

Application of Reverse Vertex Magic Labeling of a Graph



Masthan Raju. U, Sharief Basha.S

Abstract: Graph labeling is a currently emerging area in the research of graph theory. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the labels of edges are distinct positive integers and for each vertex v the sum of the labels of all edges incident with v is the same for every vertex v in the given graph, then the labeling of the graph is called magic labeling. There are several types of magic labeling defined on graphs. In this paper we consider vertex magic labeling and group magic labeling of graphs as an application of magic labeling. We solve a problem of finding number of computers/workstations to be allocated to each department in a company under certain conditions.

Keywords : vertex magic labeling, group magic labeling.

I. INTRODUCTION

Labeling of graphs is a special area in Graph Theory. Joseph A.Gallion is defined by different types of magic labeling are defined in graphs [6]. Originally Sedlacek has defined magic graph as a graph whose edges are labeled with distinct nonnegative integers such that the sum of the labels of the edges incident to a particular vertex is the same for all vertices. The notion of a vertex-magic total labeling is introduced by MacDougall, Miller, Slamin, and Wallis in 1999. For a graph $G(V, E)$, an injective mapping g from $V \cup E$ to the set $\{1, 2, \dots, |V| + |E|\}$ is a vertex-magic total labeling if there is a constant k , called the magic constant, such that for every vertex v , $g(v) + \sum g(vu) = k$, where the sum is over all vertices u adjacent to v . In [9] Wallis competently studied by the vertex-magic total labeling in the monograph and vertex magic total labeling are found for family of graphs.

In a magic graph if the labels are non zero elements of a non trivial additive abelian group A , then the graph is called A -magic graph. In the literature recently A -magic graphs are studied and many results have been derived since J sedlacek introduced group magic graphs [3,4,5,6,7]. In this

paper as an application of vertex magic labeling we solve a problem to find the number of workstations/ computers to be allocated to each department in a company under certain conditions.

II. BASIC DEFINITIONS

The graphs are divided into three types here are simple, finite, and undirected.

1. The graph labeling can be expressed as by labeling the integers to the vertices or edges or both.
2. Graph labeling of the edges with integers for all vertex v . The sum of the labels of all edges coincident with v is equal for all v is called semi magic labeling.
3. The magic labeling consists of distinct positive integers of semi magic labeling.
4. The edge labels consists of positive integers then it is termed as a super magic labeling.
5. A vertex magic total labeling (VMT) of a graph $G = (V, E)$ with vertices $|V|$ and edges $|E|$ is assigns the integers from $1, 2, \dots, |V| + |E|$ to the vertices and edges of G such that addition of the labels on vertex and incident edges is constant k , independent of the choice of the vertex and k is called as magic constant. A Super vertex magic labeling (SVM) is a VMT with set of edge labels $\{1, 2, \dots, |E|\}$.
6. A graph $G = (V, E)$ is termed as magic, when A -magic for each and every abelian group A .

III. MAIN RESULTS

An application of group magic labeling and vertex magic labeling will be demonstrated in the following problems.

Problem 1:

A company wants to provide exact numbers of computers or workstations to its 5 departments D_1, D_2, D_3, D_4, D_5 . Departments are utilizing computers in such a manner that the departments D_1 and D_2 , D_2 and D_3 , D_3 and D_4 , and D_4 and D_5 are sharing few computers. Find the number of computers required for a department. Find the exact number of computers utilized for two departments.

Solution:

For each $i = 1, 2, 3, 4, 5$, we take the department D_i as the vertex v_i and if departments D_i & D_j are sharing computers then we take an edge between v_i & v_j . The graph corresponding to the given circumstance is given in figure 1.

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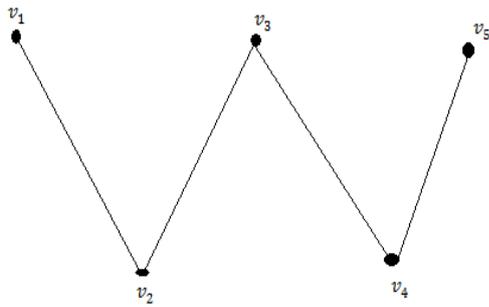


Figure 1

Consider Reverse Vertex Magic Total labeling (RVMT) of the given graph.

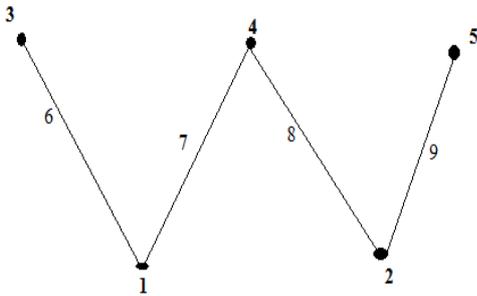


Figure 2

The sum of the labels on the vertex and the incident edges is a constant k , this k provides the exact number of computers or workstations required for a particular department. The vertex labels express the exact number of computers utilized by one department. Also it gives the edge labels by the another department. Here $k = 3$. So each department utilizes 3 computers. We catalog the vertex and edge labels as in the following table 1.

Table 1

Dept.	D_1	D_2	D_3	D_4	D_5
D_1	6	3	0	0	0
D_2	3	7	1	0	0
D_3	0	1	8	4	0
D_4	0	0	4	9	2
D_5	0	0	0	2	5

The entry in the diagonal of the table shows the exact number of computers utilized by number one department. The other table shows the number of computers utilized by the respective pair of departments.

Problem 2:

A company wants to provide equal number of computers to its 5 departments Administrations, Human resource, Logistics, Finance and Accounts. In order to reduce the idle time of the computers the company wants few computers are utilized by two departments. The computers shared by the departments are Human resource and Logistics, Human resource and Administrations, Administrations and Accounts, Accounts and Finance and Finance and Logistics. Find the exact number of computers required and also number of computer utilized by the each department. Find the number of computers utilized by one department and by two departments.

Solution:

Let us denote the departments Administrations, Human resource, Logistics, Finance and Accounts as the vertices v_1, v_2, v_3, v_4, v_5 respectively. We take an edge between vertices if there is a sharing of computers between the corresponding departments. The reverse vertex magic total labeling graph is shown in fig 3.

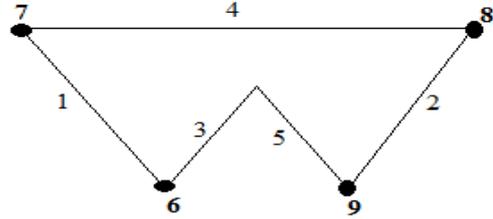


Figure 3

This RVMT labeling gives the magic constant $k = 2$. Thus 2 computers are utilized by each department. The number of computers utilized by one department and which are utilized by two departments are given in the following table 2.

Table 2

Dept.	D_1	D_2	D_3	D_4	D_5
D_1	7	1	0	0	4
D_2	1	6	3	0	0
D_3	0	3	10	5	0
D_4	0	0	5	9	2
D_5	4	0	0	2	8

The entry in the diagonal of the table shows the number of computers utilized by one department. The other entries are the number of computers utilized by the respective pair of departments.

IV. REMARKS

1. The vertex magic labeling considered in the problem 2 and problem 3 are also a reverse super vertex magic labeling (RSVM). In this RSVM labeling are smaller than the vertex labels. The computers shared by more than one department have to be installed with more software or with more equipment. So the company has to spent more money on the computers which are utilized by more than one department. If a RSVM is considered for the graph (if it exists for the graph) which is drawn according to the situation described, then the number of computers which are utilized by more than one department is minimized. Hence the amount spent by the company on those computers is minimized.
2. Suppose the company needs computers in large numbers, then consider the multiple of the labeling with any number.

V. CONCLUSION

The Reverse vertex magic total labeling to the graph is drawn based on the company requirement as obtained from the exact number of computers (or) workstations to be allotted to different department in a company which wants to provide equal number of workstations/ computers to its departments in such a way that each workstation/computer is utilized either by two or by one department.



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