



# Bingham Plastic Fluid Flow between Two Parallel Plates with Temperature Effect

P. Sudam Sekhar, Venkata Subrahmanyam Sajja, V. V.Radhakrishna Murthy

**Abstract:** Fluid flow between two parallel plates, both are at rest has been considered in this work. It describes the theoretical analysis of Bingham Plastic fluid model between the plates. The equation of motion, continuity equation and energy equations are solved simultaneously by assuming the plates are of same temperature. A semi analytical solution has been developed and the flow behavior and temperature effect has been discussed and represented with respect to different lubricant parameter.

**Keywords:** Parallel plates, Hydrodynamic, Bingham plastic, Temperature effect.

## I. INTRODUCTION

As far as non-Newtonian fluids are concern, Bingham Plastic fluid model is used widely by the researchers due to its potentiality of describing many fluids and simplicity of getting a compatible results. Since the use of ideal fluid to the present development of the science and technology known as rheology, the limitations forced on the quantity of factors influencing the dissemination of shear stress in a given fluid flow have been loose methodically. In this connection the traditional hydrodynamics have gotten new consideration. The fluid between parallel plates in motion is of great interest in hydrodynamical machines. Practically it is involved in polymer processing, compression and injection modeling and various mechanical applications. Many heavy weight lifting machine like hydraulic machines uses this concept. T. Sarpkaya [1], discussed the analytical solution of equation and used the non-Newtonian fluids between two parallel plates. Velocity profile with respect to different parameter has been discussed. Szeri, A. Z [2] examined the fluid flow between two parallel heated plates and analyzed the temperature effect. Mustafa, M., T[3] et al discussed heat and mass transfer in a viscous fluid flow and squeezing effect. Mohammad Mehdi Rashidi [4] et al flow of inviscus fluid between two parallel plates and solved the governing equation analytically. Siddiqui, A. M.[5], et al also discussed the fluid flow model between two parallel plates and analyzed the heat and mass transfer. The investigation of the movement of non-Newtonian fluids in the nonappearance and additionally within the sight of an attractive field has applications in numerous regions. Fluid flow between two parallel plates has got extensive applications in

heavily loaded machine including power generators and pumps. Since the gap between two parallel plates is very small, choosing of fluid impact the results significantly. In this problem Bingham plastic fluid model has been considered to analyze the flow behavior and temperature of the lubricant between the two parallel plates.

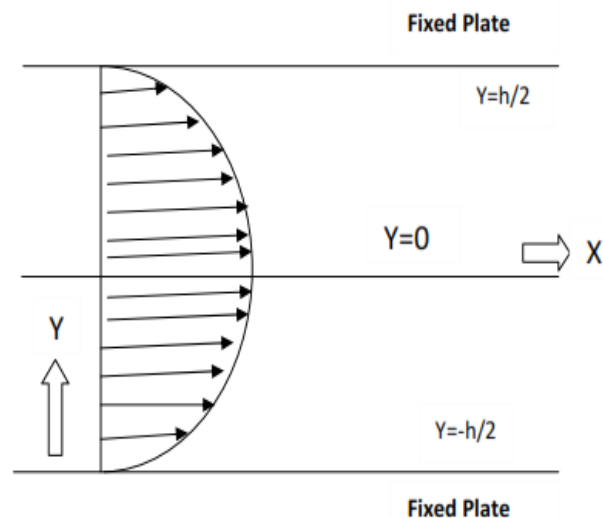
## II. MATHEMATICAL FORMULATION

The governing mathematical equations of the fluid flow under some common ssumptions used by Rao, A. Subba, et al [6] (2016), Abbas, W[7] (2016) and Raja, A. H [8] (2015) are written as -

$$\frac{dp}{dx} = \frac{d\tau}{dy} \tag{1}$$

$$\nabla \cdot u = 0 \tag{2}$$

where  $\tau = \tau_0 + \mu \frac{du}{dy}$



**Fig.1: Velocity Distribution Of The Fluid Flow Between Two Stationary Plates.**

The boundary conditions are

$$u = 0 \text{ at } y = -h/2$$

$$u = 0 \text{ at } y = h/2$$

$$\frac{du}{dy} = 0 \text{ at } y = 0$$

(3)

The velocity profiles for the given region are

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## Bingham Plastic Fluid Flow between Two Parallel Plates with Temperature Effect

$$\frac{du}{dy} \geq 0, \quad -\frac{h}{2} < y < 0$$

$$\frac{du}{dy} \leq 0, \quad 0 < y < \frac{h}{2}$$

The velocity of the fluid may be calculated from the equation (1) using the above boundary conditions

$$u_1 = \left( \frac{1}{\mu} \frac{dp}{dx} \right) \left( \frac{y^2}{2} - \frac{h^2}{8} \right), \quad -\frac{h}{2} < y < 0 \quad (4)$$

Similarly,

$$u_2 = \left( \frac{1}{\mu} \frac{dp}{dx} \right) \left( \frac{y^2}{2} - \frac{h^2}{8} \right), \quad 0 < y < \frac{h}{2} \quad (5)$$

The velocity distribution across a section of the parallel plate is parabolic in nature as shown in Fig.1 and  $\mu$ ,  $\frac{dp}{dx}$ , and 'h' are constants.

Also the velocity is same in both the region as it is symmetric and the velocity of the fluid is maximum at  $y=0$ .

$$u_{\max} = \frac{1}{8} \left( -\frac{h^2}{\mu} \frac{dp}{dx} \right) \quad (5.1)$$

Now, the total discharge of the fluid is given by

$$Q = \int_{-h/2}^{h/2} u \, dy$$

$$Q = \left( -\frac{1}{\mu} \frac{dp}{dx} \right) \left( \frac{h^3}{12} \right) \quad (6)$$

From the continuity equation one can get  $\frac{\partial Q}{\partial x} = 0$

$$\frac{dp}{dx} = \mu \left( c + \frac{h^3}{12} \right) \quad (7)$$

### 2. The Energy Equation

$$k \left( \frac{d^2 T}{dy^2} \right) + \tau \left( \frac{du}{dy} \right) = 0 \quad (8)$$

Solving above equation with the following boundary conditions

$T_1 = T_l$  at  $y = -h/2$  and  $T_2 = T_l$  at  $y = h/2$ , this

shows that both plates are with the same temperature.

Also by using interface temperature condition

$T_1 = T_2$  at  $y = 0$  and interface temperature gradient

$$\frac{dT_1}{dy} = \frac{dT_2}{dy} \text{ at } y = 0.$$

$$T = \frac{\tau_0}{\mu} \frac{y^2}{2} + \frac{\mu}{k} u + cy + d \quad (9)$$

Let  $T_m$  denotes the temperature at the middle of the channel,

i.e.  $T_1 = T_2 = T_m$  at  $y = 0$  at  $y=0$

$$T_m - T_l = \frac{\tau_0 h^2}{8\mu} + \left( \frac{1}{k} \frac{dp}{dx} \right) \left( \frac{y^2}{2} - \frac{h^2}{8} \right) \quad (10)$$

This result is same for both the regions  $-h/2 < y < 0$  and  $0 < y < h/2$ .

### III. RESULT AND DISCUSSION

The equation of motion and the energy equations are solved numerically. The following values are assumed for computation purpose.

$$\tau_0 = 0.1, \quad y/h = 1, \quad \mu = 0.001, \quad T_l = 0.4, \quad p = 1, 2$$

The obtained result is discussed clearly informs of figures. The velocity of the fluid 'u' increases with  $y/h$  in the lower region, and decreases with  $y/h$  in the upper region for different values of P as shown in Fig. 2 and Fig.3. Further, the temperature  $T_m$  of the fluid increases with  $y/h$  in the lower region, and decreases with  $y/h$  in the upper region as shown in Fig. 4. This result is in good agreement with the previous findings of Liu, K. F [9](1990) and Joshi, Sunil C., et al.[10] (2002). The temperature profile presented in Fig. 5 is taken when lower plate is adiabatic and upper plate is at constant temperature. These results are significantly following the results in the literature with good agreement. This semi-analytical approach of solution is quite rare in the literature.

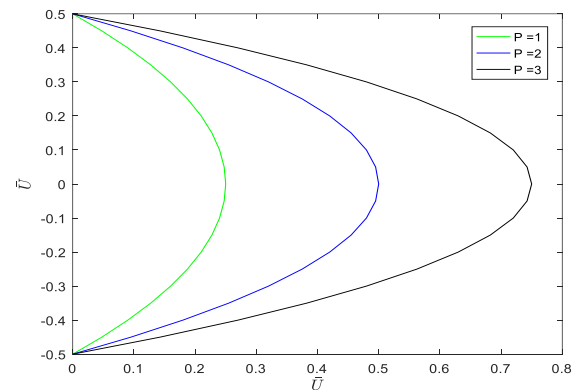


Fig. 2: Velocity profile for Newtonian case

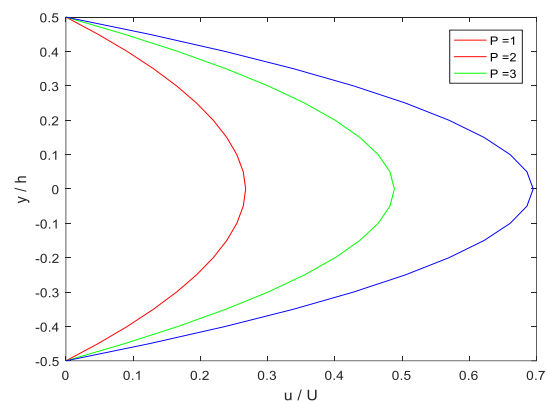


Fig.3: Velocity profile for non-Newtonian case

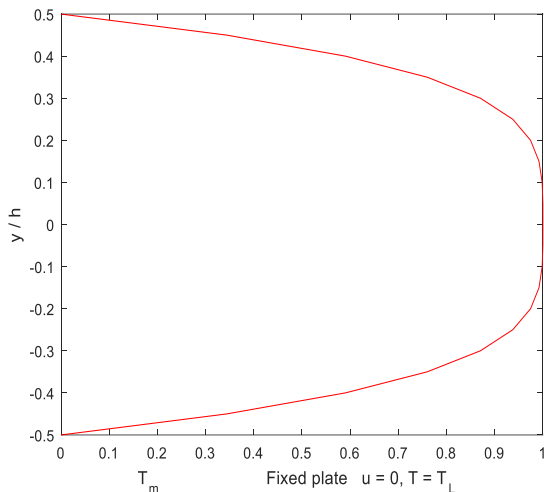


Fig.4: Temperature profile when two plates having same temperature

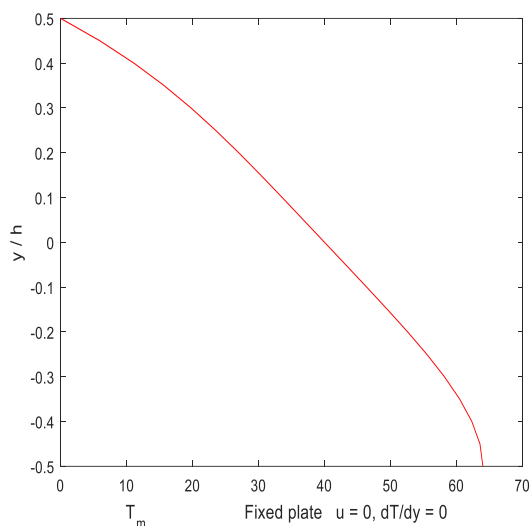


Fig. 5: Temperature profile when lower plate is adiabatic and upper plate is at constant temperature

#### IV. CONCLUSION

The steady state Bingham plastic fluids flow between two parallel plates is considered throughout the problem with incompressibility, total discharge of fluid, average velocity, shear stress and pressure head loss, velocity distributions of fluids. This problem deals with the Poiseuille flow between two plates at rest mentioning maximum velocity and ratio of maximum velocity to average velocity. The velocity distributions of fluids as well as temperature distribution are calculated mathematically and are plotted using MATLAB under constant and zero pressure gradients.

#### Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

#### REFERENCES

1. Sarpkaya, Turgut. "Flow of non-Newtonian fluids in a magnetic field." *AICHE Journal* 7.2 (1961): 324-328.
2. Szeri, A. Z., and K. R. Rajagopal. "Flow of a non-Newtonian fluid between heated parallel plates." *International Journal of Non-Linear Mechanics* 20.2 (1985): 91-101.

3. Mustafa, M., T. Hayat, and S. Obaidat. "On heat and mass transfer in the unsteady squeezing flow between parallel plates." *Meccanica* 47.7 (2012): 1581-1589.
4. Mohammad Mehdi Rashidi, Hamed Shahmohamadi, and Saeed Dinarvand, "Analytic Approximate Solutions for Unsteady Two-Dimensional and Axisymmetric Squeezing Flows between Parallel Plates," *Mathematical Problems in Engineering*, vol. 2008, Article ID 935095, 13 pages, 2008.
5. Siddiqui, A. M., et al. "Homotopy perturbation method for heat transfer flow of a third grade fluid between parallel plates." *Chaos, Solitons & Fractals* 36.1 (2008): 182-192.
6. Rao, A. Subba, et al. "Non-Similar Computational Solution for Boundary Layer Flows of Non-Newtonian Fluid from an Inclined Plate with Thermal Slip." *Journal of Applied Fluid Mechanics* 9.2 (2016).
7. Abbas, W., Hazem Ali Attia, and Mostafa AM Abdeen. "Non-Darcy effect on non-Newtonian Bingham fluid with heat transfer between two parallel plates." *Bulgarian Chemical Communications* 48.3 (2016): 497-505.
8. Raja, A. H., S. A. Patel, and R. P. Chhabra. "Laminar forced convection heat transfer from a two-dimensional transverse plate in Bingham plastic fluids." *International Journal of Heat and Mass Transfer* 83 (2015): 690-709.
9. Liu, K. F., and C. C. Mei. "Approximate equations for the slow spreading of a thin sheet of Bingham plastic fluid." *Physics of Fluids A: Fluid Dynamics* 2.1 (1990): 30-36.
10. Joshi, Sunil C., et al. "Power law fluids and Bingham plastics flow models for ceramic tape casting." *Journal of Materials Processing Technology* 120.1-3 (2002): 215-225.

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