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Haar' s Measure using Triangular Fuzzy Finite Topological Group

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Abstract: *In this paper, A new approach is used to apply Haar's measure theory to triangular fuzzy number theory for comprehending and generalizing the uniqueness of invariant measure when there are uncertainty and risk. If* $\tilde{T}$ *is a triangular fuzzy finite Topological group and* $\tilde{X}$ *is its subgroup,* $\tilde{X}$ *also being a triangular fuzzy number, then* $\frac{\mu(\tilde{x})}{\mu(\tilde{T})}$

Keywords: *Haar's measure, Invariant measure, Topological group, Triangular fuzzy number etc.*

I. INTRODUCTION

This Invariant measure plays an essential role in numerous fields of mathematical sciences, for instance, the uncertainty principle identified with Probability introduced in (R.M Dudley, 2002) does exclude any announcement around an invariant measure, but rather the measure assumes a critical part in demonstrating the probability theorem, as appearing in this paper, the structure of probability additionally offer ascent to probability distributions invariant measure. It is fascinating to perceive the method of generalizing the measure theory to probability distributions.

The general hypothesis of measure and integration was conceived in the mid twentieth century. It is currently a vital tool in significantly various fields of mathematical sciences, including functional analysis, partial differential equations, harmonic analysis, probability theory and dynamical frameworks. Surely, it has turned into a combined theory. Many different topics can agreeably accompany a treatment of this theory. The companionship between integration and functional analysis and, in particular, between integration and weak convergence, has been fostered here: this is important, for instance, in the analysis of nonlinear partial differential equations.

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II. PRELIMINARIES

1. Definition:
An invariant is the one which has no change under a set of transformations.

2. Triangular Fuzzy Number
The fuzzy system of numeration that typically utilized in applications is that the triangular (shaped) fuzzy numbers [8].

2.1 Fuzzy set: [3]

A fuzzy set L must the three axioms,

i . $\tilde{L}$ is a ordinary set.

ii. ${}^{\alpha}\tilde{L}$ is closed interval , for all $\alpha \in [0,1]$

iii. $\tilde{L}$, ${}^{0+}\tilde{L}$ is bounded.

2.2 Triangular Fuzzy Number: [13]

A fuzzy numbers delineated with three points as:
$\tilde{L} = (l_1, l_2, l_3)$
This illustration is taken as membership rule and holds the subsequent axioms

(i) Increasing function is l_1 to l_2

(ii) Decreasing function is l_2 to l_3

(iii) $l_1 \le l_2 \le l_3$

$$\mu_L(x) = \begin{cases} 0, & for\ \ x < l_1 \\ \dfrac{x - l_1}{l_2 - l_1} & for\ \ l_1 < x < l_2 \\ \dfrac{l_3 - x}{l_3 - l_1} & for\ \ l_2 < x < l_3 \\ 0, & for\ x > l_3 \end{cases}$$

2. 3. A Triangular fuzzy number is positive is defined as $\tilde{L} = (l_1, l_2, l_3)$, here $l_i > 0, i = 1,2,3$

2.4 A Triangular fuzzy number is negative is defined as $\tilde{L} = (l_1, l_2, l_3)$, here $l_i < 0, i = 1,2,3$

2.5. Two triangular fuzzy numbers $\tilde{L}$ and $\tilde{M}$ are identically equal, that is $\tilde{L} = \tilde{M}$, if and only if $l_1 = m_1$, $l_2 = m_2$ and $l_3 = m_3$

III. HAAR'S MEASURE

3.1 **Definition of Left Haar's measure:**

A Left Triangular fuzzy Haar's measure $\tilde{\mu}$ on topological group $\tilde{T}$ is Radon measurable and is invariant under left translation

(i.e) $\mu(\tilde{t}\tilde{x}) = \mu(\tilde{x}) \qquad \forall \tilde{t} \in \tilde{T}$

$$\mu(\tilde{t}\tilde{x}) = \mu((t_1t_2t_3)(x_1x_2x_3)) = \mu((t_1x_1, t_2x_2, t_3x_3))$$

$$= \mu(t_1x_1)\mu(t_2x_2)\mu(t_3x_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

3.2 Definition of Right Haar's measure:

A Right Triangular fuzzy Haar's measure μ on topological group $\tilde{T}$ is Radon measurable and is invariant under right translation

(i.e) $\mu(\tilde{x}\tilde{t}) = \mu(\tilde{x}) \qquad \forall \tilde{t} \in \tilde{T}$

where $\tilde{x}\, and\, \tilde{T}$ are triangular fuzzy numbers.

$$\mu(\tilde{x}\tilde{t}) = \mu((x_1x_2x_3)(t_1t_2t_3)) = \mu((x_1t_1, x_2t_2, x_3t_3))$$

$$= \mu(x_1t_1)\mu(x_2t_2)\mu(x_3t_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

3.3 Lemma:

The measure of a subgroup of a Triangular fuzzy invariant finite topological group is invariant

Assumption: Let Triangular fuzzy Topological group be finite

Let Triangular fuzzy invariant finite topological group be $\tilde{T}$

$\mu(\tilde{a}\tilde{T}) = \mu(\tilde{T}\tilde{a}) = \mu(\tilde{T})$ by Triangular fuzzy Haar's measure

Identity:

The identity element of $\tilde{T}$ be $'\tilde{e}'$

Here $\tilde{e}$ is the triangular number so, $\tilde{e} = (e_1e_2e_3)$

$$\mu(\tilde{e}\tilde{x}) = \mu(\tilde{x}\tilde{e}) = \mu(\tilde{x})$$

$$\mu(\tilde{e}\tilde{x}) = \mu((e_1, e_2, e_3)(x_1, x_2, x_3)) = \mu((e_1x_1, e_2x_2, e_3x_3))$$

$$= \mu(e_1x_1)\mu(e_2x_2)\mu(e_3x_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

$$\mu(\tilde{x}\tilde{e}) = \mu((x_1, x_2, x_3)(e_1, e_2, e_3)) = \mu((x_1e_1, x_2e_2, x_3e_3))$$

$$= \mu(x_1e_1)\mu(x_2e_2)\mu(x_3e_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

$$\mu(\tilde{e}\tilde{x}) = \mu(\tilde{x}\tilde{e}) = \mu(\tilde{x})$$

So identity satisfied.

Therefore the triangular fuzzy measurable group of a subgroup is invariant.

3.4 Theorem:

Measure of a subgroup of a triangular fuzzy finite topological group divides the measure of the groups.

Proof:

Consider $\tilde{x}$ as the subgroup of $\tilde{T}$ (Here $\tilde{x} \leq \tilde{T}$)

and let $\tilde{x}\, of\, \tilde{T}$ be finite

If i) $\tilde{x} = \tilde{T}$ it is obviously proved.

ii) $\tilde{x} \neq \tilde{T}$

$$\mu(\tilde{x}) = \tilde{m} \;,\; \mu(\tilde{T}) = \tilde{n}$$

$$\mu(x_1, x_2, x_3) = (m_1, m_2, m_3)$$

$$\mu(t_1, t_2, t_3) = (n_1, n_2, n_3)$$

Every Triangular fuzzy left Haar's measure is equal to right Haar's measure of x in T.

Since $\mu(\tilde{e}\tilde{x}) = \mu(\tilde{x})$

$$\mu(\tilde{e}\tilde{x}) = \mu((e_1e_2e_3)(x_1x_2x_3)) = \mu((e_1x_1, e_2x_2, e_3x_3))$$

$$= \mu(e_1x_1)\mu(e_2x_2)\mu(e_3x_3)$$
$$= \mu(x_1)\mu(x_2)\mu(x_3)$$
$$= \mu(x_1x_2x_3)$$
$$= \mu(\tilde{x})$$

as $\tilde{x}$ is the right Haar's measure of x in T.

Likewise $\mu(\tilde{x}\tilde{a}), \mu(\tilde{x}\tilde{b}), \mu(\tilde{x}\tilde{c}), \mu(\tilde{x}\tilde{d}), \ldots\ldots\ldots$ are right Haar's measures of x in T

So

$$\mu(\tilde{x}\tilde{a}) = \mu(\tilde{x}\tilde{b}) = \mu(\tilde{x}\tilde{c}) = \mu(\tilde{x}\tilde{d}) = \ldots\ldots = \mu(\tilde{x}) = \tilde{m}$$

$$\mu((x_1x_2x_3)(a_1a_2a_3)) = \mu((x_1x_2x_3)(b_1b_2b_3)) = \mu((x_1x_2x_3)(c_1c_2c_3)) = \ldots\ldots = (m_1m_2m_3)$$

$$\Rightarrow \mu(x_1a_1, x_2a_2, x_3a_3) = \mu(x_1b_1, x_2b_2, x_3b_3) = \mu(x_1c_1, x_2c_2, x_3c_3) = \ldots\ldots\ldots = (m_1m_2m_3)$$

$$\mu(x_1a_1) = \mu(x_1b_1) = \mu(x_1c_1) = \ldots\ldots\ldots = m_1$$

$$\mu(x_3a_3) = \mu(x_3b_3) = \mu(x_3c_3) = \ldots\ldots\ldots = m_3$$

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Assume that $\tilde{k}$ be the number of distinct Haar's measure of $\tilde{x}$ in $\tilde{T}$ division of $\tilde{T}$

Always the right Haar's measure of disjoint sets induces a

$$\mu(\tilde{T}) = \mu(\tilde{x}\tilde{a}) + \mu(\tilde{x}\tilde{b}) + \mu(\tilde{x}\tilde{c}) + ... \qquad [k \ \ times]$$

$$\mu(t_1t_2t_3) = \mu((x_1x_2x_3)(a_1a_2a_3)) + \mu((x_1x_2x_3)(b_1b_2b_3)) + \mu((x_1x_2x_3)(c_1c_2c_3)) + ...$$

$$= \mu(x_1a_1, x_2a_2, x_3a_3) + \mu(x_1b_1, x_2b_2, x_3b_3) + \mu(x_1c_1, x_2c_2, x_3c_3) + ...$$

$$= \mu(x_1a_1)\mu(x_2a_2)\mu(x_3a_3) + \mu(x_1b_1)\mu(x_2b_2)\mu(x_3b_3) + \mu(x_1c_1)\mu(x_2c_2)\mu(x_3c_3) + ...$$

$$\mu(t_1)\mu(t_2)\mu(t_3) = [\mu(x_1a_1) + \mu(x_1b_1) + \mu(x_1c_1) + ...],[\mu(x_2a_2) + \mu(x_2b_2) + \mu(x_2c_2) + ...],$$
$$[\mu(x_3a_3) + \mu(x_3b_3) + \mu(x_3c_3) + ...]$$

$$\mu(t_1) = [\mu(x_1a_1) + \mu(x_1b_1) + \mu(x_1c_1) + ... \qquad k\, times]$$

$$\mu(t_2) = [\mu(x_2a_2) + \mu(x_2b_2) + \mu(x_2c_2) + ... \qquad k\, times]$$

$$\mu(t_3) = [\mu(x_3a_3) + \mu(x_3b_3) + \mu(x_3c_3) + ... \qquad k\, times]$$

$$n_1 = m_1 + m_1 + m_1 + ...ktimes \qquad \Rightarrow n_1 = k_1m_1(k \ \ timesof\, m_1)$$

$$n_2 = m_2 + m_2 + m_2 + ...ktimes \qquad \Rightarrow n_2 = k_2m_2(k \ \ timesof\, m_2)$$

$$n_3 = m_3 + m_3 + m_3 + ...ktimes \qquad \Rightarrow n_3 = k_3m_3(k \ \ timesof\, m_3)$$

$\mu(x_1)$ Divides $\mu(t_1)$

$\mu(x_2)$ Divides $\mu(t_2)$

$\mu(x_3)$ Divides $\mu(t_3)$

(i.e) $\mu(\tilde{x}) Divides \mu(\tilde{T})$

Hence triangular fuzzy finite topological group divides the measure of the groups

IV. CONCLUSION

Applying Haar's measure theory to triangular fuzzy measure theory is simple to know and generalize the invariant uniqueness in the real life situations. Hence, Triangular fuzzy measure of a subgroup of a triangular fuzzy finite subgroups divides the measure of the groups

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