

Noise Standard Deviation Estimation for Additive White Gaussian Noise Corrupted Images using SVD Domain

Sridhar P, R.R Sathiya

Abstract: During denoise an image; noise level estimation is one of the most important key factors. The accurate noise level estimation is needed before processing the image. The prior knowledge of noise level estimation is also used for restoring the image without degradation. In this proposed work, the noise level is estimated by observed singular values on noisy images. The proposed work has two new methods for addressing the main challenges of the noise level estimation. 1. The tail magnitude value of the noisy images singular values has high compare with signal image. This aspect is used for estimate the noise level. 2. The visual based Gaussian noise estimation is used for pre-processing the many 2D- signals processing application which enhance the range of this work. The experimental result for this noise level estimation provides reliable and also applicable for real time images/frames and some special images such as cartoon. The proposed work is needed a simple processing unit for implementing in hardware and results are more accurate. It can be used to pre-processing all kinds of real time images.

Keywords –Noise level estimation, Gaussian noise, noise standard deviation, de noising

I. INTRODUCTION

During image acquisition and transmission noise is inevitable. It alters the intensity of the pixel as a random manner which causes unpleasant to see and process the image data. The noise generation sources are vision sensor with its circuit, acquisition equipment such as digital scanner and cameras, analog to digital converter (quantizer), transmission medium. Once the image/frame is corrupted by noise it cannot process easily [2]-[4]. However further step forward pre-processing the image such as noise level estimation and suitable filter to remove the noise after that processing the image [1]. Further the noise estimation is the preliminary step before reliable denoising. The noise estimation in an image is a very tedious task. The other works such as motion estimation, high resolution, and extraction of feature are benefits by noise level estimation [15]-[19]. The Gaussian distribution is used mostly for noise estimation. Gaussian noise model such as noise in amplifier, finite account of particle which has a less energy causes shot noise, the photographic noise. The noise estimation is hard for an image. The brightness or texture of the image is varied by noise. The survey of denoising works clearly indicates consider noise as zero mean additive white Gaussian noise [5]-[8].

The noisy image is mathematically represented by equation (1).

$$I_N(i, j) = I_O(i, j) + N(i, j) \quad (1)$$

Where $I_O(i, j)$ represents the true image, $N(i, j)$ is the Gaussian noise. The Gaussian noise distribution is given by equation (2)

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \quad (2)$$

Where σ indicates standard deviation of the Gaussian noise and μ is the mean value. In this case mean value is zero so the Gaussian noise estimation, the measuring parameter is σ . The two main issues for measuring the Gaussian noise of the separate image. [9]-[11](1) how to collect the dataset for estimation noise in an image (2) The various image such real time image, texture type and standard image how to create the adaptive approach for estimating the noise. The Gaussian noise estimation approaches can be classified into 3 types. (1) Filtering methods (2) Transform based approach (3) Patch based works. In filter based approaches, the noise can be removed by low pass filter then estimation of the noise is measured by standard deviation between true image and noisy image. The limitation of this filter based method has not adaptive for all types of images. The primary difficulties of this method the different content of the image consider as noise but commonly we are not considering like that. This provides small original image signal influence on the noise estimation. Histogram based noise estimation [11] has more computational complexity and more no of parameters using. In patch based method images are separated into blocks. The noise level is measured in each block which has homogeneous. The homogeneous block consider as a smooth part of the image add with noise. The homogeneity is relatively connected with the real world images. The identification of homogeneity block based method [12] is a difficult task. Commonly noise estimation block based approaches are used for estimation of noise. Even though this kind approach is easy, noisy estimation is varying predominantly depending upon the type of input image and level of noise.

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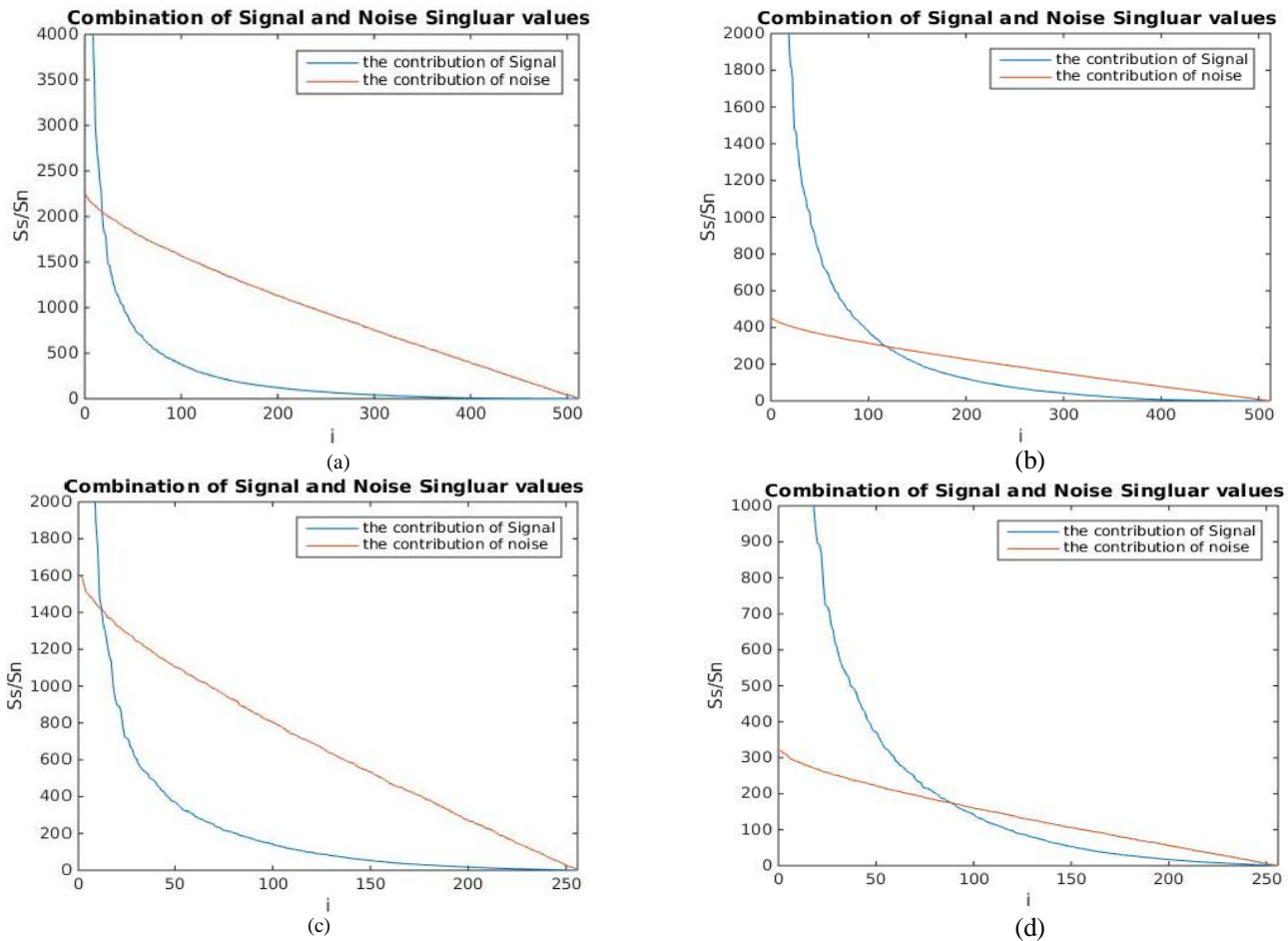


Fig.1 The signal and noise contribution of singular values in SVD domain for Lena Image (a) 512x512 $\sigma = 50$ (b) 512x512 $\sigma = 10$ (c) 256x256 $\sigma = 50$ (d) 256x256 $\sigma = 10$

Filtering approaches provides the good results for large amount of noise. But they need large memory and high computation costs.

In the transform based work, the frequently used approach is a Mean Absolute Deviation (MAD) [13].The noise is estimated by using the equation (3).

$$\hat{\sigma}_n = \frac{\text{median}(|HH|)}{0.6745} \quad (3)$$

Where HH indicates the high frequency sub band coefficients of wavelet transform. The interpretation of this equation noise signal is lying in a high frequency subband. But the estimated output is more than actual value of the noise. The high frequency subband is influenced by noise compare to signal. Consider the case which is more image content compare to noise; this case estimation is less accuracy. The little modification was performed which yields the better results. This estimation is performing is based Donoho's approach.

The mention challenges in transform domain cases and break the limitation in the existing system, we explore the techniques for noise estimation using singular value decomposition (SVD). The SVD transform domain has well covered in recognition and restoration problems in images. The SVD based noise estimation is strong influence for noise estimation and gives less significance in original image data.

The various kind of dataset is tested against the proposed work which provides reliable result for different level standard deviation of Gaussian noise with zero mean.

II NOISE INFLUENCE IN SVD DOMAIN

A. Singular values of Additive white Gaussian Noise

With the help of Linear algebra the theory of SVD in which the rectangular matrix A is decomposed into three matrices. It has an orthogonal matrix U, diagonal element matrix S and one more orthogonal matrix V.

$$A = U \times S \times V^T \quad (4)$$

Where $U^T U = I_{mm}$; $V^T V = I_{nn}$ I_{mm} and I_{nn} indicate the identity square matrices. The dimension of A is mxn. The column of U and V matrices are orthonormal and S is the diagonal matrix which square roots are eigen values of the AA^T .The

Columns of orthogonal matrix are orthonormal Eigen vector. The singular values of diagonal matrix S is denoted by descending order. The singular values are $S(i)(i = 0,1,2,\dots \dots r)$ and then $S(1) > S(2) > \dots S(r)$

The signal and noise is denoted by notation S_s and S_n and its SVD transform is denoted by equation (5) and (6)

$$S_s = U^{-1} \times I_O \times V = U^T \times I_O \times V \quad (5)$$

$$S_n = U^{-1} \times N \times V = U^T \times N \times V \quad (6)$$

In Fig (2) are shown different test image's singular values and its noise levels. The test images size is 512x512. All test images are standard grayscale images. The singular value is increasing when adding Gaussian noise to an image and this was indicated in [20]. Conversely, if the higher the noise standard deviation, the magnitude of the singular values is larger. The earlier singular values are not influenced by increasing the noise level. But the noise can be identified by the later singular values part. This is the important part of the Gaussian noise estimation.

B. Noise Analysis

Additive white Gaussian noise is represented by N of $m \times n$ size and its variance σ^2 . The SVD transform of the noise matrix is

$$N = U \times S_n \times V^T \quad (7)$$

$$\text{Variance}(\sigma^2) = \frac{1}{mn} \sum_{i=1}^r S_n^2(i)$$

The number of tail end singular values is denoted by M . The function of noise standard deviation is the average of these last M singular values and it's estimated by

$$L_M(\sigma) = \frac{1}{M} \sum_{i=r-M+1}^r S_n(i) \quad (8)$$

Where $1 \leq M \leq r$ if $M=2$, the end of two singular values will be taken. If $M=r$, all M values will be considered. The noise function $L_M(\sigma)$ is linearly dependent on standard deviation. It can be indicated by equation (9) and it was satisfied superposition principle.

$$\begin{cases} L_M(c\sigma) = c L_M(\sigma) \\ L_M(c_1\sigma + c_2\sigma_1) = c_1 L_M(\sigma) + c_2 L_M(\sigma_1) \end{cases} \quad (9)$$

Where c_1, c_2, c are constants.

Table 1 shows the linear relationship between the function $L_M(\sigma)$ and σ . The linear relationship is given by equation (10).

$$L_M(\sigma) = c\sigma \quad (10)$$

Where 'c' is the slope or proportionality constant. It can change by the chosen value of M . The chosen value of $M < r/4$ the function $L_M(\sigma)$ is not a linear relationship with the standard deviation. The resolution of the image is 512x512 added with additive white Gaussian noise we consider 128 values to 512th singular values for showing the linear relationship. For different resolution of image and proper slope value of 'c' it provides the linear relationship. The real world image sequences are entirely different from the standard dataset. Since, the equation (10) is not satisfied the function $L_M(\sigma)$. We can modify the equation (10) for real world image is

$$L_M = \sum_{i=r-M+1}^r S_i = c\sigma + \beta \quad (11)$$

β is related to the content of the image. We combine the equation (8) and (11) it gives the equation (12)

$$L_M = \sum_{i=r-M+1}^r S_i(i) = c\sigma + \beta \quad (12)$$

The function L_M can be divided into two components based on the signal and noise in image content. L_{Ms} is the

signal component and L_{Mn} is the noise component. It can be represented by equation (13) and (14)

$$L_{Ms} = \sum_{i=r-M+1}^r S_s(i) \quad (13)$$

$$L_{Mn} = \sum_{i=r-M+1}^r S_n(i) \quad (14)$$

We examine the signal and noise contribution for Standard Lena Image with 512x512. The chosen singular values are from 128 to 512. The noise distribution function is almost parallel to image with noise function. But the signal content is eventually different from this two noise function. The signal function is almost constant with respect to coefficients. It shows in Fig.3

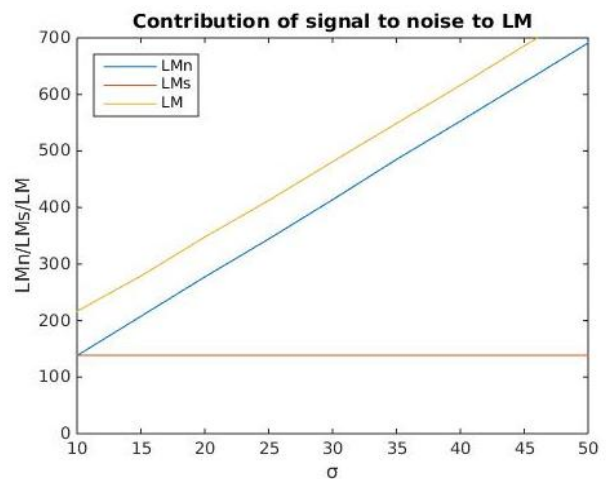


Fig.3 Signal and Noise Contribution of L_M

The equation (19) is related with the Fig.3. The signal with noise function or noise function 'c' values are slope of the function and β value depends on the content of the image.

The complex image has higher value of β .

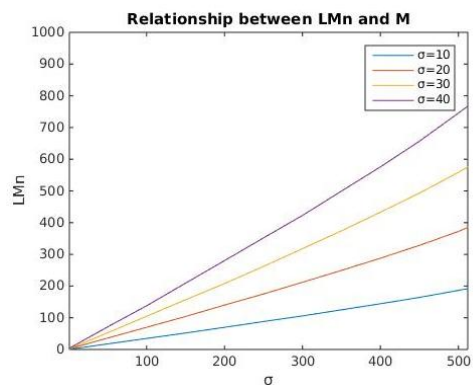


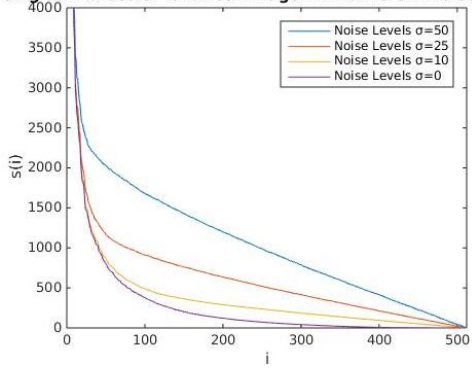
Fig.4 Linear relationship between Singular values and Noise function for different noise levels of 512X512 image

We should not take the M value is larger. Since the starting singular values the signal part is influenced in the noisy image. Noise function L_M is calculated based on the M parameter value. The M value should not choose too small. It does not provide the sufficient data noise estimation. It causes reliability and accuracy issues of the noise estimation.

Noise Standard Deviation Estimation for Additive White Gaussian Noise Corrupted Images using SVD Domain

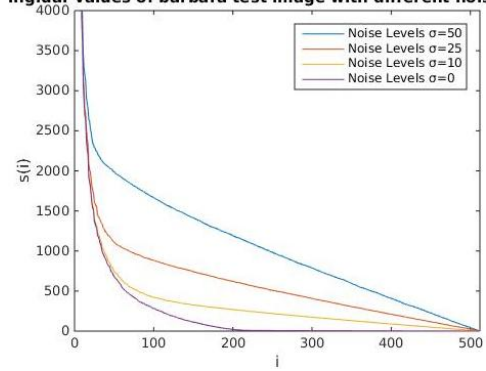
The various experimental tests was performed from different images at different resolution we

Singular values of lena test image with different noise level



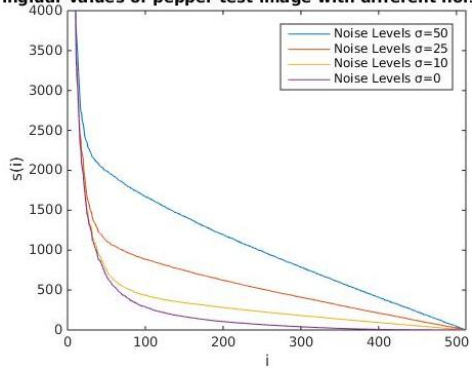
(a)

Singular values of barbara test image with different noise level



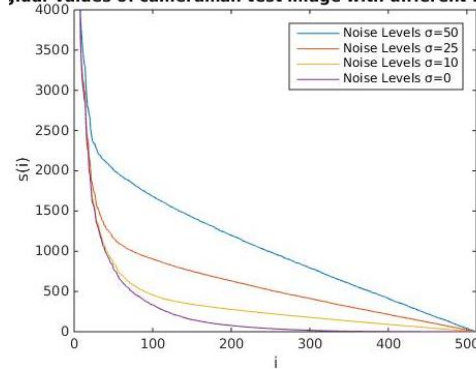
(b)

Singular values of pepper test image with different noise level



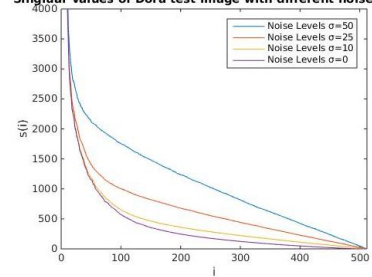
(c)

Singular values of cameraman test image with different noise level



(d)

Singular values of Dora test image with different noise level



(e) (f)

Fig.2.Singular values of various test images for different Gaussian noise level column (a) Lena (512x512)

image and its singular values. column (b) Barbara (512x512) image and its singular values. column (c) Pepper image and fix the M parameter range from $\left[\frac{r}{4} \text{ to } \frac{4r}{5}\right]$. In Fig.5 provides

the graph between standard deviation and the noise function for different resolution test imssages. The content does not influence the noise function. In Fig.5 (b) the test images size has 256x256.If the noise standard deviation is increased the L_M is also increased. So the parameter M value is chosen ($M=3r/4$).Fig.5 (a) shows the crowd, Lena, pepper and Blank test images graphs. The test result shows that the blank image signal Furthermore, the content of the test images is varying. So the line of each test image parallel but

its singular values. column (d) Cameraman image and its singular values. (e-f) Dora image and its singular values intercept is changed. The Fig.5 gives the clear about linear relationship between noise standard deviation and L_M . The intercept value β is based on the content of the image. The Fig.5 (b) shows blank image has less content. So compare the other test images the blank image is located lower position. In Fig.6 shows the chosen value of the M parameter how to affect the estimation of noise. The chosen M value is too small the noise function is vibrated else the M is too large the function L_M and c are non-linear.

Table-I: Using the slope values of 'c' and $M=3R/4$, the linear relationship between L_M and σ can be seen for different resolution

	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=25$	$\Sigma=30$	$\sigma=35$	$\sigma=40$	$\sigma=45$	$\sigma=50$	c
$L_{MN}(512 \times 512)$	137.9833	207.5290	277.2149	344.2644	413.6893	484.9244	552.6644	621.6843	690.9194	12.86
$L_{MS}(256 \times 256)$	97.4532	150.3232	189.2242	242.3334	256.4858	347.2265	389.2456	445.1226	495.2364	9.97
$L_M(128 \times 128)$	216.3034	278.8576	347.6419	412.4439	480.9524	548.6564	616.1221	685.8564	754.2832	6.98

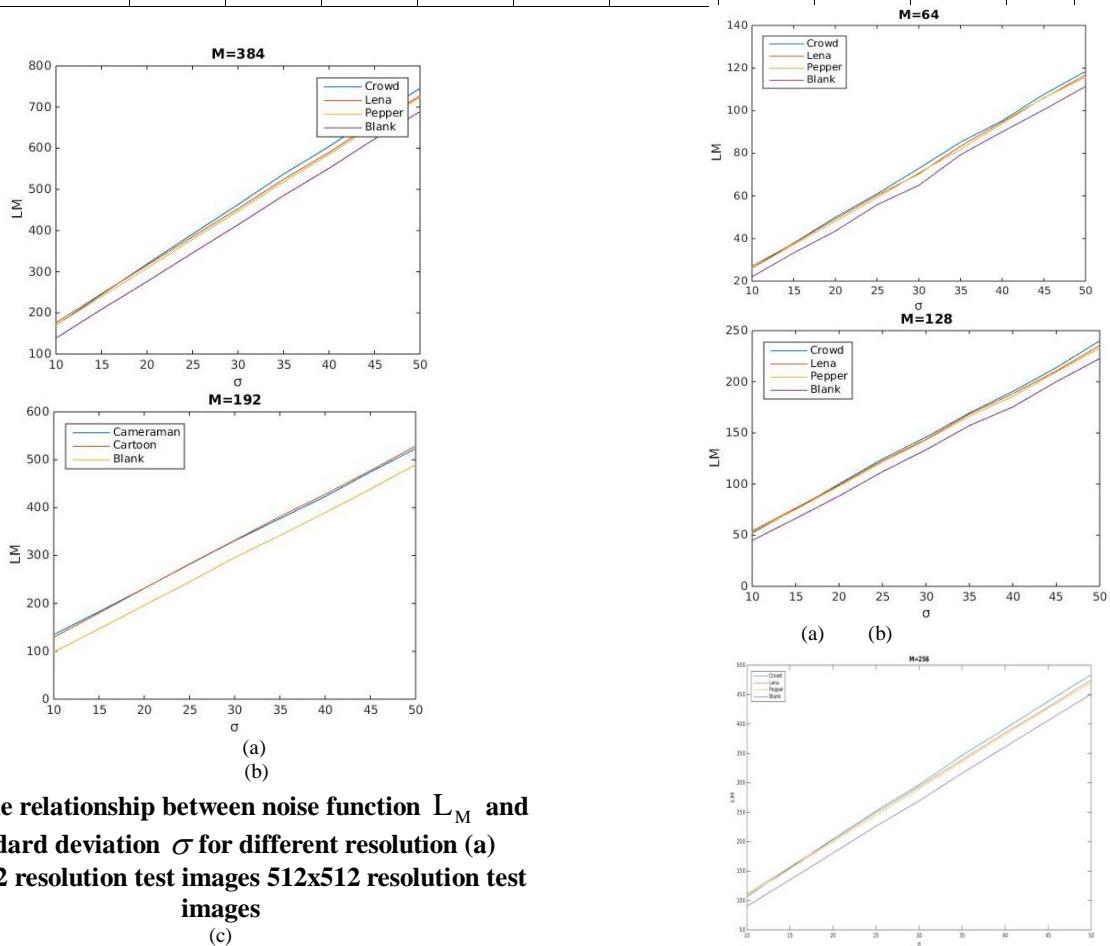
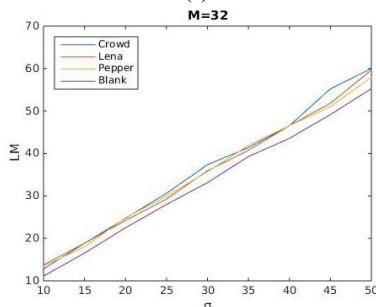
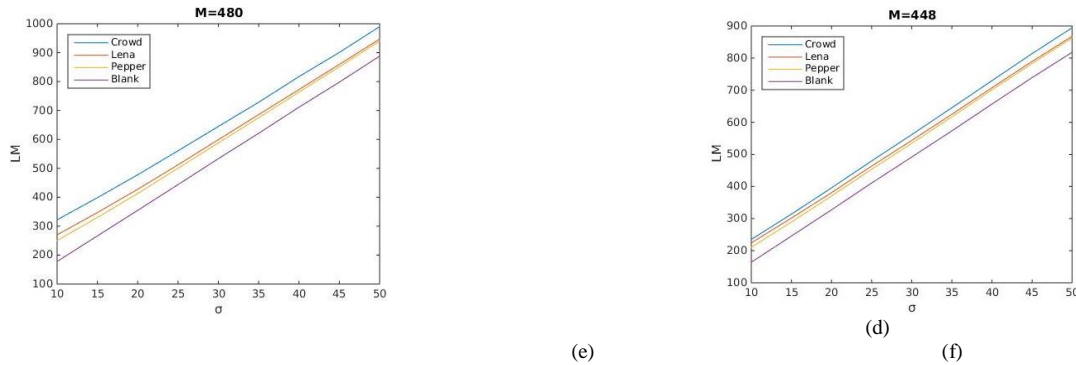


Fig.5 the relationship between noise function L_M and standard deviation σ for different resolution (a) 512x512 resolution test images 512x512 resolution test images





(e) (f)
Fig .6 the relationship between parameter M and slope c value

III. PROPOSED NOISE STANDARD DEVIATION ESTIMATION ALGORITHM USING SINGULAR VALUE DECOMPOSITION DOMAIN

The analysis results give the relationship between parameter value M and noise function and linearity of the noise standard deviation. This analysis creates the platform for noise estimation. The different test images used at different resolution levels and fixed the M parameter to maintain the linearity for noise estimation. The different image size the chosen value of M is the only factor for affecting the slope value c. For instance from Table I, if the image size has 512X512 the chosen M value is 3r/4 i.e. M=384 and slope c is 12.86.If the size 256x256 the chosen M parameter is 192 and slope is 9.97.The analysis provides the conclusion for choosing the M parameter value range of

$[\frac{r}{4}, \frac{3r}{5}]$ for reliable calculation. The other size of image the following steps provide the way for calculating the slope c values.

1. Calculating the noise function of additive white Gaussian noise corrupted image at different deviation level we tabulated the values such as Table I
2. Calculating the slope c value by least square method.
 The slope c value is not dependent on image content. Due to development of the embedded system, nowadays different resolution cameras are available in the market for various purposes. The calculation of the slope value for different resolution test image is estimated by method such as Table I. Noise standard deviation level is calculated by using equation (11) we find the image content related parameter β .But this is very difficult to find the accurate β value. The proposed algorithm uses 4 steps for estimating the Gaussian noise level.
 1. The parameter M value have to choose Properly.
 2. Apply the Singular value Decomposition of noise corrupted image
 3. Calculate the value of L_M
 4. Estimating the noise standard deviation

Table-II: Estimation of Noise standard deviation (σ) for various test images at different noise level (average value of 100 times test)

Images	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=25$	$\sigma=30$	$\sigma=35$	$\sigma=40$	$\sigma=45$	$\sigma=50$
Lena	9.8315	14.9106	19.9450	24.9956	30.0751	35.1242	40.0321	45.1530	50.2167
Peppers	9.6652	14.6668	19.6797	24.7306	29.7179	34.7337	39.7923	44.7708	49.7558
Crowd	7.6176	12.9417	18.1816	23.3610	28.4766	33.6353	38.7309	43.8501	48.9776
Barbara	9.1786	14.4172	19.4980	24.6122	29.6503	34.6896	39.7497	44.7619	49.7390
Cameraman	9.3022	14.4623	19.5854	24.7041	29.7218	34.8377	39.9021	44.8898	49.9531
Cartoon	8.5879	13.7858	18.9598	24.1058	29.1829	34.3145	39.3237	44.3515	49.465
Dora	11.4242	16.0352	20.8403	25.7705	30.7491	35.7404	40.7908	45.8461	50.8522
Blank	9.9395	14.9266	19.9019	24.8837	29.8564	34.9422	39.8510	44.8797	49.8594

Table-III: Estimated values deviation of 100 tests at different noise level

Images	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=25$	$\sigma=30$	$\sigma=35$	$\sigma=40$	$\sigma=45$	$\sigma=50$
Lena	0.1198	0.1547	0.2018	0.2215	0.2358	0.3020	0.330	0.3876	0.4432
Peppers	0.1454	0.1613	0.1811	0.2234	0.2257	0.2941	0.3687	0.4296	0.4132
Crowd	0.1254	0.1535	0.1875	0.1961	0.2488	0.2900	0.3436	0.4111	0.4477
Barbara	0.1146	0.1481	0.1609	0.2358	0.2817	0.3112	0.3598	0.4078	0.4725
Cameraman	0.1215	0.1421	0.1833	0.2108	0.2488	0.3167	0.3169	0.3918	0.4532

Cartoon	0.1332	0.1405	0.2016	0.2227	0.2300	0.2839	0.3645	0.3610	0.4335
Dora	0.1370	0.1670	0.1761	0.2307	0.2871	0.9473	0.3479	0.4172	0.6358
Blank	0.1234	0.1490	0.1905	0.1998	0.2696	0.2407	0.3328	0.4185	0.4723

IV EXPERIMENTAL RESULTS AND ANALYSIS

The proposed work is tested different types of images such as standard image and real world image Dora. The estimated value of L_M is the mean of tail ended singular values (75 % of singular values i.e. $3r/4$). The proposed

Table-IV: Noise Estimation Comparison

Noise Level	Proposed Method	Algorithm 1 [13]	Algorithm 2 [14]
10	0	0.9	1.38
15	-0.1	0.75	1.5
20	0.01	0.58	0.9
25	-0.12	0.5	0.8
30	0.2	0.22	0.7
35	0.15	0.16	0.6
40	0.1	-0.1	0.58
45	0.01	-0.3	0.4
50	0.18	-0.35	0.2

parameter M and c is used for estimating the noise standard deviation. Noise occurs when scanning the drawn image for making the cartoon film. This proposed work helps to estimate the noise to the extent of the problem and it helps cartoon related noise estimation. Table II and Table III are the result of statistical measure of noise estimation of all kinds of images. Table II shows the noise estimation of average of the 100 times different additive white Gaussian noise zero mean and same variance of noise corrupted image. Table III shows the deviation from the mean for 100 times testing. The experimental work tells the estimation of Gaussian noise in images is accurate and reliable for all real time images. The proposed work is compared with other the other two noise estimation algorithms. The performance

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method using Lena, Barbara, Peppers, Crowd, Blank Dora images at different resolution level. The Fig.1 isolates the signal and noise content of specific range of singular values. We analyse the simulation results from Fig.2 to Fig 5 how the

parameter is noise standard deviation estimation

error $\delta = \hat{\sigma} - \sigma$. In Table IV shows the comparison analysis using two wavelet based works. Algorithm 1 [13] D.L Donoho proposed wavelet coefficient based noise estimation. Algorithm 2 [14] is modified version of wavelet domain noise estimation. The two algorithms provide the good noise estimation for high standard deviation noise level. But minimum noise standard deviation these two works more deviation was there.

V. CONCLUSION

Singular value decomposition is mathematical tool it has been important for signal processing application for a long time. We used SVD tool for additive white Gaussian noise estimation. In this tool signal and noise are separately distinguished. The experimental results show our proposed methods are outperforms the estimation of noise with the other existing methods. It provides better estimation results. Computer vision, pattern recognition, image processing application noise estimation is important for us to know in advance. These works such as video analytics and denoising are needed for noise estimation. The proposed method good ground work for image processing application. A lot of algorithms don't provide us favourable results due to noise in an videos or frames. It is initiative for denoising

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