on 'm' machines and our objective is to assign the jobs to the machines at the minimum cost or maximum profit under the assumption that each machine can perform each job but with varying degree of efficiencies. The assignment problem can be stated in the form of $m \times n$ matrix (c_{ij}) called a cost matrix or effectiveness matrix where c_{ij} is the cost of assigning ith machine to the jth job.

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II. MAIN RESULTS

Definition : Consecutive Integers With Residue Partition

A partition of a positive interger n is a finite non- decreasing sequence $\lambda = (\lambda_1, \lambda_2, ...\lambda_k) = (1, 2, 3, ...,k, \alpha)$ where $(1 \le \alpha \le k)$ such that $\sum_{i=1}^{k} \lambda_i + \alpha = n$ and l is the length of the partition [13], [14], [15].

1. Consecutive Integers With Residue Partitioning Algorithm For Balanced Problems

Step 1 :- To find the length of the maximum cost.

Case (i) : In a minimization assignment problems, select the maximum cost from the given problem and partitioning this using consecutive integer with residue partitioning and find the length l of the maximum cost then go to step 2.

Case (ii) : In a maximization case, convert the given maximization problems into minimization by subtract all the elements from maximum element and then partition this maximum element using consecutive integer with residue partitioning and find the length l of the maximum cost then go to step 2.

Step 2 :- Add this length l with the given balanced assignment problem. Select the minimum element from each row and divide the remaining elements. Round off the element to the nearest integer. This will create atleast one ones in each rows and in terms of ones for each row do assignment, otherwise go to step 3.

Step 3 :- Select the minimum cost element of each column and divide the remaining elements. Round off the element to the nearest integer. This will create atleast one ones in each columns and in terms of ones for each column do assignment, otherwise go to step 4.

Step 4 :- Cover all the ones by drawing a minimum number of straight lines. If the number of drawn lines is less than n, then the complete assignments is not possible, then go to step 5. If the number of lines is exactly equal to n, then the complete assignment is obtained.

Step 5 :- Select the smallest element not covered by the straight lines and divide each element of the uncovered rows or columns. This will create some new one to this row (or) column. Repeat the steps 4 and 5 until an optimum assignment is attained.

2. Consecutive Integers With Residue Partitioning Algorithm For Unbalanced Problems

Step 1 :- To find the length l of the maximum cost using consecutive integer with residue partitioning . Convert the given unbalanced assignment problem into a balanced one by adding dummy rows or dummy columns as a length of the maximum cost element in the cost matrix depending upon whether m < n or m > n.

Step 2 :- Select the minimum element from each row and divide the remaining elements. Round off the element to the nearest integer. This will create atleast one ones in each rows and in terms of ones for each row do assignment and finally subtract the value of l based on the number of dummy rows or dummy columns from the optimum assignment cost otherwise go to step 3.

Step 3 :- Select the minimum cost element of each column and divide the remaining elements. Round off the element to the nearest integer. This will create atleast one ones in each columns and in terms of ones for each column do assignment and finally subtract l based on the number of dummy rows or dummy columns from the optimum assignment cost otherwise go to step 4.

Step 4 :- Cover all the ones by drawing a minimum number of straight lines. If the number of drawn lines is less than n, then the complete assignment is not possible, then go to step 5. If the number of lines is exactly equal to n, then the complete assignment is obtained.

Step 5 :- Select the smallest element not covered by the straight lines and divide each element of the uncovered rows or columns. This will create some new one to this row (or) column. Repeat the steps 4 and 5 until an optimum assignment is attained.

Illustration 1 : Minimization Problem

Consider the following assignment problem

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{pmatrix} 10 & 5 & 6 \\ 8 & 7 & 11 \\ 0 & 9 & 10 \end{pmatrix} \end{array}$$

Assign the three jobs to the three machines so as to minimize the total cost.

(1). Hungarian method : Select the smallest cost element in each row and subtract this from all the elements of the corresponding row, we get

$$\begin{pmatrix} 5 & 0 & 1 \\ 1 & 0 & 4 \\ 0 & 9 & 10 \end{pmatrix}$$

Select the smallest cost element in each column and subtract this from all the elements of the corresponding column. The resultant matrix is

$$\begin{pmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 9 & 9 \end{pmatrix}$$

Do assignment here

$$\begin{pmatrix} 5 & 0 & (0) \\ 1 & (0) & 3 \\ (0) & 9 & 9 \end{pmatrix}$$

each row and each column contains exactly one assignment (exactly one encircled zero) the current assignment is optimal.

$\therefore$ The optimum assignment schedule is given by $A \to III, B \to II, C \to I$. The minimum assignment cost = 6 + 7 + 0 = Rs.13.

(2). Consecutive Integer With Residue Partitioning Algorithm :

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{pmatrix} 10 & 5 & 6 \\ 8 & 7 & 11 \\ 0 & 9 & 10 \end{pmatrix} \end{array}$$

The maximum element is 11. Its partition is $11 = 1 + 2 + 3 + 4 + 1$. Length of the partition by $l = 5$. Add this length to the given matrix. The resultant matrix is,

$$\begin{pmatrix} 15 & 10 & 11 \\ 13 & 12 & 16 \\ 5 & 14 & 15 \end{pmatrix}$$

Select the minimum element from each row and divide the remaining elements by this minimum element and round off the integer and do assignment interms of ones.

$$\begin{pmatrix} 2 & 1 & (1) \\ 1 & (1) & 1 \\ (1) & 3 & 3 \end{pmatrix}$$

$\therefore$ The optimum assignment schedule is given by $A \to III, B \to II, C \to I$. The minimum assignment cost = 6 + 7 + 0 = Rs.13.

Illustration 2 : Maximization Problem

Consider the following assignment problem

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$$\begin{array}{c} \\ P \\ Q \\ R \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 51 & 53 & 54 \\ 47 & 50 & 48 \\ 49 & 50 & 60 \end{pmatrix} \end{array}$$

Assign the three jobs to the three machines so as to maximize the total cost.

(1). Hungarian method : Convert the given maximization problem into minimization by subtracting all the elements from 60.

The given problem becomes

$$\begin{pmatrix} 9 & 7 & 6 \\ 13 & 10 & 12 \\ 11 & 10 & 0 \end{pmatrix}$$

Select the smallest cost element in each row and subtract this from all the elements of the corresponding row, we get

$$\begin{pmatrix} 3 & 1 & 0 \\ 3 & 0 & 2 \\ 11 & 10 & 0 \end{pmatrix}$$

Select the smallest cost element in each column and subtract this from all the elements of the corresponding column. The resultant matrix is

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 8 & 10 & 0 \end{pmatrix}$$

Do assignment here

$$\begin{pmatrix} (0) & 1 & 0 \\ 0 & (0) & 2 \\ 8 & 10 & (0) \end{pmatrix}$$

each row and each column contains exactly one assignment (exactly one encircled zero) the current assignment is optimal.

$\therefore$ The optimum assignment schedule is given by $P \to 1, Q \to 2, R \to 3.$ The minimum assignment cost = 51 + 50 + 60 = Rs.161.

(2). Consecutive Integer With Residue Partitioning Algorithm :

$$\begin{array}{c} \\ P \\ Q \\ R \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 51 & 53 & 54 \\ 47 & 50 & 48 \\ 49 & 50 & 60 \end{pmatrix} \end{array}$$

Select the maximum element and partition this using consecutive integer with residue partitioning. It is given by $60 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 5$.

Denote the length of the partition by l, here $l = 11$.

Convert the given maximization problem into minimization by subtracting all the elements from 60. The given problem becomes

$$\begin{pmatrix} 9 & 7 & 6 \\ 13 & 10 & 12 \\ 11 & 10 & 0 \end{pmatrix}$$

Add the length $l = 11$ in the minimization matrix we have

$$\begin{pmatrix} 20 & 18 & 17 \\ 24 & 21 & 23 \\ 22 & 21 & 11 \end{pmatrix}$$

Select the minimum element from each row and divide the remaining elements by this minimum element and round off the integer and do assignment interms of ones

$$\begin{pmatrix} (1) & 1 & 1 \\ 1 & (1) & 1 \\ 2 & 2 & (1) \end{pmatrix}$$

$\therefore$ The optimum assignment schedule is given by $P \to 1, Q \to 2, R \to 3.$ The minimum assignment cost = 51 + 50 + 60 = Rs.161.

Illustration 3 : Unbalanced Problem

Consider the following assignment problem

$$\begin{array}{c} \\ X \\ Y \end{array}\begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix} \end{array}$$

Assign the three jobs to the three machines so as to minimize the total cost.

(1). Hungarian method : Convert the given unbalanced assignment problem into balanced one by adding dummy row with zero cost elements. The balanced cost matrix is given by,

$$\begin{array}{c} \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Select the smallest cost element in each row and subtract this from all the elements of the corresponding row, we get

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

Select the smallest cost element in each column and subtract this from all the elements of the corresponding column. The resultant matrix is

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

In this resultant matrix, we shall make the assignment in rows and columns having single zero.

$$\begin{pmatrix} (0) & 1 & 2 \\ \cancel{0} & 1 & 3 \\ \cancel{0} & (0) & \cancel{0} \end{pmatrix}$$

Since there are some rows and columns without assignment, the current assignment is not optimal. Cover all the zeros by drawing a minimum number of straight lines. Choose the smallest cost element not covered by these straight lines.

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

Here 1 is the smallest cost element not covered by these straight lines. Subtract this 1 from all the uncovered elements, add this 1 to those elements which lie in the intersection of these straight lines and do not change the remaining elements which lie on the straight lines and do assignment. We get

$$\begin{pmatrix} (0) & 0 & 1 \\ 0 & (0) & 2 \\ 1 & 0 & (0) \end{pmatrix}$$

$\therefore$ The optimum assignment schedule is given by $X \to I, Y \to II, Z \to III.$ The minimum assignment cost = 2 + 6 + 0 = Rs.8.

(2). Consecutive Integer With Residue Partitioning Algorithm :

$$\begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix} \end{array}$$

Select the maximum element and partition this using consecutive integer with residue partitioning. It is given by $8 = 1 + 2 + 3 + 2$. Denote the length of the partition by l, here $l = 4$.

Convert the given unbalanced assignment problem into balanced one by adding dummy row as the value of length of the maximum cost element . The balanced cost matrix is given by,

$$\begin{array}{c} \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 4 & 4 & 4 \end{pmatrix} \end{array}$$

Select the minimum element from each row and divide the remaining elements by this minimum element and round off the integer and do assignment interms of ones

$$\begin{pmatrix} (1) & 2 & 2 \\ 1 & (1) & 2 \\ 1 & 1 & (1) \end{pmatrix}$$

$\therefore$ The optimum assignment schedule is given by $X \to I, Y \to II, Z \to III.$ The minimum assignment cost = 2 + 6 + 4 = 12 - l = Rs.8.

III. Conclusion

The algorithm appears to be the same as the Hungarian method, but on dividing the element from each row by the selection of minimum cost element, the calculation is made easier to find the optimum assignment cost. But to get the optimum assignment cost we adopt a slightly different procedure. This algorithm may be extended to any other problem which can be represented as network.

REFERENCES

1. G. E. Andrews, The Theory of Partitions, Encyclopedia of Mathematics and Its Applications. Vol.2. Reading, Mass.: Addison-Wesley, 1976.
2. George E. Andrews, Number Theory, W. B. Saunders Company, London
3. Ivan Niven, Herbert S. Zuckerman, Hugh L. Montgomery, An Introduction to the theory of Numbers, 5th Edition, John Wiley & Sons, Inc. New York.
4. Hokyu Lee and Seung Kyung Park, The Double Complete Partitions of Integers, Commun. Korean Math. Soc. 17(2002), No.3, pp. 431 – 437.
5. S. K. Park, Complete Partitions, Fibonacci Quart., to appear.
6. Hokyu Lee and Seung Kyung Park, The Double Complete Partitions of Integers, Commun. Korean Math. Soc. 17(2002), No.3, pp. 431 – 437.
7. Oystein J. Rodseth, Journal of integer sequences, Vol. 10 (2007).
8. Kantiswaroop, Gupta P.K and Manmohan, Opereations Research, Sultan Chand & Sons, New Delhi, 2008.
9. Prem Kumar Gupta, D. S. Hira, Problems in Operations research, S. Chand and Company Limited, 2004.
10. Hamdy A. Taha, Operations Research : An Introduction, 10th Edition, Pearson Publications, 2017.
11. Gupta R. K., Linear Programming, Krishna Prakashan Media(P) Ltd., 2009.
12. V. Sundaresan, K. S. Ganapathy Subramanian, K. Ganesan, Resource Management Techniques, A. R. Publications, June 2013.
13. P. Geetha, A. Gnanam, Shortest Path Problems Using Complete Partitions, International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 12 (2018) pp. 10478-10481 © Research India Publications.
14. P. Geetha, A. Gnanam, Prim's Algorithm Combined With Partitions, International Journal of Scientific Research and Review, ISSN NO: 2279-543X, Volume 8, Issue 5, 2019, pp 1593 – 1599.
15. S. Vimala, S. Krishna Prabha, Assignment Problems with fuzzy costs using Ones Assignment Method, IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 5 Ver. V (Sep. - Oct.2016), PP 85-89.

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