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Mechanical Property Variation of a Rotating Cantilever FGSW Beam under Parametric Excitation

S.N.Padhi, K.S.Raghuram, T.Rout, V.Naga Sudha, Gill Santosh Kumar

Abstract: ***We report here the dynamic stability of functionally graded sandwich (FGSW) rotating cantilever Timoshenko beams under parametric excitation. Power law with various indices as well as exponential law were used to find out the properties along the thickness of the FGSW beam. The stability boundaries were established using Floquet's theory. The equation of motion was governed by Hamilton's principle and solved by Finite element method. The power index was optimized for uniform variation of shear modulus along the thickness of FGSW beam.The shear modulus variation along the thickness of the FGSW beam was compared both by power law and exponential law.It has been confirmed that the Exponential distribution of constituent phases renders better stability compared to power law distribution of the phases in the functionally graded material(FGM).***

Keywords: Exponential law, FGSW beam, Power law, Shear modulus, Stability.

I. INTRODUCTION

Functionally Graded Sandwiched structures find their use in spacecrafts, machinery and automobile industries because they have their high strength and stiffness compared to their low weight. In modern engineering, the FGSW beams have gradually substituted the large weighed metallic beams. Rotating FGSW beam structures are commonly found in engineering applications, including robotics, turbine blades, and helicopter rotors. Vibration of rotating structures has become a commonly occurring phenomenon. The vibration becomes severe because of resonance which causes heavy mechanical damage. Therefore, the stability and dynamic behavior of these rotating structures are of great practical importance to eliminate the problems of resonance. In actual practice, the rotating components mentioned above are usually pre-twisted and of asymmetric cross-section. However, beams of uniform cross-section under rotation can be used as a simple model and compared at par with the actual rotating structures to investigate the stability and dynamic behavior. The research on functionally graded materials (FGMs) is rapidly growing because of their continuously varying material properties, which give great advantages over the conventional homogeneous and layered materials. The major issues in conventional laminated composite materials, such as debonding, huge residual stress, locally large plastic deformations can be eliminated by using FGM. An FGM can be made as a good substitute for the rotating beam material. Many researchers have worked on the stability of the rotating beams. Stafford and Giurdiutiu [1] have developed a simplified model of helicopter blade considering shear deformation and rotary inertia corrections and investigated the natural frequencies using transfer matrix method. Dokumaci [2] has presented his work on the effects of pre-twist, ratio of bending rigidities and loading angle on the unstable zones of pre-twisted blades under lateral parametric excitation. Celep [3] studied the dynamic stability of pre-twisted column. The author showed that combination resonances of the sum type may exit or disappear depending on the pre-twist angle and rigidity ratio of the cross-section in case of simply supported columns. Abbas [4] used finite element method to determine the effect of rotational speed and root flexibility on the stability of a rotating Timoshenko beam. Ishida et al. [5] have investigated the parametrically excited oscillations of rotating shaft under a periodic axial force. They have observed that an elastic shaft with a disk exhibits only difference type combination resonance. Chen and Ku [6] have revealed from their investigation of cantilever shaft-disc system that gyroscopic couple can enlarge the principal regions of dynamic instability. Kosmatka [7] has developed a simple two-node Timoshenko beam element for the matrices of linear flexural ,incremental stiffness, mass, and force based upon Hamilton's principle, where interdependent cubic and quadratic polynomials are used for the transverse and rotational displacements, respectively. He reported that the buckling load and natural frequencies of axially-loaded isotropic and composite beams can be found for a variety of -lengths and boundary conditions. Lin and Hsiao [8] has derived the governing equations for linear vibration of a rotating Timoshenko beam by D'Alembert's principle and the virtual work principle and investigated the effect of Coriolis force on the natural frequency of rotating beams with different angular velocity, hub radius and slenderness ratio. Yang, Jiang and Chen [9] have investigated flexible motion of a uniform rotating Euler-Bernoulli beam and found that the vibration control can be improved by combining positive position feedback and momentum exchange feedback control laws. Wang and Wereley [10] have proposed a method based on a spectral finite element technique and reported that only one single spectral finite element is enough to obtain any modal frequency or mode shape, which is as accurate or better than other approaches.

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***Correspondence Author(s)**
Dr.S.N.Padhi, Dept.of Mechanical Engg., Koneru Lakshmaiah Education Foundation, Guntur, India.
Dr.K.S.Raghuram, Department of Mechanical engineering, Vignan's Institute of Information Technology, Visakhapatnam, A.P., India.
***Dr.T. Rout,** Dept.of Mechanical Engg., Parala Maharaja Engineering College, Berhampur,India.

Mohanty, Dash and Rout [11] have carried out the free vibration analysis of functionally graded ordinary (FGO) and functionally graded sandwich (FGSW) rotating cantilever beam and found that the effect of property distribution laws on the frequencies is predominant for lower values of rotary inertia parameter and for higher values of rotational speed parameter and hub radius parameter .DeValve and Pitchumani [12] have studied the damping behaviour of carbon nanotubes (CNT's) embedded in the matrix of fiber-reinforced composite materials used in rotating structures and performed a parametric study to examine the effects of various beam geometries, angular speed profiles, and CNT damping values on the vibration settling times of the numerically simulated beams. Padhi, Choudhury and Rout [13] evaluated the static and dynamic behavior of simply supported sigmoid functionally graded ordinary (SFGO) beam using finite element method and observed that an SFGO beam with lower power index is a better beam as per as parametric instability is concerned. Rafiee, Nitzsche, and Labrosse [14] reviewed about rotating composite beams analytical, semi-analytical and numerical studies dealing with dynamical problems involving adaptive/smart/intelligent materials (e.g. piezoelectric materials, electrorheological fluids, shape memory alloys, etc.), damping and vibration control, advanced composite materials (e.g. functionally graded materials and nanocomposites), complicating effects and loadings (e.g. added mass, tapered beams, initial curve and twist, etc.), and experimental methods. Padhi, Rout and Raghuram studied on stability of functionally graded ordinary (FGO) rotating cantilever Timoshenko beam and reported that the properties drawn by Exponential distribution confirms better stability compared to properties drawn by power law.

Though many researchers have reported on static and dynamic stability of ordinary beams plentily, the literature on dynamic stability of functionally graded rotating beams reported are not enough to the best of the authors' knowledge. In the present article, a functionally graded rotating ordinary beam with fixed-free support condition is considered for dynamic stability analysis.

II. FORMULATION

A functionally graded sand witch(FGSW) beam with top skin as alumina, bottom skin as steel and core as FGM is shown in Fig. 1(a). The beam fixed at one end free at the other end is subjected to a pulsating axial force $P(t) = P_s + P_t \cos \Omega t$, acting along its undeformed axis. The static component of the axial force is P_s. The amplitude and frequency of the dynamic component of the force are P_t and Ω respectively, and t is time. The coordinate system of the typical two noded finite element used to derive the governing equations of motion is shown in Fig. 1(b). The mid-longitudinal(x-y) plane is chosen as the reference plane for expressing the displacements as shown in Fig. 1(b). The thickness coordinate is measured as 'z' from the reference plane. The displacement vector of a point on the reference plane and along the longitudinal axis is expressed as

$$\{\bar{u}\} = [u \; w \; \phi]^T \qquad (1)$$

Here, u , w and ϕ are respectively the axial displacement transverse displacement and rotation of cross-sectional plane with respect to the un-deformed configuration. Figure 1(c) shows a two nodded beam finite element having three degrees of freedom per node.

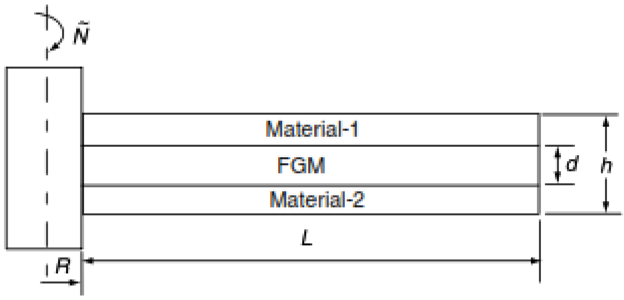

Figure 1(a). FGSW Cantilever beam under rotation.

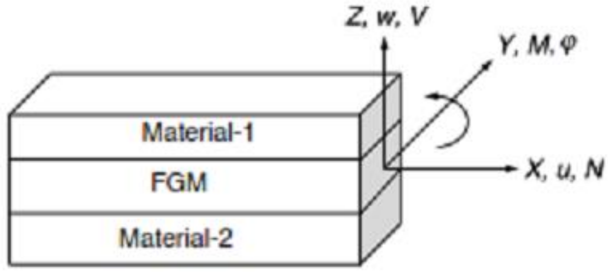

Figure 1(b) Forces and displacement coordinate system of the FGSW beam element.

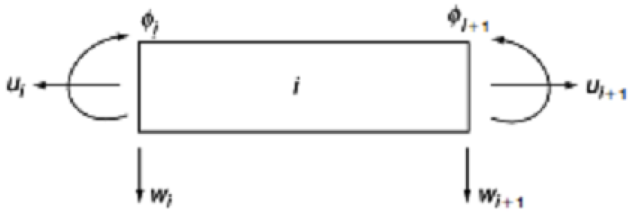

Figure 1(c) Degrees of freedom of i^{th} element of FGSW beam.

A. Shape functions

According to first order Timoshenko beam theory the displacement fields can be expressed as

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$$U(x,y,z,t) = u(x,t) - z\phi(x,t),$$
$$W(x,y,z,t) = w(x,t), \tag{2}$$

U and W are axial and transverse displacement of a material point respectively.

The material properties of the FGM that varies along thickness are assumed to follow exponential law given by

$$R(z) = R_t \exp(-e(1-2z/h)), \tag{3}$$

$e = \frac{1}{2}\log\left(\frac{R_t}{R_b}\right)$, and power law given by

$$R(z) = (R_t - R_b)\left(\frac{z}{h}+\frac{1}{2}\right)^n + R_b \tag{4}$$

Here, $R(z)$ denotes a material property such as, E , G , ρ etc., R_t and R_b denote the values of the properties at topmost and bottommost layer of FGSW beam respectively, and n is an index. The variation of rigid modulus (G) of FGSW beam along thickness as per power law with different indices and a comparison between power law and exponential law is shown in Fig. 2(a) and Fig. 2(b) respectively.

The shape function is derived following the article by Mohanty et. al [11]

The displacement vector shown in eqn. (1) can be expressed in terms of shape function as follows.

$$\{\bar{u}\} = [\aleph(x)]\{\hat{u}\} \tag{5}$$

Here, the nodal displacement vector is

$$\{\hat{u}\} = \left[u_i \;\; w_i \;\; \phi_i \;\; u_{i+1} \;\; w_{i+1} \;\; \phi_{i+1}\right]^T \tag{6}$$

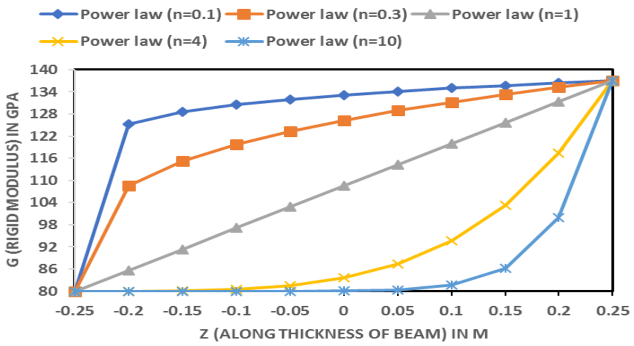

Fig. 2(a) Variation of Rigid modulus along thickness of steel-aluminium FGSW beam with steel-rich bottom according to power law with various indices.

B. Element elastic stiffness matrix

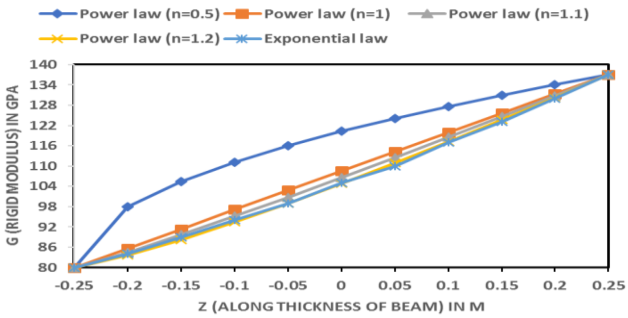

Fig. 2(b) Variation of Rigid modulus along thickness of steel-aluminum FGSW beam with steel-rich bottom according to power law at n = 0.5 and exponential law.

The element elastic stiffness matrix is given by the relation

$$[k_e]\{\hat{u}\} = \{F\} \tag{7}$$

Here, $\{F\} = \lfloor N_x \;\; V_y \;\; M_{xy} \rfloor$ is the nodal load vector, $[k_e]$ is the required element elastic stiffness matrix and $N_x \; V_y \; M_{xy}$ are respectively axial force, shear force and bending moment acting on the beam.

C. Element mass matrix

The element mass matrix is given by

$$T = \frac{1}{2}\{\dot{\hat{u}}\}^T [m]\{\dot{\hat{u}}\} \tag{8}$$

D. Element centrifugal stiffness matrix

The centrifugal force on ith element of the beam can be expressed as

$$F_c = \int_{x_i}^{x_i+l}\int_{-h/2}^{h/2} b\rho(z)\tilde{\mathrm{N}}^2(R+x)dzdx \tag{9}$$

Where x_i is the distance of i[th] node from axis of rotation, $\tilde{\mathrm{N}}$ is angular velocity of beam in rad/s and R is the radius of hub. Work done by the centrifugal force is given by

$$W_c = \frac{1}{2}\int_0^l F_c\left(\frac{dw}{dx}\right)^2 dx = \frac{1}{2}\{\hat{u}\}[k_c]\{\hat{u}\} \tag{10}$$

Here, the centrifugal element stiffness matrix is

$$[k_c] = \int_0^l F_c[\aleph'_w]^T[\aleph'_w]dx \tag{11}$$

E. Element geometric stiffness matrix

The work done by an axial load P can be expressed as

$$W_p = \frac{1}{2}\int_0^l P\left(\frac{\partial w}{\partial x}\right)^2 dx \tag{12}$$

Substituting the value of w from eq. (6) into eq. (12) the work done can be expressed as

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$$W_p = \frac{P}{2}\int_0^l \{\hat{u}\}^T [\aleph'_w]^T [\aleph'_w]\{\hat{u}\}dx = \frac{P}{2}\{\hat{u}\}[k_g]\{\hat{u}\} \quad (13)$$

$[k_g]$ is called the element geometric stiffness matrix.

III. GOVERNING EQUATION OF MOTION

The element equation of motion for a beam is obtained by using Hamilton's principle.

$$\delta\int_{t_1}^{t_2}(T - S + W_c - W_p)dt = 0 \quad (14)$$

Substituting Eqns (7, 8, 10 and 13) into Eqn (14) the equation of motion for the beam element is obtained as follows

$$[m]\{\ddot{\hat{u}}\} + [[k_{ef}] - P(t)[k_g]]\{\hat{u}\} = 0 \quad (15)$$

$$[m]\{\ddot{\hat{u}}\} + [[k_{ef}] - P^{\oplus}(\alpha + \beta_d\cos\Omega t)[k_g]]\{\hat{u}\} = 0 \quad (16)$$

$$[k_{ef}] = [k_e] + [k_c] \quad (17)$$

Here, $[k_{ef}]$ is the effective stiffness matrix. Assembling the element matrices the equation in global matrix form which is the equation of motion for the beam, can be expressed as

$$[M]\{\ddot{\hat{U}}\} + [[K_{ef}] - P^{\oplus}(\alpha + \beta_d\cos\Omega t)[K_g]]\{\hat{U}\} = 0 \quad (18)$$

$[M]$, $[K_{ef}]$, $[K_g]$ are global mass, effective stiffness and geometric stiffness matrices respectively and $[\hat{U}]$ is global displacement vector. Equation (19) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The periodic solutions for the boundary between the dynamic stability and instability zones can be obtained from Floquet Theory as described by Mohanty et. Al.[ ]

$$\left([K_{ef}] - (\alpha \pm \beta_d/2)P^{\oplus}[K_g] - \frac{\Omega^2}{4}[M]\right)\{\hat{U}\} = 0 \quad (19)$$

Equation (19) represents an eigenvalue problem for known values of $P^{\oplus}$, α, β_d. $P^{\oplus}$ is the critical buckling load of a homogeneous steel beam of same dimensions as FGSW beam This equation gives two sets of eigenvalues (Ω) binding the regions of instability due to the presence of plus and minus sign. The instability boundary can be determined from the solution of the equation

$$\left|[K_{ef}] - (\alpha \pm \beta_d/2)P^{\oplus}[K_g] - \frac{\Omega^2}{4}[M]\right| = 0 \quad (20)$$

Choosing $\Omega = \left(\frac{\Omega}{\omega_1}\right)\omega_1$, eq. (20) can be rewritten as

$$\left|[K_{ef}] - (\alpha \pm \beta_d/2)P^{\oplus}[K_g] - \left(\frac{\Omega}{\omega_1}\right)^2\frac{\omega_1^2}{4}[M]\right| = 0 \quad (21)$$

The solution of eq. (21) will give two sets of values of $\left(\frac{\Omega}{\omega_1}\right)$ for given values of α, β_d, $P^{\oplus}$, and ω_1. The plot between β_d and $\left(\frac{\Omega}{\omega_1}\right)$ will give the regions of dynamic instability.

IV. RESULTS AND DISCUSSION

A steel-alumina functionally graded sandwich (FGSW) rotating cantilever beam of length 1m and width 0.1m is considered for the parametric study. The bottom and top skin of the beam are steel and alumina respectively, whereas the core is the mixture of alumina and steel with bottom layer rich in steel. Both the top and bottom skin are of same thickness. The thickness of the core (d) is 0.3 times of total thickness (h). Figure 3 shows the effect of property distribution laws in the core on the dynamic stability behavior of the FGSW beam. It is observed that exponential distribution of properties along the thickness of the core causes shifting of the instability regions for both the modes (Fig.3(a) and Fig.3(b)) away from dynamic load factor axis. Therefore, it ensures less sensitiveness to parametric instability.

The material properties of the constituent phases of the beam are as follows.

Properties of steel: E =2.1x10^{11} Pa, G =0.8x10^{11} Pa ρ =7.85x10^{3}kg/m^3

Properties of alumina E =3.9x10^{11} Pa, G =1.37x10^{11} Pa, ρ =3.9x10^{3}kg/m^3,

The shear correction factor is chosen as $k=(5+\nu)/(6+\nu)=0.8667$, where ν the poisson's ratio is assumed as 0.3. The additional data for dynamic stability analysis are static load factor α =0.1, Critical buckling load, $P^{\oplus}$ =6.49x10^7 N, and fundamental natural frequency ω_1 =1253.1 rad/s.

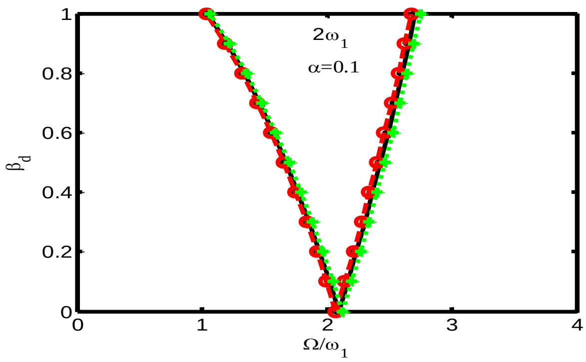

Figure 3(a). Effect of property distribution on first mode instability region of steel-alumina FGSW beam for δ=0.1, s =0.2, ν =1.15 (*n=1.5, °n=2.5, +exp. law)

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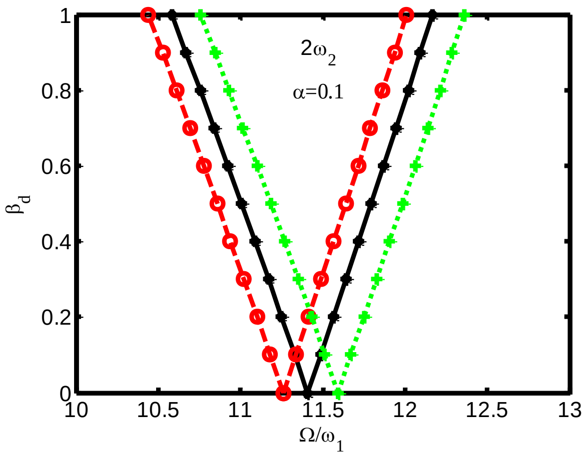

Figure 3(b). Effect of property distribution on second mode instability region of steel-alumina FGSW beam for δ=0.1, s =0.2, ν =1.15 (*n=1.5, °n=2.5, +exp. law)

V. CONCLUSION

Finite element method was used to study the dynamic stability analysis of functionally graded sandwich (FGSW) rotating cantilever beams. The variation of shear or rigid modulus along the thickness of FGSW beam was studied at different power index to optimize the power index at which the variation of rigid modulus is uniform along the thickness of the FGSW beam. The power index also was optimized for which the variation of rigid modulus along the thickness of FGSW beam remains same both by power law and exponential law. The effect of property distribution on parametric instability of the FGSW beams is investigated on both first mode and second mode. Exponential distribution of material properties ensures better dynamic stability compared to power law distribution of properties for FGSW beams on both first mode and second mode.

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