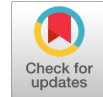


On The Metric Index of Oxide Networks

R. Nithya Raj, F. Simon Raj



Abstract : Let w_i be a vertex of a connected simple graph G and (v_1, v_2) be a pair of vertices in G . Let $d(v_1, v_2)$ be the distance between v_1 and v_2 . A vertex w_i is said to resolve v_1 and v_2 if $d(w_i, v_1) \neq d(w_i, v_2)$. A set of vertices W of G is called a settling set of G if every pair of vertices (v_i, v_j) resolved by atleast one vertices $w_j \in W$. A settling set of G with least cardinality is called metric premise of G . The cardinality of metric premise is called metric index of G . In this paper metric index of oxide network is investigated.

Catchphrases: Metric premise, Metric index, Oxide network, Interconnection Network.

I. INTRODUCTION

The metric index of a graph was first investigated by Harry and Melter [1]. They acclimated properties of the metric dimension of trees. Melter and Tomescu [2] studied the metric index problems for grid graphs, khuller et.al comprehensive melter and Tomescu's Results. They have exhibited the metric index of dimensional d of a network graph is d [3]. The metric index of bipartite graph is an NP complete [4]. Slater [5] and later [6] contributed another name for metric premise as settling set. Slater baptized the quantity of component in a settling set of the graph as land mark of the graph. He clarify the utilization of metric premise in loran and Sonar station. Instated of metric premise Chartand et.al used the word of least settling [7]. The metric index problem investigated for leaves and net graph [3], Petersen graph [8], Honeycomb Network, Hexagonal system [9], Circullent and Harry graphs [10], Enhanced Hyper solid shapes [11], Silicate stars [12], Triangular oxide network [13], stat of david networks [14]. In 2014 Dachang Xu [15] et.al given the conclusion the metric dimension of Hex Derived Network is either 3 or 4. In 2019 Zehui shao [16] et.al verified the metric index of Hex derived network is 4 by using vector coloring scheme. The metric dimension have many other applications such as robot route, design acknowledgement [3]. Network revolution and check [17]. In this section, we elucidate about the oxide network. This Oxide network drawn from removing all silicon nodes from the silicate network of dimension ω . In polymer and pharmaceutical industries are running by using the application of oxide network [18]. The number of vertices and edges in a oxide networks of dimension ω is $9\omega^2 + 3\omega$ and $18\omega^2$ respectively [13,19]. and other specification of this oxide network having vertices of degree 2 and 4 only. By using this X Y and Z channel coordinate system any vertex (x_i, y_j, z_k) in the Oxide network satisfies the condition $y_j = x_i + z_k$.

Oxide Network :

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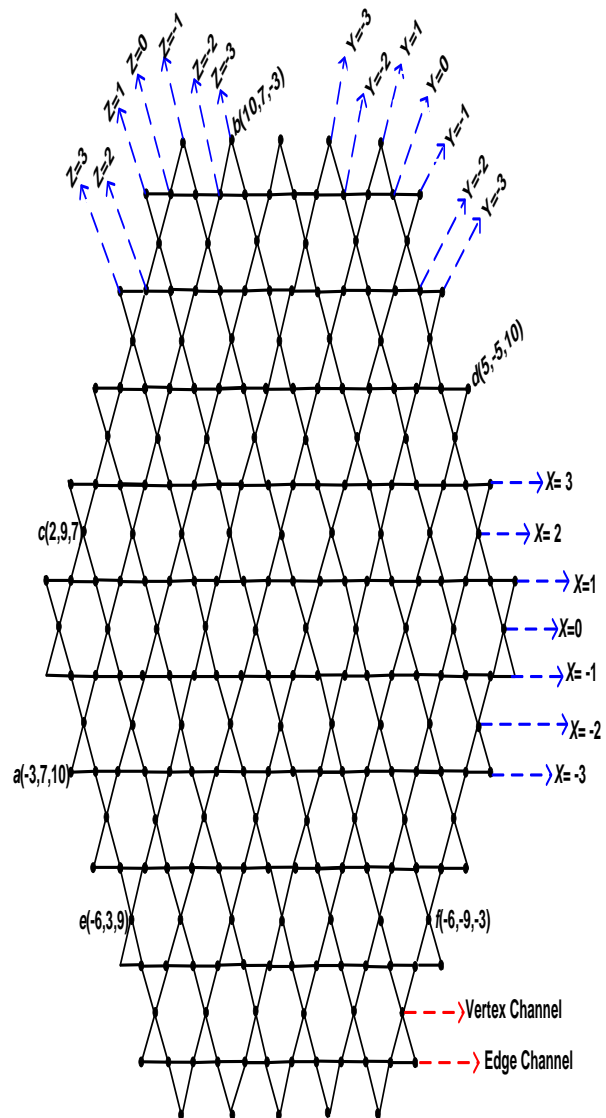


Figure 1: Oxide Network of Dimension 5

Proposed methodology.

In this paper we use graph coordinate system to prove the main result.

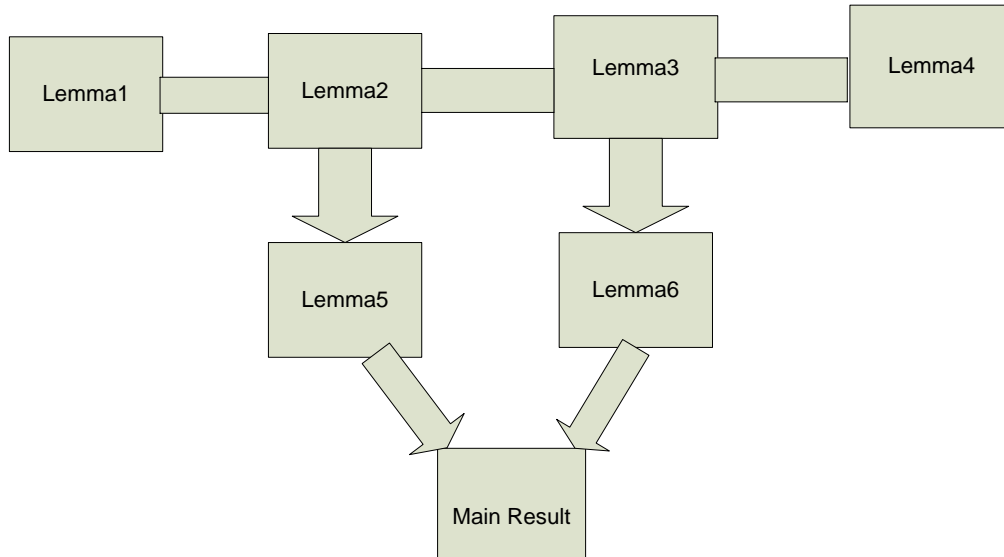
The distance between any two nodes $u = (x_1, y_1, z_1)$ and $v = (x_2, y_2, z_2)$ in the oxide networks

is equal to

$$\begin{cases} \frac{1}{2}\{|x_2 - x_1| + |y_2 - y_1| + |z_2 - z_1| + 2\} & \text{if } u \text{ and } v \text{ lies on the same vertex channel} \\ \frac{1}{2}\{|x_2 - x_1| + |y_2 - y_1| + |z_2 - z_1|\} & \text{otherwise} \end{cases}$$

In Figure 1, the distance between edge channels $a(-3,7,10)$ and $b(10,7,-3)$ are 13, the distance between vertex channel $c(2,9,7)$ and edge channel $d(5,-5,-10)$ is 17, the distance between vertex channels $e(-6,3,9)$ and $f(-6,-9,-3)$ are 13.

Flow Chart.



Main Result Theorem

Let G be a Oxide network of dimension ω , then $\dim(G) = \begin{cases} 2 & \omega = 1 \\ 3 & \omega \geq 2 \end{cases}$

Proof:

Any pair of vertices in the oxide network will come under one of the categories discussed below the lemma from (1-6). Therefore any pair of vertices in the oxide network can be resolved by $M = \{\alpha, \beta, \gamma\}$. Hence the metric dimension of oxide network is 2 for $\omega = 1$ and metric dimension of oxide network is 3 for $\omega \geq 2$. We prove this Theorem we need the following lemma.

Lemma 1:

Let $A = \{u_r(-r-1, -1, r) / 2 \leq r \leq 2\omega - 1\}$ and $B = \{v_r(-r-1, -r, 1) / 2 \leq r \leq 2\omega - 1\}$, $u_i \in A$ and $v_i \in B$ then $\{\alpha, \beta\}$ is not a resolving set for (u_i, v_i) .

Proof:

Let $\alpha = (2\omega - 1, 2\omega, 1)$ and $\beta = (2\omega - 1, -1, -2\omega)$ where ω is the dimension of oxide network

Let $u = (x_1, y_1, z_1)$ & $v = (x_2, y_2, z_2)$ be any two points in $OX(\omega)$.

$$\begin{aligned} d(u, v) &= \frac{1}{2}\{|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|\} \\ d(u_i, \alpha) &= \frac{1}{2}\{|-r-1-2\omega+1| + |-1-2\omega| + |r-1|\} \\ &= \frac{1}{2}\{|-(r+1)-2\omega+1| + |1+2\omega| + |r-1|\} \\ &= \frac{1}{2}\{|-(r+1)-2\omega+1| + |2\omega+1+r-1|\} \text{ (since } r-1 \geq 1 \text{ implies } r \text{ is positive)} \\ &= \frac{1}{2}\{|-(r+1)-2\omega+1| + |2\omega+r|\} \\ d(v_i, \alpha) &= \frac{1}{2}\{|-(r+1)-2\omega+1| + |-r-2\omega| + |1-1|\} \\ &= \frac{1}{2}\{|-(r+1)-2\omega+1| + |r+2\omega|\} \\ &= d(u_i, \alpha) \\ d(u_i, \beta) &= \frac{1}{2}\{|-(r+1)-2\omega+1| + |-1+1| + |r+2\omega|\} \\ &= \frac{1}{2}\{|r+2\omega| + |r+2\omega|\} \\ &= |r+2\omega| \end{aligned}$$

$$\begin{aligned} d(v_i, \beta) &= \frac{1}{2} \{|-(r+1) - 2\omega + 1| + | -r + 1| \\ &\quad + |1 + 2\omega|\} \\ &= \frac{1}{2} \{|r + 2\omega| + |r - 1| + |1 + 2\omega|\} \\ &= \frac{1}{2} \{|r + 2\omega| + |r - 1 + 1 + 2\omega|\} \\ &= \frac{1}{2} \{|r + 2\omega| + |r + 2\omega|\} \\ &= |r + 2\omega| \\ &= d(u_i, \beta) \end{aligned}$$

Hence the lemma.

Lemma 2:

Let $C = \{u_l(2r, 2\omega - 1, 2\omega - 1 - 2r)/0 \leq r \leq \omega - 2 \text{ \& } \omega \geq 2\}$
 $D = \{v_l(2r + 1, 2\omega, 2\omega - 2r - 1)/0 \leq r \leq \omega - 2 \text{ \& } \omega \geq 2\}$
 $E = \{u_r(2r, -(2\omega - 1 - 2r), -(2\omega - 1)/0 \leq r \leq \omega - 2 \text{ \& } \omega \geq 2\}$
 $F = \{v_r(2r + 1, -(2\omega - 1 - 2r), -2\omega)/0 \leq r \leq \omega - 2 \text{ \& } \omega \geq 2\}$
 $u_l \in C$, $v_l \in D$, $u_r \in E$ & $v_r \in F$, then $\{\alpha, \beta\}$ is not a resolving set for $(u_l, v_l) \& (u_r, v_r)$.

Proof

Let $\alpha = (2\omega - 1, 2\omega, 1) \& \beta = (2\omega - 1, -1, -2\omega)$

$$d(u_i, \alpha) = \frac{1}{2} \{|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|\} \rightarrow 1$$

Here $u_l = (2r, 2\omega - 1, 2\omega - 1 - 2r)$, $\alpha = (2\omega - 1, 2\omega, 1)$

$$\begin{aligned} d(u_i, \alpha) &= \frac{1}{2} \{|2r - 2\omega + 1| + |2\omega - 1 - 2\omega| \\ &\quad + |2\omega - 1 - 2r - 1|\} \\ &= \frac{1}{2} \{|2r - 2\omega + 1| + |-1| + |2\omega - 2r - 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1| + |1| + |2\omega - 2r - 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1 + 1| + |2\omega - 2r - 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r| + |2\omega - 2r - 2|\} \end{aligned}$$

Distance between any two points $v_l \& \alpha$ lie on same vertex channel is

$$d(v_i, \alpha) = \frac{1}{2} \{|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| + |2|\} \rightarrow 2$$

Here $v_l = (2r + 1, 2\omega, 2\omega - 2r - 1)$, $\alpha = (2\omega - 1, 2\omega, 1)$

$$\begin{aligned} d(v_i, \alpha) &= \frac{1}{2} \{|2r + 1 - 2\omega + 1| + |2\omega - 2\omega| \\ &\quad + |2\omega - 2r - 1 - 1| + |2|\} \\ &= \frac{1}{2} \{|2r - 2\omega + 2| + |2\omega - 2r - 2| + |2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2| + |2\omega - 2r - 2| + |2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2 + 2| + |2\omega - 2r - 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r| + |2\omega - 2r - 2|\} \end{aligned}$$

$$= d(u_i, \alpha)$$

Using equation 1, we can find $d(u_l, \beta) \& d(v_l, \beta)$.

Here $u_l = (2r, 2\omega - 1, 2\omega - 1 - 2r)$, $\beta = (2\omega - 1, -1, -2\omega)$

$$\begin{aligned} d(u_i, \beta) &= \frac{1}{2} \{|2r - 2\omega + 1| + |2\omega - 1 + 1| \\ &\quad + |2\omega - 1 - 2r + 2\omega|\} \\ &= \frac{1}{2} \{|2r - 2\omega + 1| + |2\omega| + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1| + |2\omega| + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|4\omega - 2r - 1| + |4\omega - 2r - 1|\} \\ &= |4\omega - 2r - 1| \end{aligned}$$

To find $d(v_l, \beta)$, here $v_l = (2r + 1, 2\omega, 2\omega - 2r - 1)$, $\beta = (2\omega - 1, -1, -2\omega)$

$$\begin{aligned} d(v_i, \beta) &= \frac{1}{2} \{|2r + 1 - 2\omega + 1| + |2\omega + 1| \\ &\quad + |2\omega - 2r - 1 + 2\omega|\} \\ &= \frac{1}{2} \{|2r - 2\omega + 2| + |2\omega + 1| \\ &\quad + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2| + |2\omega + 1| \\ &\quad + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2 + 2\omega + 1| + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|4\omega - 2r - 1| + |4\omega - 2r - 1|\} \\ &= |4\omega - 2r - 1| \\ &= d(u_i, \beta) \end{aligned}$$

Using equation 1, we can find $d(u_r, \alpha) \& d(v_r, \alpha)$ where

$u_r = (2r, -(2\omega - 1 - 2r), -(2\omega - 1))$, $\alpha = (2\omega - 1, 2\omega, 1)$

$$\begin{aligned} d(u_r, \alpha) &= \frac{1}{2} \{|2r - 2\omega + 1| \\ &\quad + |-2\omega + 1 + 2r - 2\omega| \\ &\quad + |-2\omega + 1 - 1|\} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \{|2r - 2\omega + 1| + |-4\omega + 2r + 1| + |-2\omega|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1| + |4\omega - 2r - 1| + |2\omega|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1 + 2\omega| + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|4\omega - 2r - 1| + |4\omega - 2r - 1|\} \\ &= |4\omega - 2r - 1| \end{aligned}$$

Then $v_r = (2r + 1, -(2\omega - 2r - 1), -2\omega)$

$$\begin{aligned}
 1), -2\omega), \alpha &= (2\omega - 1, 2\omega, 1) \\
 d(v_r, \alpha) &= \frac{1}{2} \{ |2r + 1 - 2\omega + 1| \\
 &\quad + |-2\omega + 2r + 1 - 2\omega| \\
 &\quad + |-2\omega - 1| \} \\
 &= \frac{1}{2} \{ |2r - 2\omega + 2| + |-4\omega + 2r + 1| \\
 &\quad + |2\omega + 1| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 2| + |4\omega - 2r - 1| \\
 &\quad + |2\omega + 1| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 2 + 2\omega + 1| + |4\omega - 2r - 1| \} \\
 &= \frac{1}{2} \{ |4\omega - 2r - 1| + |4\omega - 2r - 1| \} \\
 &= |4\omega - 2r - 1| \\
 &= d(u_r, \alpha)
 \end{aligned}$$

Using equation 1, we can find $d(u_r, \beta)$

$$u_r = (2r, -(2\omega - 1 - 2r), -(2\omega - 1)) \quad , \quad \beta = (2\omega - 1, -1, -2\omega)$$

$$\begin{aligned}
 d(u_r, \beta) &= \frac{1}{2} \{ |2r - 2\omega + 1| \\
 &\quad + |-2\omega + 1 + 2r + 1| \\
 &\quad + |-2\omega + 1 + 2\omega| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 1| + |2\omega - 2r - 2| + |1| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 1 + 1| + |2\omega - 2r - 2| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r| + |2\omega - 2r - 2| \}
 \end{aligned}$$

Using equation 2, find $d(v_r, \beta)$

$$\text{where } v_r = (2r + 1, -(2\omega - 2r - 1), -2\omega) \quad , \quad \beta = (2\omega - 1, -1, -2\omega)$$

$$\begin{aligned}
 d(v_r, \beta) &= \frac{1}{2} \{ |2r + 1 - 2\omega + 1| \\
 &\quad + |-2\omega + 2r + 1 + 1| \\
 &\quad + |-2\omega + 2\omega| + |2| \} \\
 &= \frac{1}{2} \{ |2r - 2\omega + 2| + |-2\omega + 2r + 2| + |2| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 2| + |2\omega - 2r - 2| + |2| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 2 + 2| + |2\omega - 2r - 2| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r| + |2\omega - 2r - 2| \} \\
 &= d(u_r, \beta)
 \end{aligned}$$

$\therefore \{\alpha, \beta\}$ is not a resolving set for $(u_l, v_l) \& (u_r, v_r)$. Hence proved.

Lemma 3:

Let $G = \{u_l(-2r, 2\omega - 1 - 2r, 2\omega - 1)\}$, $H = \{v_l(-(2r + 1), 2\omega - 1 - 2r, 2\omega)\}$,
 $I = \{u_r(-2r, -(2\omega - 1), -(2\omega - 1 - 2r))\}$,
 $J = \{v_r(-(2r + 1), -2\omega, -(2\omega - 1 - 2r))\}$
 where $0 \leq r \leq \omega - 2$ & $\omega \geq 2$, $u_l \in G, v_l \in H, u_r \in I, v_r \in J$, then $\{\gamma, \eta\}$ is not a resolving set for $(u_l, v_l) \& (u_r, v_r)$.

Proof:

We know that if u & v not lie on the same vertex channel then

$$d(u, v) = \frac{1}{2} \{ |x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| \} \rightarrow 1$$

and if u & v lie on the same vertex channel then

$$d(u, v) = \frac{1}{2} \{ |x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| + |2| \} \rightarrow 2$$

Using equation 1, find $d(u_l, \gamma) \& d(v_l, \gamma)$ where

$$u_l = (-2r, 2\omega - 1 - 2r, 2\omega - 1), \gamma = (-(2\omega - 1), -2\omega, -1)$$

$$\begin{aligned}
 d(u_l, \gamma) &= \frac{1}{2} \{ |-2r + 2\omega - 1| \\
 &\quad + |2\omega - 1 - 2r + 2\omega| \\
 &\quad + |2\omega - 1 + 1| \}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \{ |2\omega - 2r - 1| + |4\omega - 2r - 1| + |2\omega| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 1 + 2\omega| + |4\omega - 2r - 1| \} \\
 &= \frac{1}{2} \{ |4\omega - 2r - 1| + |4\omega - 2r - 1| \} \\
 &= |4\omega - 2r - 1|
 \end{aligned}$$

$$\begin{aligned}
 d(v_l, \gamma) &= \frac{1}{2} \{ |-2r - 1 + 2\omega - 1| \\
 &\quad + |2\omega - 2r - 1 + 2\omega| \\
 &\quad + |2\omega + 1| \}
 \end{aligned}$$

$$\text{where } v_l = (-(2r + 1), 2\omega - 2r - 1, 2\omega) \quad , \quad \gamma = (-(2\omega - 1), -2\omega, -1)$$

$$\begin{aligned}
 d(v_l, \gamma) &= \frac{1}{2} \{ |2\omega - 2r - 2| + |4\omega - 2r - 1| \\
 &\quad + |2\omega + 1| \} \\
 &= \frac{1}{2} \{ |2\omega - 2r - 2 + 2\omega + 1| + |4\omega - 2r - 1| \} \\
 &= \frac{1}{2} \{ |4\omega - 2r - 1| + |4\omega - 2r - 1| \} \\
 &= |4\omega - 2r - 1| \\
 &= d(u_l, \gamma)
 \end{aligned}$$

Using equation 1, find $d(u_l, \eta)$ where

$$u_l = (-2r, 2\omega - 1 - 2r, 2\omega - 1), \quad \eta = (-(2\omega - 1), 1, 2\omega)$$

$$\begin{aligned} d(u_l, \eta) &= \frac{1}{2} \{|-2r + 2\omega - 1| \\ &\quad + |2\omega - 2r - 1 - 1| \\ &\quad + |2\omega - 1 - 2\omega|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1| + |2\omega - 2r - 2| + |-1|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1 + 1| + |2\omega - 2r - 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r| + |2\omega - 2r - 2|\} \end{aligned}$$

Using equation 2, find $d(v_l, \eta)$ where

$$v_l = (-(2r + 1), 2\omega - 2r - 1, 2\omega), \quad \eta = (-(2\omega - 1), 1, 2\omega)$$

$$\begin{aligned} d(v_l, \eta) &= \frac{1}{2} \{|-2r - 1 + 2\omega - 1| \\ &\quad + |2\omega - 2r - 1 - 1| + |2\omega - 2\omega| \\ &\quad + |2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2| + |2\omega - 2r - 2| + |2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2 + 2| + |2\omega - 2r - 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r| + |2\omega - 2r - 2|\} \\ &= d(u_l, \eta) \end{aligned}$$

Using equation 1, find $d(u_r, \gamma)$ where

$$u_r = (-2r, -(2\omega - 1), -(2\omega - 1 - 2r)), \quad \gamma = (-(2\omega - 1), -2\omega, -1)$$

$$\begin{aligned} d(u_r, \gamma) &= \frac{1}{2} \{|-2r + 2\omega - 1| \\ &\quad + |-2\omega + 1 + 2\omega| \\ &\quad + |-2\omega + 1 + 2r + 1|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1| + |1| + |-2\omega + 2r + 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1 + 1| + |2\omega - 2r - 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r| + |2\omega - 2r - 2|\} \end{aligned}$$

Using equation 2, find $d(v_r, \gamma)$ where

$$v_l = (-(2r + 1), -2\omega, -(2\omega - 1 - 2r)), \gamma = (-(2\omega - 1), -2\omega, -1)$$

$$\begin{aligned} d(v_r, \gamma) &= \frac{1}{2} \{|-2r - 1 + 2\omega - 1| \\ &\quad + |-2\omega + 2\omega| \\ &\quad + |-2\omega + 1 + 2r + 1| + |2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2| + |-2\omega + 2r + 2| + |2|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2 + 2| + |-2\omega + 2r + 2|\} \\ &= \frac{1}{2} \{|2\omega - 2r| + |2\omega - 2r - 2|\} \\ &= d(u_r, \gamma) \end{aligned}$$

Using equation 1, find $d(u_r, \eta)$ & $d(v_r, \eta)$ where

$$u_r = (-2r, -(2\omega - 1), -(2\omega - 1 - 2r)), \quad \eta = (-(2\omega - 1), 1, 2\omega)$$

$$\begin{aligned} d(u_r, \eta) &= \frac{1}{2} \{|-2r + 2\omega - 1| + |-2\omega + 1 - 1| \\ &\quad + |-2\omega + 1 + 2r - 2\omega|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1| + |-2\omega| + |-4\omega + 2r + 1|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 1 + 2\omega| + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|4\omega - 2r - 1| + |4\omega - 2r - 1|\} \\ &= |4\omega - 2r - 1| \end{aligned}$$

$$\begin{aligned} d(v_r, \eta) &= \frac{1}{2} \{|-2r - 1 + 2\omega - 1| + |-2\omega - 1| \\ &\quad + |-2\omega + 1 + 2r - 2\omega|\} \end{aligned}$$

where $v_r = (-(2r + 1), -2\omega, -(2\omega - 1 - 2r)), \eta = (-(2\omega - 1), 1, 2\omega)$

$$\begin{aligned} d(v_r, \eta) &= \frac{1}{2} \{|2\omega - 2r - 2| + |2\omega + 1| \\ &\quad + |-4\omega + 2r + 1|\} \\ &= \frac{1}{2} \{|2\omega - 2r - 2 + 2\omega + 1| + |4\omega - 2r - 1|\} \\ &= \frac{1}{2} \{|4\omega - 2r - 1| + |4\omega - 2r - 1|\} \\ &= |4\omega - 2r - 1| \\ &= d(u_r, \eta) \end{aligned}$$

Hence the lemma.

Lemma 4

Let $S_1 = \{u_i(r + 1, r, -1)/2 \leq r \leq 2\omega - 1\}$

$S_2 = \{v_i(r + 1, 1, -r)/2 \leq r \leq 2\omega - 1\}$

where $u_i \in S_1$ and $v_i \in S_2$ then $\{\gamma, \eta\}$ is not a resolving set for (u_i, v_i) where $\gamma = (-(2\omega - 1), -2\omega, -1)$ and $\eta = (-(2\omega - 1), 1, 2\omega)$

Proof:

The distance between u_i and γ is

$$\begin{aligned} d(u_i, \gamma) &= \frac{1}{2} \{|r + 1 + 2\omega - 1| + |r + 2\omega| \\ &\quad + |-1 + 1|\} \\ &= \frac{1}{2} \{|2\omega + r| + |2\omega + r|\} \\ &= |2\omega + r| \end{aligned}$$

The distance between v_i and γ is

$$\begin{aligned} d(v_i, \gamma) &= \frac{1}{2} \{|r + 1 + 2\omega - 1| + |1 + 2\omega| \\ &\quad + |-r + 1|\} \\ &= \frac{1}{2} \{|2\omega + r| + |2\omega + 1| + \\ &\quad |r - 1|\} \end{aligned}$$

$= \frac{1}{2}\{|2\omega + 1| + |2\omega + 1|\}$ (since $r - 1 \geq 1$ implies r is positive)

$$= d(u_i, \gamma)$$

The distance between u_i and η is

$$\begin{aligned} d(u_i, \eta) &= \frac{1}{2}\{|r + 1 + 2\omega - 1| + |r - 1| \\ &\quad + |-1 - 2\omega|\} \\ &= \frac{1}{2}\{|2\omega + r| + |r - 1| + |2\omega + 1|\} \\ &= \frac{1}{2}\{|2\omega + r| + |2\omega + r|\} \\ &= |2\omega + r| \end{aligned}$$

The distance between v_i and η is

$$\begin{aligned} d(v_i, \eta) &= \frac{1}{2}\{|r + 1 + 2\omega - 1| + |1 - 1| \\ &\quad + |-r - 2\omega|\} \\ &= \frac{1}{2}\{|2\omega + r| + |2\omega + r|\} \\ &= |2\omega + r| \\ &= d(u_i, \eta) \end{aligned}$$

Hence the Lemma.

Lemma 5:

If $\{\alpha, \beta\}$ is not resolving u and v then both $\{\gamma\}$ and $\{\eta\}$ must resolve u and v .

Proof:

By using lemma 2, $\{\alpha, \beta\}$ is not a resolving set for

- (i) $u = u_l$ and $v = v_l$ for the same value of r .
- (ii) $u = u_r$ and $v = v_r$ for the same value of r

Now we have to show that $\{\gamma\}$ and $\{\eta\}$ must resolve u and v . From lemma 2 we have

$$u_l = (2r, 2\omega - 1, 2\omega - 1 - 2r)$$

$$v_l = (2r + 1, 2\omega, 2\omega - 2r - 1)$$

$$u_r = (2r, -(2\omega - 1 - 2r), -(2\omega - 1))$$

$$v_r = (2r + 1, -(2\omega - 1 - 2r), -2\omega) \quad \text{where } 0 \leq r \leq \omega - 2 \text{ \& } \omega \geq 2$$

Also we have $\eta = (-(2\omega - 1), -2\omega, -1)$, $\gamma = (-(2\omega - 1), 1, 2\omega)$

The distance between u_l and γ is

$$\begin{aligned} d(u_l, \gamma) &= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 1 + 2\omega| \\ &\quad + |2\omega - 1 - 2r + 1|\} \\ &= \frac{1}{2}\{|2\omega + 2r - 1| + |4\omega - 1| + \\ &\quad |2\omega - 2r|\} \\ &= \frac{1}{2}\{|4\omega - 1| + |4\omega - 1|\} \\ &= |4\omega - 1| \end{aligned}$$

The distance between v_l and γ is

$$\begin{aligned} d(v_l, \gamma) &= \frac{1}{2}\{|2r + 1 + 2\omega - 1| + |2\omega + 2\omega| \\ &\quad + |2\omega - 1 - 2r + 1|\} \\ &= \frac{1}{2}\{|2\omega + 2r| + |4\omega| + |2\omega - 2r|\} \\ &= \frac{1}{2}\{|4\omega| + |4\omega|\} \\ &= |4\omega| \\ &\neq d(u_l, \gamma) \end{aligned}$$

The distance between u_l and η is

$$\begin{aligned} d(u_l, \eta) &= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 1 - 1| \\ &\quad + |2\omega - 1 - 2r - 2\omega|\} \\ &= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 2| + |2r + 1|\} \\ &= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega + 2r - 1|\} \\ &= |2\omega + 2r - 1| \end{aligned}$$

The distance between v_l and η is

$$\begin{aligned} d(v_l, \eta) &= \frac{1}{2}\{|2r + 1 + 2\omega - 1| + |2\omega - 1| \\ &\quad + |2\omega - 2r - 1 - 2\omega|\} \\ &= \frac{1}{2}\{|2r + 2\omega| + |2\omega - 1| + \\ &\quad |2r + 1|\} \\ &= \frac{1}{2}\{2|2\omega + 2r|\} \end{aligned}$$

$\neq d(u_l, \eta)$.

The distance between u_r and γ is

$$\begin{aligned} d(u_r, \gamma) &= \frac{1}{2}\{|2r + 2\omega - 1| + |2\omega - 1 - 1| \\ &\quad + |-2\omega + 1 + 2r + 2\omega|\} \\ &= \frac{1}{2}\{|2\omega + 2r - 1| + |2r + 1| + |2\omega - 2|\} \\ &= \frac{1}{2}\{|2\omega + 2r - 1| + |2\omega + 2r - 1|\} \\ &= |2\omega + 2r - 1| \end{aligned}$$

The distance between v_r and γ is

$$\begin{aligned} d(v_r, \gamma) &= \frac{1}{2}\{|2r + 1 + 2\omega - 1| \\ &\quad + |-2\omega + 2r + 1 + 2\omega| \\ &\quad + |-2\omega + 1|\} \\ &= \frac{1}{2}\{|2\omega + 2r| + |2r + 1| + |2\omega - 1|\} \\ &= \frac{1}{2}\{|2\omega + 2r| + |2\omega + 2r|\} \\ &= |2\omega + 2r| \\ &\neq d(u_r, \gamma) \end{aligned}$$

The distance between u_r and η is

$$\begin{aligned} d(u_r, \eta) &= \frac{1}{2} \{|2r + 2\omega - 1| \\ &\quad + |-2\omega + 1 + 2r - 1| \\ &\quad + |-2\omega + 1 - 2\omega|\} \\ &= \frac{1}{2} \{|2r + 2\omega - 1| + |2\omega - 2r| + |4\omega - 1|\} \\ &= \frac{1}{2} \{|4\omega - 1| + |4\omega - 1|\} \\ &= |4\omega - 1| \end{aligned}$$

The distance between v_r and η is

$$\begin{aligned} d(v_r, \eta) &= \frac{1}{2} \{|2r + 1 + 2\omega - 1| \\ &\quad + |-2\omega + 2r + 1 - 1| \\ &\quad + |-2\omega - 2\omega|\} \\ &= \frac{1}{2} \{|2\omega + 2r| + |2\omega - 2r| + |4\omega|\} \\ &= \frac{1}{2} \{|4\omega| + |4\omega|\} \\ &= |4\omega| \\ &\neq d(u_r, \eta) \end{aligned}$$

Hence the Lemma.

Lemma 6:

If $\{\gamma, \eta\}$ is not resolving u and v then both $\{\alpha\}$ and $\{\beta\}$ will resolve u and v .

Proof:

By using Lemma 3 we will get $\{\gamma, \eta\}$ is not resolve by u and v .

Enough To Prove: $\{\alpha\}$ and $\{\beta\}$ will resolve u and v . Where u and v denoted by

$$\begin{aligned} u_l &= (-2r, 2\omega - 1 - 2r, 2\omega - 1) \quad , \quad v_l = \\ &= (-(2r + 1), 2\omega - 2r - 1, 2\omega) \\ u_r &= (-2r, -(2\omega - 1), -(2\omega - 1 - 2r)), \quad v_r = \\ &= (-(2r + 1) - 2\omega, -(2\omega - 1 - 2r)) \\ \alpha &= ((2\omega - 1), 2\omega, 1) \quad \text{and} \quad \beta = ((2\omega - 1), -1, -2\omega) \end{aligned}$$

The distance between u_l and α is

$$\begin{aligned} d(u_l, \alpha) &= \frac{1}{2} \{|-2r - 2\omega + 1| \\ &\quad + |2\omega - 1 - 2r - 2\omega| \\ &\quad + |2\omega - 1 - 1|\} \\ &= \frac{1}{2} \{|2\omega + 2r - 1| + |2r + 1| \\ &\quad + |2\omega - 2|\} \\ &= \frac{1}{2} \{|2\omega + 2r - 1| + |2\omega + 2r - 1|\} \\ &= |2\omega + 2r - 1| \end{aligned}$$

The distance between v_l and α is

$$\begin{aligned} d(v_l, \alpha) &= \frac{1}{2} \{|-2r - 1 - 2\omega + 1| \\ &\quad + |2\omega - 1 - 2r - 2\omega| \\ &\quad + |2\omega - 1|\} \\ &= \frac{1}{2} \{|2\omega + 2r| + |2r + 1| + \\ &\quad |2\omega - 1|\} \\ &= \frac{1}{2} \{|2\omega + 2r| + |2\omega + 2r|\} \\ &\neq d(u_l, \alpha) \end{aligned}$$

The distance between u_l and β is

$$\begin{aligned} d(u_l, \beta) &= \frac{1}{2} \{|-2r - 2\omega + 1| \\ &\quad + |2\omega - 1 - 2r + 1| \\ &\quad + |2\omega - 1 + 2\omega|\} \\ &= \frac{1}{2} \{|2\omega + 2r - 1| + |2\omega - 2r| + |4\omega - 1|\} \\ &= \frac{1}{2} \{|4\omega - 1| + |4\omega - 1|\} \\ &= |4\omega - 1| \end{aligned}$$

The distance between v_l and β is

$$\begin{aligned} d(v_l, \beta) &= \frac{1}{2} \{|-2r - 1 - 2\omega + 1| \\ &\quad + |2\omega - 2r - 1 + 1| \\ &\quad + |2\omega + 2\omega|\} \\ &= \frac{1}{2} \{|2\omega + 2r| + |2\omega - 2r| + |4\omega|\} \\ &= \frac{1}{2} \{|4\omega| + |4\omega|\} \\ &= |4\omega| \\ &\neq d(u_l, \beta) \end{aligned}$$

The distance between u_r and α is

$$\begin{aligned} d(u_r, \alpha) &= \frac{1}{2} \{|-2r - 2\omega + 1| \\ &\quad + |-2\omega + 1 - 2\omega| \\ &\quad + |-2\omega + 1 + 2r - 1|\} \\ &= \frac{1}{2} \{|2\omega + 2r - 1| + |4\omega - 1| + |2\omega - 2r|\} \\ &= \frac{1}{2} \{|4\omega - 1| + |4\omega - 1|\} \\ &= |4\omega - 1| \end{aligned}$$

The distance between v_r and α is

$$\begin{aligned} d(v_r, \alpha) &= \frac{1}{2} \{|-2r - 1 - 2\omega + 1| \\ &\quad + |-2\omega - 2\omega| \\ &\quad + |-2\omega + 1 + 2r - 1|\} \\ &= \frac{1}{2} \{|2\omega + 2r| + |4\omega| + |2\omega - 2r|\} \\ &= \frac{1}{2} \{|4\omega| + |4\omega|\} \\ &= |4\omega| \\ &\neq d(u_r, \alpha) \end{aligned}$$

The distance between u_r and β is

$$\begin{aligned} d(u_r, \beta) &= \frac{1}{2} \{ |-2r - 2\omega + 1| \\ &\quad + |-2\omega + 1 + 1| \\ &\quad + |-2\omega + 1 + 2r + 2\omega| \} \\ &= \frac{1}{2} \{ |2\omega + 2r - 1| + |2\omega - 2| + |2r + 1| \} \\ &= \frac{1}{2} \{ |2\omega + 2r - 1| + |2\omega + 2r - 1| \} \\ &= |2\omega + 2r - 1| \end{aligned}$$

The distance between v_r and β is

$$\begin{aligned} d(v_r, \beta) &= \frac{1}{2} \{ |-2r - 1 - 2\omega + 1| + |-2\omega + 1| \\ &\quad + |-2\omega + 1 + 2r + 2\omega| \} \\ &= \frac{1}{2} \{ |2\omega + 2r| + |2\omega - 1| + |2r + 1| \} \\ &= \frac{1}{2} \{ |2\omega + 2r| + |2\omega + 2r| \} \\ &= |2\omega + 2r| \\ &\neq d(u_r, \beta) \end{aligned}$$

Hence the Lemma.

Discussion : In this paper we have presented distance between the vertex of the Oxide network and metric index of the Oxide network.

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