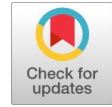


# Critical path through Interval valued Hexagonal Fuzzy Number



Ghousia Begum S, N. Jose Parvin Praveena, A. Rajkumar

**Abstract:** This critical path method is very useful method in the project network to succeed the multifaceted jobs. The main benefit of this technique is to decrease the completion time of the jobs. In this paper the Interval valued Hexagonal fuzzy number is applied to determine the critical path. The procedure for critical path method is described. Finally Numerical example is presented.

**Index Terms:** Critical Method, Hexagonal fuzzy number, Shortest Path, Floats.

## I. INTRODUCTION

The expression of Fuzzy number in a fuzzy set defines a fuzzy interval in the real number. This interval becomes a fuzzy set as its boundary is ambiguous. The representation of a fuzzy interval is done by two end points  $a_1$  and  $a_3$  and peak point  $a_2$  as  $[a_1, a_2, a_3]$ . The  $a$ -cut operation can be also applied to the fuzzy number. If  $a$ -cut interval for fuzzy number is denoted as  $A$  as  $A_\alpha$ , the obtained interval  $A_\alpha$  is defined as  $A_\alpha = [a_1', a_3']$ . A fuzzy number may emerge in a practical description in the variation of the knowledge on correct value of some measurements when the level of our confidence varies. The concept of fuzzy numbers and its arithmetic operations were first presented and inspected by (Zadeh, 1972) and others. (Dubois Prade, 1978) introduced the notion of a fuzzy number as a fuzzy subset of the real line. The definition of Intuitionistic fuzzy number is suggested by (Burillo, 1994). The Hexadecagonal Intuitionistic fuzzy number was introduced by (Rajkumar, 2017) and used it for reliability evaluation. Also (Jose, 2017) described a interval valued Hexadecagonal fuzzy number to used to find out the failure rate of single cylinder, based on a  $a$ -cut method. (Selvam, 2016) described the dodecagonal fuzzy number and their operational laws. We used Interval valued Hexagonal fuzzy number in this article.. If uncertainty arises in 6 different points, in such cases this Interval valued Hexagonal fuzzy numbers are applied. Critical path method is one of the useful methods to handle complex projects. This article consists of the following sections. Section one designates the definitions of fuzzy number and Interval valued Hexagonal Fuzzy number. Section two explains the procedure to find the critical path. Numerical example is given in section three. Section four concludes the paper.

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## II. PRELIMINARIES

### Definitions

An interval valued fuzzy set  $\tilde{F}$  on  $R$  is given by  $\tilde{F} = \{ \{y, [\mu_{\tilde{F}^L}(y), \mu_{\tilde{F}^U}(y)]\} \} \forall y \in R$

Where  $0 \leq \mu_{\tilde{F}^L}(y) \leq \mu_{\tilde{F}^U}(y) \leq 1$  and  $\mu_{\tilde{F}^L}(y), \mu_{\tilde{F}^U}(y) \in [0,1]$  denoted by

$$\mu_{\tilde{F}}(y) = [\mu_{\tilde{F}^L}(y), \mu_{\tilde{F}^U}(y)], y \in R \text{ or}$$

$\tilde{F} = [\tilde{F}^L, \tilde{F}^U]$  An Interval valued Hexagonal fuzzy number is defined by

$$\tilde{F} = [\tilde{F}^L, \tilde{F}^U] = \left[ \begin{matrix} (f_1, f_2, f_3, f_4, f_5, f_6) \\ (f_1', f_2', f_3', f_4', f_5', f_6') \end{matrix} \right] \text{ Where}$$

$f_1, f_2, \dots, f_6, f_1', f_2', \dots, f_6'$  are real numbers.

### Notations for basic planning computation

$m - n$  = Activity (m, n) with tail event m and head event n

$E_T$  = Earliest incidence time of event m

$L_T$  = Latest acceptable incidence time of event n

$L_{mn}$  = Length of activity

$(E_S)_{mn}$  = EST of activity (m,n)

$(E_F)_{mn}$  = EFT of activity (m,n)

## III. METHODOLOGY

### ALGORITHM To Estimate The Critical Path And The Floats

Step-1 Consider the earliest incident time of the initial event as zero.

Step-2 The maximum of the EFT of all activities ending into that event is considered as the earliest event time for event m.

$E_n = \text{Max} \{ (E_F)_{mn} \}$  for all the immediate predecessor of (mn)

$E_n = \text{Max} \{ E_m + D_{mn} \}$  For the last event the EST and the LFT are same.

Step-4 The least of the LST of all events originating from that event is called the latest event time.

$L_m = \min_n \{ (L_S)_{mn} \}$  for all the immediate successors of (mn)



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$$= \min_n \{ (L_F)_{mn} - L_{mn} \}$$

$$= \min_n \{ L_n - L_{mn} \}$$

The floats are calculated as follows:

Step-5 The difference between the LFT and the EFT or the difference between the LST and the

EST of the activity is taken as Total Float (TF).  $(TF) = (L_F)_{m,n} - (E_F)_{m,n}$

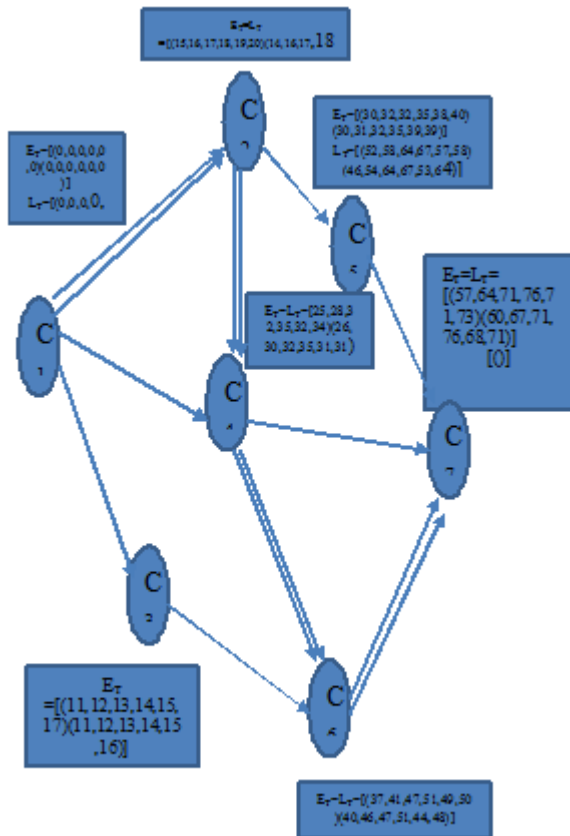
The FF of an activity is that portion of the TF which can be used for rearrangement that activity without disturbing the subsequent activity.

$(FF)_{m,n} = (TF)_{m,n} - (L_T - E_T)$  of the event n Independent float of an activity is calculated using the following formula.  $(IF)_{m,n} = (FF)_{m,n} - (L_T - E_T)$  of the event n. The negative value of IF is considered as zero. Activities which are having zero fuzzy total float are making fuzzy critical path. Consider a network with nodes C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub>, C<sub>4</sub>, C<sub>5</sub> and C<sub>6</sub>. The length of the events are in hours.

**Table: 1 Length of Activity**

Activity	Duration in hours
C <sub>1</sub> - C <sub>2</sub>	Approximately between 15-20 hours ((15,16,17,18,19,20),(14,16,17,18,19,20))
C <sub>1</sub> - C <sub>3</sub>	Approximately between 11-17 hours ((11,12,13,14,15,17),(11,12,13,14,15,16))
C <sub>1</sub> - C <sub>4</sub>	Approximately between 5-10 hours ((5,6,7,8,8,10),(5,7,7,8,9,10))
C <sub>2</sub> - C <sub>5</sub>	Approximately between 15-20 hours ((15,16,15,17,19,20),(16,15,15,17,20,19))
C <sub>4</sub> - C <sub>7</sub>	Approximately between 5-10 hours ((5,7,9,10,6,7),(6,8,9,10,7,5))
C <sub>6</sub> - C <sub>7</sub>	Approximately between 20-25 hours ((20,23,24,25,22,23),(20,21,24,25,24,23))
C <sub>5</sub> - C <sub>7</sub>	Approximately between 5-15 hours ((5,6,7,9,14,15),(14,13,7,9,15,7))
C <sub>3</sub> - C <sub>6</sub>	Approximately between 6-10 hours ((6,7,10,9,8,6),(8,7,10,9,7,8,))
C <sub>2</sub> - C <sub>4</sub>	Approximately between 10-15 hours ((10,12,15,17,13,14),(12,14,15,17,12,11))
C <sub>4</sub> - C <sub>6</sub>	Approximately between 11-17 hours ((12,13,15,16,17,16),(14,16,15,16,13,17))

### IV. BLOCK DIAGRAM FOR CRITICAL PATH



V. RESULT ANALYSIS

Activity	C1-C2	C1-C3	C1- C4	C2- C4	C2- C5	C3- C6	C4- C6	C4- C7	C5- C7	C6- C7
Earliest Start	0	0	0	((15,16,17,18,19,20)(14,16,17,18,19,20))	((15,16,17,18,19,20)(14,16,17,18,19,20))	((11,12,13,14,15,16,17,18,19,20))	((25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50))	((25,28,31,34,37,40,43,46,49,52,55,58,61,64,67,70,73,76,79,82,85,88,91,94,97,100))	(30,32,32,35,38,40)(30,31,32,33,35,39,39))	((37,41,47,51,49,50)(40,46,47,51,44,48))
Earliest finish	((15,16,17,18,19,20)(14,16,17,18,19,20))	((11,12,13,14,15,16,17,18,19,20))	((5,6,7,8,8,10)(5,7,7,8,9,10))	((25,28,32,35,32,34)(26,30,32,35,31))	((30,32,33,38,40)(30,31,33,25,39,39))	((17,19,23,23,31,32,33,39,42))	((37,41,47,51,49,50))	((30,35,41,45,38,41)(32,38,41,45,38,36))	((35,38,39,44,52,55)(44,44,39,44,54,46))	((57,64,71,76,71,73)(60,67,71,76,68,71))
Latest start	0	((20,22,22,4,28,26,27)(21,23,25,27,22,1,27,24,28,22,24))	((20,22,25,27,24,24)(21,23,25,27,22,1,27,24,28,22,24))	((15,16,17,18,19,20)(14,16,17,18,19,20))	((37,42,9,50,38,38,39,49,50,33,45))	((31,34,37,42,41,44,32,39,37,40))	((25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50))	((52,57,62,66,65,66)(54,59,62,66,61,66))	((52,58,64,67,57,58)(46,54,64,67,53,64))	((37,41,47,51,49,50)(40,46,47,51,44,48))
Latest finish	((15,16,17,18,19,20)(14,16,17,18,19,20))	((31,34,37,42,41,44,39,37,40))	((25,28,32,35,32,34)(26,30,32,35,31))	((15,16,17,18,19,20)(14,16,17,18,19,20))	((52,58,64,67,57,58)(46,54,64,67,53,64))	((37,41,47,51,49,50))	((37,41,47,51,49,50))	((57,64,71,76,71,73)(60,67,71,76,73))	((57,64,71,76,71,73)(60,67,71,76,68,71))	((57,64,71,76,71,73)(60,67,71,76,68,71))
Total float	0	((20,22,22,4,28,26,27)(21,23,25,27,22,1,27,24,28,22,24))	((20,22,25,27,24,24)(21,23,25,27,22,1,27,24,28,22,24))	0	((22,26,32,32,19,18)(16,23,32,14,25))	((20,22,4,28,26,27)(21,23,32,14,25))	0	((27,29,30,31,33,32)(28,29,30,31,30,35))	((22,26,32,32,19,18)(16,23,32,2,14,25))	0
Free float	0	0	((20,22,25,27,24,24)(21,23,25,27,22,1,27,24,28,22,24))	0	0	((20,22,4,28,26,27)(21,23,32,14,25))	0	((27,29,30,31,33,32)(28,29,30,31,30,35))	((22,26,32,32,19,18)(16,23,32,2,14,25))	0



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Independent float	0	0	((20,22,25,27,24,24)(21,23,25,27,24,24))	0	0	(20,22,24,28,26,27)(21,27,24,28,26,27)	((27,29,30,31,33,32)(28,29,30,31,30,35))	((27,29,30,31,33,32)(28,29,30,31,30,35))	((22,26,32,31,19,18)(16,23,32,32,14,25))	0
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The critical path is  $C_1 - C_2 - C_4 - C_6 - C_7$  The least time required to complete the project is (57, 64, 71, 76, 71, 73)(60, 67, 71, 76, 68, 71)]

### VI. CONCLUSION:

In this paper Critical path is calculated through total floats, EST and LFT of the activities. The duration of the activities is approximate values. So vagueness arises here. In such cases the Interval valued Hexagonal fuzzy number is applied.

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