η-Open Sets in Topological Spaces

D. Subbulakshmi, K. Sumathi, K. Indirani

Abstract: ***In this paper a new class of open sets in a topological space called η-open sets is defined and the inclusive relationship of this set with the other existing sets like semi open, pre-open, α-open, r-open sets are discussed.In addition to this the concept of η-interior, η-closure, η-boundary, η-exterior, η-derived, η-border, η-neighbourhood, η-dense, η-residual are also introduced.***

Keywords:η-open, η-closed, η-interior, η-closure, η-boundary, η-exterior, η-derived, η-border, η-neighbourhood, η-dense, η-residual.

I. INTRODUCTION

In recent years a number of generalizations of open sets have been developed by many mathematicians. In 1963, Levine [3] introduced the notion of semi-open sets in topological spaces. In 1984, Andrijevic [1] introduced some properties of the topology of α-sets. In 2016, Sayed and Mansour introduced[6] new near open set in Topological Spaces. Motivated by various open and closed sets discussed in the previous literature, in this paper η-open sets using the concept of semi open and α-open set in topological spaces are introduced. This paper also defines the η-interior, η-closure, η-boundary, η-exterior, η-derived, η-border, η-neighborhood, η-dense, η-residual.

II. PRELIMINARIES

Definition :2.1

A subset A of topological space(X,τ) is called
(i) α-open [1] if A⊆int(cl(int(A)))
(ii) pre-open [4] if A⊆int(cl(A))
(iii) semi-open [3] if A⊆cl(int(A))
(iv) regular open [5] if A=int(cl(A))
(v) β-open [2] (or semi pre open) if A⊆ (cl(int(cl(A)))

Proposition : 2.1
For any two subsets A,B of a space(X,τ)the following statements are true :
(i) scl(A)= A∪int(cl(A)),sint(A)= A∩ cl(int(A))
(ii) αcl(A) = A∪cl(int(cl(A))), αint(A) = A∩ int(cl(int(A)))
(iii) pcl(A)= A∪cl(int(A)),pint(A)= A∩int(cl(A))
(iv) spcl(A) = A∪int(cl(int(A))), spint(A) = A∩cl(int(cl(A)))
(v) X \ (int(A)) = cl(X \ (A)), int(X \A) = X \ cl(A).

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III. MAIN RESULTS

Definition: 3.1
Let (X,τ) be a topological space. Then a subset A of X is said to be
(i). an η-open set if A ⊆int(cl(int(A)))∪ cl(int(A)).
(ii).an η-closed set if A ⊇cl(int(cl(A))) ∩ int(cl(A)).
The family of allη-open set (resp. η-closed set) subsets of a space(X,τ) is denoted by η-o(X) (resp. η-c(X)).

Proposition: 3.1
(i). Every open set is an η-open set.
(ii). Every α-open set is an η-open set.
(iii) Every r-open set is an η-open set.

Remark: 3.1
The converse of the above results need not be true as seen from the following example.

Example: 3.1
Let X={a, b, c},τ ={X, φ,{a}, {c}, {a, c}}. A = {a, b} is an η-open set but not open, α-open, r- open set.
Note: 3.1
Every semi-open set is an η-open set.

Lemma: 3.1
Intersection of two η-open sets need not be an η-open set.

Example: 3.2
Let X={a, b, c},τ ={X, φ,{a}, {c}, {a, c}}. Here the sets {a, b} and {b, c} are η-open sets,
but {a, b} ∩{b, c} = {b} is not an η-open set.

Lemma: 3.2
The finite union of η-open sets is an η-open set.
Proof :
Let $\{A_\alpha\}_{\alpha\epsilon\Delta}$ be a family of η-open sets in a space (X ,τ), then
$A_\alpha \subset int(cl(int(A_\alpha))) \cup cl(int(A_\alpha)), \forall \alpha\epsilon\Delta$
(where Δ = 1, 2 ...... n)
now,
$\cup_{\alpha\epsilon\Delta} A_\alpha \subset \cup_{\alpha\epsilon\Delta}\{int(cl(int(A_\alpha))) \cup cl(int(A_\alpha))\}$
$= [\cup_{\alpha\epsilon\Delta}\{int(cl(int(A_\alpha)))\}] \cup [\cup_{\alpha\epsilon\Delta}\{cl((A_\alpha))\}]$
$\subset$
$[int\{cl(\cup_{\alpha\epsilon\Delta}(int(A_\alpha)))\}] \cup [cl\{\cup_{\alpha\epsilon\Delta}(int(A_\alpha))\}]$
$\subset$
$[int\{cl((int\cup_{\alpha\epsilon\Delta}(A_\alpha)))\}] \cup$
$[cl\{(int(\cup_{\alpha\epsilon\Delta}(A_\alpha))\}]$

$\Rightarrow \cup_{\alpha\epsilon\Delta} A_\alpha$ is also an η-open set.

Definition: 3.2
Let (X,τ) be topological space. Then:
(1) The union of all η-open sets of X contained in A is called η-interior of A and is denoted by η-int(A).
(2) The intersection of all η-closed sets of X containing in A is called η-closure of A and is denoted by η-cl(A).
Theorem: 3.1
Let (X,τ) be a topological space and A⊂X, then the following statements are equivalent:
(i) A is anη-open set,
(ii) A=αint(A)∪sint(A)
Proof:
(i)→ (ii). Let A be anη-open set.Then A ⊆int(cl(int(A))) ∪ cl(int(A))[By proposition 2.1].
αint(A)∪sint(A)=(A∩int(cl(int(A)))∪(A∩cl(int(A))) = A∩(int(cl(int(A)))∪cl(int(A)))= A
(ii)→ (i). Suppose that A = αint(A) ∪sint(A).[By proposition 2.1]
A = (A∩int(cl(int(A)))∪(A∩cl(int(A)))⊂int(cl(int(A)))∪cl(int(A)). Therefore, A is anη-open.

Remark: 3.2
Let (X,τ) be topological space and A⊂X, then the following statement are equivalent:
(i) A is an η-closed set,
(ii) A=αcl(A)∩scl(A).

Theorem: 3.2
Let A be a subset of a space (X,τ). Then ηcl(A)=αcl(A)∩scl(A)
Proof:
Let A ⊂ X. Where (X, τ) is a topological spaces.Since ηcl(A) is an η-closed set
ηcl(A) ⊃ cl(int(cl(ηcl(A)))) ∩ int(cl(ηcl(A))) ⊃ cl(int(cl(A)) ∩ int(cl(A)) [By definition 3.1]
A∪ηcl(A) ⊃A∪(cl(int(cl(A))) ∩ int(cl(A))) ⟹ ηcl(A)⊃(A∪(cl(int(cl(A))) ∩(A∪int(cl(A))))
⊃ αcl(A) ∩ scl(A) ----I [By proposition 2.1] alsoηcl(A) ⊂ αcl(A) and ηcl(A)⊂scl(A) then
ηcl(A)⊂αcl(A)∩ scl(A) ----II. From I and II ηcl(A)= αcl(A) ∩ scl(A).

Remark: 3.3
Let A be a subset of a space (X,τ). Then ηint(A)=αint(A)∪sint(A).

Theorem :3.3
Let A be a subset of a space (X,τ). Then
(i) A is an η-open set if and only if A =η-int(A)
(ii) A is an η-closed set if and only if A=η-cl(A)
Proof:
(i) Let A be an η-open set. Then by theorem (3.1), A=αint(A)∪sint(A) and by theorem (3.2), we have A =η-int(A). Conversely, let A =η-int(A) Then by theorem(3.2),A= αint(A)∪sint(A) and by theorem (3.1), A is an η-open A= αint(A)∪sint(A).
(ii) Let A be an η-closed set. Then by theorem (3.1), A = αcl(A) ∩ scl(A) and by theorem (3.2), we have A = η-cl(A). Conversely, let A= η-cl (A) Then by theorem (3.2), A=αcl(A)∩scl(A) and by theorem (3.1), A is an η-closed set.

Theorem: 3.4
Let A and B be a subsets of a space(X,τ). Then the following are true
(i) η-cl(X\ A) = X \η-int(A).
(ii) η-int(X\ A) = X \η-cl(A).
(iii) If A ⊆B, then η-cl (A)⊆η-cl (B)
(iv) x∈η-cl(A)if and only if there exists anη-open set U and x∈U such that U ∩ A ≠ϕ.
(v) x∈η-int(A) if and only if there exists anη-open set G and x∈G such that x∈G⊆A
(vi) η-cl(η-cl(A))=η-cl(A)and η-int(η-int(A))=η-int(A).
(vii) η-cl(A)∪η-cl(B)⊆η-cl(A∪B)and η-int(A)∪η-int(B)⊆ηint(A∪B)
(viii) η-int(A∩B)⊆η-int(A)∩η-int(B), η-cl(A∩B)⊆η-cl(A)∩η-cl(B)
Proof:
(i) Since (X\A) ⊆X, [By theorem 3.3] η-cl (X\A) = αcl (X \ A)∩scl(X\ A) [By proposition 1.1]
η-cl(X \ A) = (X \ αint(A)) ∩ (X \ sint(A)) = X \ (αint(A) ∪sint(A)) [By theorem 3.3],
ηcl(X \A) = X \ηint(A).
(ii) Since (X\A)⊆X, [By theorem 3.3] η-int(X\A)= αint(X \ A) ∪ sint(X\A) [By proposition 1.1]
η-int(X \ A) = (X \ αcl(A)) ∪ (X \ scl(A)) = X \ (αcl(A) ∩ scl(A)) [By theorem 3.3]
η-int(X \ A)= X \ η-cl(A).
(iii) Since, η-cl(A)= αcl(A)∩scl(A) and A ⊆B, η-cl(A)= αcl(A)∩scl(A)⊆αcl(B)∩scl(B)=η-cl(B)
(iv) Let x∉η-cl(A)then x∉∩F where F is η-closed with A ⊂F, so x ∉X\∩F and X\∩F is a η-open set containing x and hence(X\∩F)∩A⊆(X\∩F)∩(∩F)= φ. Conversely, suppose that exists anη-open set containing x with A ∩ U=ϕ. Then A⊆X\ U and X \ U is an η-closed. Hence x∉η-cl(A).
(v) Necessity: Let x∈η-int(A). Then x ∈∪{G:G is η-open G ⊆A} and hence there exists an η-open set G such that x∈G⊆A
Sufficiency: Let G be an η-open set such that x∈G⊆A. Then A=∪{G: x∈G} which is the union of η-open set. Therefore, x∉η-cl(A).
(vi) Since η-cl(η-cl(A)) = αcl(ηcl(A))∩scl(ηcl(A)). [By theorem 3.3]
αcl(αcl(A)∩ scl(A))∩scl(αcl(A)∩scl(A))⊆(αcl(A)∩ αcl(scl(A)))∩scl(αcl(A)∩scl(A))= αcl(A)∩ scl(A)=ηcl(A). Hence η-cl(η-cl(A))⊆η-cl(A). But, η-cl(A)⊆η-cl(η-cl(A)). Therefore, η-cl(η-cl(A))=η-cl(A).
(vii) Since A⊆A∪B and B⊆A∪B.we have η-cl (A) ⊆η-cl(A∪B) and η-cl (B) ⊆η-cl(A∪B). Therefore,η-cl (A) ∪η-cl (B)⊆η-cl(A∪B). A ⊆(A∪B)and B⊆(A∪B).we have η-int(A) ⊆
η-int(A∪B) and η-int(B) ⊆η-int(A∪B). Therefore,η-int(A) ∪η-int(B) ⊆η-int(A∪B).
(viii) Since A ⊇A∩B and B ⊇A∩B we have η-cl (A) ⊇η-cl(A∩B) and η-cl (B) ⊇η-cl(A∩B),
η-cl(B) ⊇η-cl ((A∩B).Therefore, η-cl (A) ∩ η-cl (B)⊇ηcl(A∩B) and A⊇(A∩B) and B⊇(A∩B). We have η-int(A) ⊇η-int(A∩B) and
η-int(B)
⊇η-int(A∩B).Therefore

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η-int(A)∩η-int(B) ⊇η-int(A∩B).

Remark:3.4
The inclusion relation in part (vii),(viii) of the above theorem cannot be replaced by equality as shown by the following example.

Example: 3.3
Let X={a,b,c,d} with topology τ={X,Φ, {a}, {b},{a, b},{a,b,c}}.
(i) If A={a,b},B={d} and (A∪B)={a,b,d}, then η-int(A)= {a, b},η-int(B)=ϕ and η-int(A∪B)={a,b, d}. So, η-int(A∪B)⊄η-int(A) ∪η-int(B)
(ii) If C={a}, D ={b} and (C∪D)= {a, b} then η-cl(C)={a}, η-cl(D)= {b}and η-cl(C∪D)= X, therefore, η-cl (C) ∪η-cl (D)⊄η-cl(C∪D).

Example: 3.4
Let X={a,b,c,d} with topology τ={X,Φ, {b},{c, d},{b, c, d}} then
(i) If A={a, b}, B={b, c} and(A∩B)={b}, then η-cl (A)= {a, b}, η-cl (B)= Xand η-cl(A∩ B)= {b}. So, η-cl(A∩B)⊄η-cl(A) ∩ η-cl(B)
(ii) If C={a, b}, D={a, c, d} and (C∩D)={a}, then η-int(C)={a, b}, η-int(D)= {a, c, d}and η-int(C∩D)=ϕ. η-int(C) ∩ η-int(D) ⊄η-int(C∩D)

IV. Some Topological Operations

Definition: 4.1
Let (X,τ) be a topological space and A⊂X. Then the η-boundary of A (briefly, η-b(A)) is given by η-b(A))=η-cl (A)) ∩ η-cl(X/A).

Example :4.1
Let X={a,b,c,d} with topology τ={X,Φ, {a},{b, d},{a, b, d}}. For the set, A={a, b, d}.
η-b(A))={c}.

Theorem:4.1
If A is a subset of a space (X,τ), then the following statement are hold:
(i) η-b(A)) =η-b(X \ A)).
(ii) η-b(A)) =η-cl(A)) \η-int(A).
(iii) η-b(A))∩η-int(A)=Φ.
(iv) η-b(A))∪η-int(A)=η-cl(A).
Proof:
(i) Since η-b(A)) = η-cl(A)) ∩ η-cl(X \ A) = η-b(X \ A)) = η-cl(X \ A))∩ η-cl(A)
(ii) Since η-b(A)) = η-cl(A) \ η-int(A) = η-cl(A))∩ η-cl(X \ A)= η-cl(A)) ∩ (X \ η-int(A))= (ηcl(A) ∩ X) \ (η-cl(A)∩ η-int(A))= η-cl(A)\ η-int(A).
(iii) By using(2)η-b(A)) ∩ η-int(A) = (η-cl(A) \ η-int(A)) ∩ η-int(A)=(η-cl(A)∩ η-int(A))\(η-int(A)∩ η-int(A))=ηint(A)\ηint(A) = Φ.
(iv) By using(3)η-b(A))∪ η-int(A) = (η-cl(A)\ η-int(A))∪ η-int(A) = (η-cl(A)∪ η-int(A))\(η-int(A) ∪ η-int(A))=η-cl(A) \ η-int(A)=ηcl(A).
Theorem : 4.2
If A is a subset of a space (X,τ), then the followingstatements are hold:
(i) A is anη-open set if and only if A∩η-b(A)=Φ
(ii) A is anη-closed set if and only if η-b(A) ⊂A
(iii) A is anη-clopenset if and only if η-b(A)=Φ.
Proof:
(i) Let A is anη-open set.Then A= η-int(A). A∩η-b(A)=η-int(A)∩η-b(A)[By theorem 4.1]
= η-int(A)∩(η-cl(A) \ η-int(A)) =(η-int(A)∩η-cl(A)) \(η-int(A)∩(η-int(A)) =Φ.
Conversely, let A∩η-b(A) = A∩(η-cl(A) \ η-int(A))[By theorem 4.1]= (A∩(η-cl(A)) \ (A∩(η-int(A)) = A \ η-int(A)=Φ. Hence A is anη-open.
(ii) Let A is a η-closed set.Then A = η-cl(A) [By theorem 4.1] but η-b(A) = (η-cl(A) \ η-int(A))
= A\ η-int(A)⊂AConversely, Let η-b(A) ⊂A. [By theorem 4.1] η-cl(A)=η-b(A))∪η-int(A)
⊂A∪η-int(A)= A. Thus η-cl(A) ⊂A and A⊂η-cl(A).Therefore, A=η-cl(A).
(iii) Let A is anη-clopenset. Then A = η-int(A), and A = η-cl(A) [By theorem 4.1]η-b(A) =
(η-cl(A)\ η-int(A))= A\ A=Φ. Conversely, Suppose that η-b(A)= Φ. Then η-b(A)=(η-cl(A)\η-int(A))= Φ. HenceAis anη-clopenset.

Definition: 4.2
Let (X,τ) be a topological space and A⊂X.Then the setX\(η-cl(A)) is called the η-exterior of A and is denoted by η-ext(A). Each point p∈X is called anη-exterior point of A, if it is a η-interior point of X\A.

Example : 4.2
Let X={a,b,c,d} with topology τ={X,Φ, {a},{b, c},{a,b, c}}. If A={a}, B={a, b}, C={ a, c} then we haveη-ext(A)={ b,c,d}, η-ext(B)=Φ and η-ext(C)=Φ
Theorem : 4.3
If A and B are two subsets of a topological space (X,τ), then the following statements are true
(i) η-ext(A)=η-int(X \ A).
(ii) η-ext(A) is η-open
(iii) η-ext(A)∩η-int(A)=Φ.
(iv) η-ext(A)∩η-b(A)=Φ.
(v) η-ext(A)∪η-b(A)=η-cl(X \A).
(vi) {η-int(A),η-b(A)and η-ext(A)} from apartition of X.
(vii) If A ⊆B, then η-ext(B)⊆η-ext(A)
(viii) η-ext(A∪B)⊆η-ext(A)∪η-ext(B).
(ix) η-ext(A∩B)⊇η-ext(A)∩η-ext(B).
(x) η-ext(X) =Φand η-ext(φ) = X.
Proof:
(i) by Definition (4.2) η-ext(A) = X \ η-cl(A) = η-int(X \ A).
(ii) From (1) η-ext(A) = η-int(X \ A).Since η-int(A) is the union of all η-open sets of X contained in A. Thus η-ext(A) is anη-open.
(iii) Since η-ext(A) ∩ η-int(A)= X \ η-cl(A)∩ η-int(A)= η-int(X \ A)∩ η-int(A)=φ
(iv) By theorem (4.1), η-ext(A) ∩ η-b(A) =η-int(X \ A) ∩ η-b(X \ A)= Φ.
(v) By theorem (4.1). η-ext(A)∪η-b(A)=η-int(X \A)∪η-b(X \A)=η-cl(X \A).
(vi) From (iii), (iv) we have η-ext(A) ∩ η-int(A) = Φ and η-ext(A) ∩ η-b(A) = Φ. Then by

theorem (4.1) then η-b(A))∩ η-int(A)=Φ.η-int(A)∪η-b(A)∪η-ext(A)= X. Hence from(v) η-ext(A)∪η-b(A)=η-cl(X \ A)then η-int(A)∪η-cl(X \ A)=η-int(A)∪X \η-int(A)= X.

(vii) Let A ⊆B then (η-cl(A))⊆(η-cl(B)) and hence X \ (η-cl(B))⊆X \ (η-cl(A)). So η-ext(B)⊆η-ext(A).

(viii) η-ext(A∪B)= X \(η-cl(A∪B))⊆X \(η-cl(A)∪(η-cl(B))⊆(X \(η-cl(A)))∪(X\(η-cl(B))) ⊆η-ext(A)∪η-ext(B)⊆η-ext(A)∪η-ext(B).

(ix) η-ext(A∩B)= X \(η-cl(A∩B))⊇X \(η-cl(A)∩(η-cl(B))⊇(X \(η-cl(A)))∩(X \(η-cl(B))) ⊇η-ext(A)∩η-ext(B)⊇η-ext(A)∩η-ext(B).

(x) η-ext(X) = X \ (η-cl(X))= X \ X=φand η-ext(φ) = X \ (η-cl(φ))= X \φ= X.

Remark : 4.1

The inclusion relation in part (v),(vi) of the above theorem cannot by replaced by equality as is shown by the following example.

Example : 4.3

Let X={a,b,c,d} with topology τ={X,Φ, {a}, {b}, {a, b}, {a, b, c}}. A={a,c}andB={c, d} then η-ext(A)={b, d},η-ext(B)= {a, b} but η-ext(A∪B) = {b}.

Therefore,η-ext(A)∪η-ext(B) ⊈ η-ext(A∪B). Also η-ext(A∩B) ={a,b, d}, hence η-ext(A∩B)⊈η-ext(A)∩η-ext(B).

Definition : 4.3 If A is a subset of a topological space (X,τ), then a point p∈X is called a η-limit point of a set A ⊂X if every η-open set G⊂X containing p, contains a point of A other than p. The set of all η-limit point of A is called an η-derived set of A and is denoted by η-d(A)

Example : 4.4

Let X={a,b, c} with topology τ={X,Φ, {a}}. A={a, c}, thenη-d(A))= {c}.

Theorem : 4.4

If A and B are two subsets of aspace (X,τ), then the following statements are hold:

(i) IfA⊂B,then η-d(A)⊂η-d(B).

(ii) A is anη-closed set if and only if it contains each of its η-limit point.

(iii) η-cl(A)= A∪η-d(A).

(iv)η-d(A∪B)⊃η-d(A)∪η-d(B)

(v) η-d(A∩B)⊂η-d(A)∩η-d(B)

Proof:

(i) By definition (4.3), we have p ∈η-d(A) if and only if G ∩ (A\{P}) ≠ ϕ, for every η-open set G containing p. But A ⊂B, then G ∩ (B \{P}) ≠ ϕ, for every η-open set G containing p. Hence, so p ∈η-d(B). Therefore η-d(A) ⊂η-d(B).

(ii) Let A be an η-closed set and p∉A then p∈(X \ A) which is an η-open, hence there exists η-open(X\A) such that(X \ A)∩A=ϕ.So p∉η-d(A), thereforeη-d(A)⊂A.Conversely, suppose that η-d(A)⊂A and p∉A. Then p∉η-d(A), hence there exists η-open setG containing p such that G ∩ A=ϕ and hence X \ A = $\bigcup_{p\in A}\{G, G \text{ is } \eta-\text{open}\}$. Therefore, A is anη-closed.

(iii)Since, η-d(A)⊂ η-cl(A) and A ⊂ η-cl(A). η-d(A)∪A⊂ η-cl(A). Conversely, Suppose that p∉η-d(A)∪A. Then p∉η-d(A), p∉A and hence there exists η-open set G containing p such that G ∩ A=ϕ. Thus p∉η-cl(A) which implies that η-cl(A)⊂η-d(A)∪A , therefore, η-cl(A)=η-d(A)∪A.

(iv)Since A ⊆A∪B and B ⊆A ∪B.We have η-d(A) ⊆ η-d(A∪B)and η-d(B) ⊆ η-d(A∪B).

Thereforeη-d(A)∪ η-d(B) ⊆η-d(A∪B).

(v)Since A ⊇ A∩ B and B ⊇ A ∩ B. We have η-d(A) ⊇ η-d(A∩B)and η-d(B)⊇ η-d(A∩B).

Thereforeη-d(A)∩ η-d(B)⊇ η-d(A∩B).

Definition : 4.4

Let (X,τ)be a topological space and A⊆X. Then the η-border of A (briefly, η-Bd(A)) is given by η-Bd(A))=A\η-int(A).

Example : 4.5

Let X={a,b,c} with topology τ={X,Φ,{a},{b},{a, b}}. If A={a,c}, B={c} then η-Bd(A)=Φ, η-Bd(B)= {c}.

Theorem : 4.5

For a subset A of a space and X, the following statements are hold:

(i) A=η-int(A)∪η-Bd(A))

(ii)η-int(A)∩η-Bd(A))=φ

(iii) η-Bd(X)) = η-Bd(φ)) =φ

(iv) η-Bd(η-int(A)) = φ

(v) η-int(η-Bd(A)) =φ

(vi) η-Bd(η-Bd(A)) = η-Bd(A))

(vii) η-Bd(A)) = A∩η-cl(X \ A)

(viii) η-Bd(A)) =η-d(X \A)

Proof:

(i) η-int(A) ∪ η-Bd(A)) =η-int(A) ∪(A\η-int(A)) = (η-int(A) ∪A) \ (η-int(A) ∪η-int(A))
= A\η-int(A) = A

(ii) η-int(A)∩η-Bd(A)) = η-int(A)∩(A\η-int(A) = (η-int(A)∩A) \ (η-int(A)∩η-int(A))
= η-int(A) \η-int(A) = φ

(iii) η-Bd(X)) = X \ η-int(X)=X \X=φand η-Bd(φ))=φ \η-int(φ) = φ \φ=φ .

(iv) η-Bd(η-int(A))=η-int(A)\η-int(A)=φ .

(v)Since,
η-int(η-Bd(A))=η-int(A\η-int(A))=η-int(A)\η-int(η-int(A))=η-int(A)\η-int(A)=φ

(vi) Since,
η-Bd(η-Bd(A))=η-Bd(A)\η-int(η-Bd(A))=η-Bd(A)\φ=η-Bd(A)

(vii) η-Bd(A)) = A\η-int(A))= A\ (X \η-cl(A))= A∩η-cl(X \ A)

(viii) η-Bd(A)) = A\η-int(A))= A\ (A\η-d(A))=η-d(X \A).

Theorem : 4.6

For a subset A of a space(X,τ), the following statements are equivalent

(i) A is an η-open,

(ii) A=η-int(A),

(iii) η-Bd(A)=ϕ.

Proof:

(i)→(ii) Obvious from Theorem (4.4).

(ii)→(iii).Suppose that A=η-int(A). Then by Definition (4.4),η-Bd(A)=η-int(A)\η-int(A)=ϕ
(iii)→(i). Let η-Bd(A)=ϕ. Then by Definition (4.4), A\η-int(A)=ϕ and hence A=η-int(A).

Definition : 4.5
A subset N of a topological space (X,τ) is called a η-neighbourhood (briefly, η-nbd.) of a point p∈X if there exists an η-open set G such thatp∈G ⊆N. The class of all η-neighbourhood of p∈X is called the η-neighbourhood system of p and denoted by η-Np.

Example :4.6
Let X={a,b,c,d} with topology τ={X,Φ,{a}, {b, d},{a,b, d}}, then η-Nc={a,c}.
Remark : 4.2
For any topological space (X,τ) and for each x∈X we have Nx⊆p-Nx⊆η-Nx.

Example : 4.7
Let X={a,b,c,d} with topology τ={X,Φ,{a}, {b, d},{a,b, d}}. We have {a,c}∈η-Nc but it is not in p-Nc and not in Nc.

Theorem : 4.7
A subset G of a topological space (X,τ) is anη-open if and only if it is an η-neighbourhood, for every point p∈G.
Proof:
Necessity: Let G be an η-open set. Then G is an η-neighbourhood for each p∈G.
Sufficiency: Let G be anη-neighbourhood, for each p∈G. Then there exists anη-open set W containing p such that p∈W⊆G, so G=∪{p:p∈W}. Therefore, G is an η-open.

Theorem : 4.8
For a topological space (X,τ). If η-Np is an η-neighbourhood systems of a point p∈X, then the following statements are hold:
(i) η-Np is not empty and p belongs to each member of η-Np
(ii) Each superset of the members of η-Np belongs to η-Np,
(iii) Each member N∈η-Np is a superset of the member W∈η-Np, where W is anη-neighbourhoodof each point p∈W.
Proof:
(i) Since X is anη-open set containing p, then X∈η-Np. So, η-Np≠ϕ. Also, if Nη-Np, then there exists anη-open set G such that p∈G⊆N.Therefore, p belongs to each member of η-Np.
(ii) Let Mbe a superset of N∈η-Np, then there exists a η-open set G such that p∈G⊆N⊆M which implies p∈G⊆M and hence, M is a η-neighbourhoodof p. Therefore, M∈η-Np
(iii) Let N be anη-neighbourhoodof p∈X, then there exists anη-open set W such that p∈W⊆N. Then by Theorem 3.4.1, W is anη-neighbourhoodof each of its points.

Definition : 4.6
For a topological space (X,τ), a subset A of X is said to be η-dense in X if and only if η-cl (A)=X. The family of all η-dense sets in (X,τ) will be denoted by η-D(X,τ).

Example : 4.8
Let X={a,b,c} with topology τ={X, Φ, {a}}.If A={a,b}, and η-cl (A)=X then η-dense in X.

Remark : 4.3
Every η-dense set in a space (X,τ) is dense in (X,τ) by the fact that η-cl (A) ⊆cl (A), while the converse may not be true.

Example : 4.9
Let X={a,b,c,d} with topology τ={X,φ, {a},{ b,d},{a, b,d}}. If A= {d}, then cl(A)=X but η-cl(A)={b,d}. Therefore, A is dense inX but not η-dense in X.

Theorem : 4.9
For a space (X,τ) and E ⊆X, the following statements are equivalent:
(i) E is anη-dense in X
(ii) If F is an η-closed set in X containing E, then, F=X
(iii) η-int(X \ E)=ϕ.
Proof:
(i)→(ii). Let Ebe anη-dense set of X. Then η-Cl(E)=X. But F is an η-closed set contains E, then η-Cl(E)⊆F and therefore F=X.
(ii)→ (iii). Since η-Cl(E) is an η-closed set contains E, By (2) we have η-Cl(E)=X.
Hence φ = X \ η-cl(E) = η-int(X \ E) .
(iii)→ (i). Since η-int(X \ E)=ϕ. Then η-Cl(E)=X. Hence E is anη-dense in X.

Proposition : 4.1
For a topological space (X,τ), if E∈η-D(X,τ), then the following statements are hold:
(i) η-b(E) = η-cl(X \ E) ,
(ii) η-ext(E) =φ.
Proof:
(i) From Definition (4.1), we have η-b(E)=η-cl(E)∩η-cl(X \E) and since E∈η-D(X,τ), then η-b(E) = η-cl(X \ E)
(ii) Also, by From Definition (4.2), η-ext(E)= X\η-cl(E) but E∈η-D(X,τ), then η-ext(E)=ϕ.

Definition : 4.7
For a space (X,τ), A⊆X is called:
(i) η-nowhere dense if int(A)⊆η-int(η-cl(A))=ϕ
(ii) η-residual if η-cl(X\A)=X or η-int (A)=ϕ
η-nowhere dense is η-residual from the fact that η-int(A)⊆η-int(η-cl(A)) for every A⊆X.

Example : 4.10
Let X={a,b,c}with topology τ={X,Φ,{a}, {b}, {a,b}} and A={c} than η-int(η-cl(A))=ϕ. and η-int (A)=ϕ so A is η-nowhere dense andη-residual.

Proposition : 4.2
A subset A of a topological space (X,τ), A⊆X is an η-nowhere dense of X if A⊆η-Cl(X \ η-cl (A)).
Proof:
Let A is an η-nowhere dense then η-int(η-cl(A))=ϕ.HenceX \ η-int(η-cl(A))) = η-cl(X \ η-cl(A)) = η-cl(η-int(X \ A)) = X ⊇A.

Theorem : 4.10

The η-boundary of each η-open (resp. η-closed) set is η-nowhere dense.
Proof:
Let A∈η-o(X) then
η-int(η-cl(η-b(A)) = η-int(η-cl(η-cl (A)) ∩ η-cl(X \ A)) = η-int(η-cl(η-cl (A)) ∩ (X \ η-int(A))
⊆η-int(η-cl (A)) ∩η-int(X \ η-int(A)) ⊆η-int(η-cl (η-int(A))) ∩ (X \ η-cl(η-int(A)))⊆η-cl (η-int(A))∩ (X \ η-cl(η-int(A)) = ϕ.
Also if A∈η-c(X) Then
η-int(η-cl(η-b(A)) = η-int(η-cl(η-cl (A)) ∩ η-cl(X \ A)) = η-int(η-cl (A)) ∩η-int(X \ η-int(A))
= η-int(η-cl (A)) ∩ (X \η-cl(η-int(A))) ⊆η-int(η-cl (A)) ∩ (X \η-cl(η-int(η-cl(A))))⊆sη-int(η-cl (A)) ∩ (X \η-int(η-cl(A))) = ϕ.

Proposition : 4.3
For a space (X,τ), A ⊆X, then the sets A∩ η-cl(X\A) and η-cl(A)∩ (X\A) are η-residual.
Proof:
Sinceη-int(A∩η-cl(X\A)) ⊆η-int(A) ∩η-int(η-cl(X\A))) ⊆η-int(A) ∩η-cl(X\A)
= η-int(A) ∩ (X \ η-int(A))= ϕ.Then A∩η-cl(X\A) is residual.Similarly, η-int(η-cl(A)∩(X \ A))⊆η-int(η-cl(A))∩η-int(X \ A)=η-cl(A)∩(X \η-cl(A))= φ,and hence η-cl(A)∩(X\A) is η-residual.

Theorem : 4.11
The η-boundary of any set contains the union of two η-residual sets.
Proof:
Let (X,τ) be a space and A⊆X. Then by Proposition (4.3), we have
(A∩η-cl(X\A)) ∪ (η-cl(A) ∩ (X\A)) = (A∩η-cl(X\A) ∪η-cl(A)) ∩ (A∩η-cl(X\A))∪(X\A) = (A∪η-cl(A))∩ (η-cl(X\A)∪η-cl(A))∩ ((A∪(X\A)∩η-cl(X\A) ∪(X\A) =η-cl(A)∩ (η-cl(A))∩(η-cl(X\A)∩ (η-cl(X\A))) ⊆η-cl(A) ∩ (η-cl(A∩(X\A)))∩ (η-cl(X\A))) =η-cl(A)∩ (η-cl(X\A)) =η-b(A).

V. CONCLUSION

In this paper we found η-open set and η-closed sets in topological spaces and deals properties of η-open set.

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7.

system presentation it is clear that, FA based PID regulator with stands an assortment of circumstances of the consignment. Nature inspired meta-heuristic methods provide plagiaristic free solution to optimize combination problems. A new meta experiential search algorithm, Firefly Algorithm is pragmatic to train BPNN to achieve fast meeting rate and to minimize the training error. FA is applied to train the NN by exploiting the objective function definitely. However, the number of exercise data, populace of fireflies and repetition number must be satisfactorily high to obtain high correctness. The proposed FA based NN controller when implemented in a two area interconnected power system ensures an improved transitory response of the system than that of the output response obtained with PID Controller examination on the values of parameters like attraction and light intensity in better-quality version of firefly algorithm is to be investigated in the near future.

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