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Enumeration of Parallel Flow Heat Exchanger using RK-4 Numerical Method

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***Abstract*: This paper presents the details of theoretical and numerical investigation of parallel flow heat exchanger. Theoretical or Empirical Formulae are used only for simpler geometry object but numerical technique can be used for complex geometry. Runge-Kutta fourth order numerical method is used here for high accuracy. Accuracy is also increased by increasing the number of elements. The numerical results had high degree of consistency and showed a perfect match with the theoretical readings. Using the parallel flow arrangement, the temperature difference between the hot and cold fluid dropped from 70 oC at the entry side to 40 oC at the exit of the heat exchanger**

***Keywords* : parallel flow heat exchanger, numerical method, theoretical method, Runge-Kutta fourth Order.**

I. INTRODUCTION

Heat exchangers which is used to exchange the heat between two fluids. The dividing wall temperature between the fluids and each fluid temperature is also changes when it passes through the heat exchanger changes along the length of the heat exchanger. In heat exchanger, an effect in a system occurs solely as a result of temperature difference between the system and some other system, the process in which the effect occurs shall be called as transfer of heat from the system of the lower temperature and so the distribution of temperature is very principle phenomenon in the heat exchanger [1].

1.1 Mathematical Analysis

A mathematical technique which makes use of the study of the variable which may be independent variable or dependent variable for solving several engineering problems. This analysis helps in determining the systematic arrangement of the variables in the physical relationship which has become the powerful tool for analyzing fluid flow and heat conduction problems. This analysis which is used to plan present experimental results and model tests in a systematic manner, which helps to making it possible to analyze the heat transfer of complex phenomenon in heat exchangers and to acquire precise results [2]. Numerical analysis is the study of algorithm which use numerical approximation for the problems of mathematical analysis.

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1.2 Numerical Analysis

Relatively simple geometric shapes is limited for the scope of analytical solution. The practical 2-Dimensional heat transfer problems involve for irregular geometric which are solved by using numerical analysis technique. The advantage of the numerical methods is which can be applied to any 2-Dimensional shape irrespective of its boundary conditions or complexity. Numerical analysis is concerned with obtaining approximate solutions while maintain reasonable bounds and errors. It is the primary method of solving complex heat transfer problems due to widespread use of digital computers [3].

1.3 Heat Exchangers

An equipment which transfer heat from hot fluid to cold fluid with minimum investment and running cost with maximum rate. The temperature of the dividing wall between the fluids also changes along the length of the exchanger when each fluid passes through the exchanger. According to the relative directions of the two fluid streams the heat exchanger are classified into three categories,

i) Parallel flow
ii) Counter flow
iii) Cross flow

Parallel flow heat exchangers:

As the name suggest, the two fluid streams flows in same direction. Both hot fluid and cold fluid enters at one end and leaves at other end. The variation of temperature of the fluid streams and flow arrangement in case of parallel flow heat exchangers as shown in figure

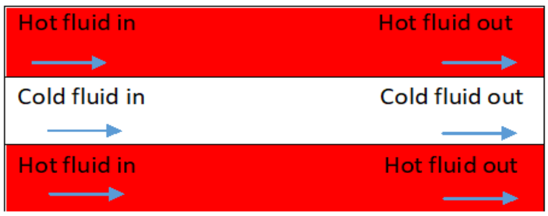

Fig 1. Parallel flow heat exchanger

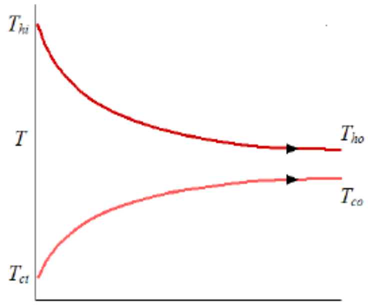

Fig 2.Temperature Distribution of Parallel flow heat exchanger

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II. HEAT EXCHANGER ANALYSIS

The heat exchanger it is necessary for designing that the total heat transfer is related with its governing parameters overall heat transfer coefficient, total surface area and fluid inlet and outlet temperatures.

Assuming that there is no heat loss to the surroundings and potential and kinetic energy changes are negligible, from the energy balance in a heat exchanger.

Heat supplied by the hot fluid,

$$Q = m_h c_{ph}(t_{h1} - t_{h2}) \quad ....(1)$$

Heat received by the cold fluid,

$$Q = m_c c_{pc}(t_{c2} - t_{c1}) \quad ....(2)$$

Total Heat transfer rate in the heat exchanger,

$$Q = UA\theta_m \quad .....(3)$$

2.1 Theoretical Model

Theoretical modeling is the numerical analyzing technique used to find the localization temperature in the parallel flow heat exchanger using the general heat transfer rate equations. Localization temperature are the temperature values of the fluid at particular distance or at exact location in the tubes of the heat exchanger. The theoretical analyzing technique make use of differential calculus, integration and basic mathematical methods to find the required quantities [4]. Following are the assumptions made:

1) There will be no heat loss to the surrounding
2) kinetic and Potential energy changes are negligible
3) The cold fluid is assumed to flow inside the thin walled tube.

Modeling for the following prescribed values

1) Inlet temperature of the cold and hot fluid
2) Mass flow rate of the cold and hot fluid
3) Specific heat of the cold and hot fluid
4) Overall heat transfer coefficient
5) Length and diameter of the thin walled tube.

From equation 1,2 and 3

$$dt_h = -\frac{dQ}{m_h c_{ph}} = -\left[U \frac{dA(t_h - t_c)}{(m_h c_{ph})}\right] \quad ....(4)$$

$$dt_c = \frac{dQ}{m_c c_{pc}} = \left[U \frac{dA(t_h - t_c)}{(m_c c_{pc})}\right] \quad ....(5)$$

$$d(t_h - t_c) = -U\, dA\, (t_h - t_c)\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d\theta}{\theta} = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right] dA$$

$$\ln\theta(x) = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right] A(x) + c \quad ....(6)$$

By applying boundary condition at x=0: $\theta_1 = (t_{h1} - t_{c1})$

$$\ln\theta_1 = c \quad ....(7)$$

$$\ln\theta(x)/\theta_1 = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right](p * x)$$

$$\theta(x) = \theta_1 e^{\left\{-U\left[\frac{1}{C_h}+\frac{1}{C_c}\right](p*x)\right\}} \quad ....(8)$$

Substituting equation 8 in equation 4

$$dt_h(x) = -\frac{U\,p}{C_h}\theta_1 e^{\left\{-U\left[\frac{1}{C_h}+\frac{1}{C_c}\right](p*x)\right\}} dx \quad ....(9)$$

Let $k = -U\left[\frac{1}{C_h} + \frac{1}{C_c}\right] p$

$$dt_h(x) = -\frac{U\,p}{C_h}\theta_1 e^{(kx)} dx$$

Integrating

$$t_h(x) = -\frac{U\,p}{C_h}\theta_1 \int_0^x e^{(kx)} dx$$

$$t_h(x) = -\frac{U\,\theta_1\,p}{C_h\,k}(e^{kx} - 1) + c \quad ......(10)$$

At x=0, $t_h = t_{h1}$

$$t_{h1} = c \quad .....(11)$$

$$t_h(x) = -\frac{U\,\theta_1\,p}{C_h\,k}(e^{kx} - 1) + t_{h1} \quad .....(12)$$

Similarly

$$t_c(x) = -\frac{U\,\theta_1\,p}{C_c\,k}(e^{kx} - 1) + t_{c1} \quad .....(13)$$

Equation 12 is Localization temperature of hot fluid and cold fluid parallel flow heat exchanger

2.2 Numerical model

In numerical method Runge-Kutta methods are an important family of implicit and explicit iterative methods, which is used in temporal discretization for the approximation of solution of ordinary differential equations. Runge_kutta methods is often referred to as "RK4", "The Runge-kutta method" or "classical Runge-Kutta method" [5], [6]... This method is usually used to improve the numerical analysis programming due to its ease.

Let an initial value problem be specified as follows

$$\dot{y} = f(t, y), \quad y(t_0) = y_0$$

Here y which is an unknown function of time (t) which would like to approximate that $\dot{y}$, the rate at which is a function of y and of t itself. At the initial time t_0 the corresponding value of y values is y_0. The function f and the data t_0, y_0 are given by,

$$k_1 = f(t_n, y_n)$$

$$k_2 = f\left(t_n + \frac{1}{2}h, y_n + \frac{h}{2}k_1\right)$$

$$k_3 = f\left(t_n + \frac{1}{2}h, y_n + \frac{h}{2}k_2\right)$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

Now pick a step size (h>0) and define

$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$

$$t_{n+1} = t_n + h$$

For n=0, 1, 2, 3…, etc.

Programming Technique

This problem is proceeded by programming language of C. C programming is a flexible, powerful, and portable programming language. Since C combines the high level language features with the assembler elements which is suitable for both applications programming and system. Now a days It is undoubtedly the widely most used general purpose language [7]-[9].

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III. FLOW CHART

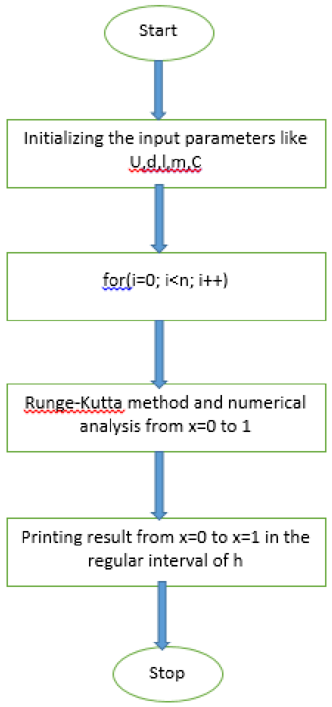

Fig 3. Flowchart of parallel flow Heat Exchanger

IV. RESULT AND DISCUSSION

Case 1: (equal heat capacity)

$m_h = 1.667e^{-02}\ kg/s$, $m_h = 1.667e^{-02}\ kg/s$

$c_{ph} = 4180\ \frac{J}{Kg\ k}$, $c_{pc} = 4180\ \frac{J}{Kg\ k}$

$t_{h(0)} = 90^0\,C$, $t_{c(0)} = 20^0\,C$

Length (l) = 5.0 m,

Diameter (d) = 0.25 m

Overall heat transfer coefficient (U) =100 W/m2k

No of steps = 10

Table1: Temperature relation between Numerical and theoretical values

S.No	X (m)	Numerical value		Theoretical value	
		t_h	t_c	t_h	t_c
1	0	90.00	20.00	90.00	20.00
2	0.2	88.46	21.54	88.46	21.54
3	0.4	86.98	23.02	86.98	23.02
4	0.6	85.57	24.43	85.57	24.43
5	0.8	84.22	25.78	84.22	25.78
6	1	82.93	27.07	82.93	27.07
7	1.2	81.70	28.30	81.70	28.30
8	1.4	80.53	29.47	80.53	29.47
9	1.6	79.40	30.60	79.40	30.60
10	1.8	78.32	31.68	78.32	31.68
11	2	77.30	32.70	77.30	32.70

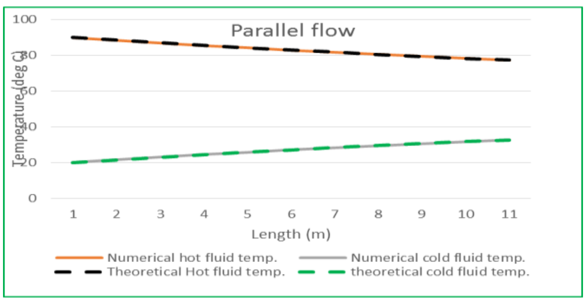

Fig 4.Temperature Distribution of Parallel flow heat exchanger

Heat rejected by the hot fluid = $m_h * c_{ph} * (t_{h1} - t_{h2})$

$= 1.6667e^{-02} * 4180 * (90 - 77.29607)$

= 885.05 W

Heat added by the cold fluid = $m_c * c_{pc} * (t_{c2} - t_{c1})$

$= 1.6667e^{-02} * 4180 * (32.7039 - 20)$

= 885.05 W

Heat given up by the hot fluid in numerical analysis = Heat picked up by the cold fluid in numerical analysis

% of correctness = 885.05/885.05= 100%

NTU method:

$C_{min} = C_{max} = m_h * c_{ph} = m_c * c_{pc} = 68\ W/K$

$C_{min}/C_{max} = 1$

Effectiveness (€)=$m_h * c_{ph} * (t_{h1} - t_{h2})/C_{min} * (t_{h1} - t_{c1})$

$= \frac{68*(90-77.29607)}{68*(90-20)}$

=0.1814

Rate of heat transfer in the heat exchanger is

Q= €*$C_{min} * (t_{h1} - t_{c1})$

= 0.1814*68*(90-20)

= 863.86 W

% of correctness=863.86/885.05 = 97.6%

Case 2: (unequal heat capacity)

$m_h = 3.333e^{-02}\ kg/s$, $m_h = 1.667e^{-02}\ kg/s$

$c_{ph} = 4180\ \frac{J}{Kg\ k}$, $c_{pc} = 2090\ \frac{J}{Kg\ k}$

$t_{h(0)} = 90^0\,C$, $t_{c(0)} = 20^0\,C$

Length (l) = 2.0 m,

Diameter (d) = 0.25 m

Overall heat transfer coefficient (U) =100 W/m2k

No of steps = 10

Table2: Temperature relation between Numerical and theoretical values

S.No	X (m)	Numerical value		Theoretical value	
		t_h	t_c	t_h	t_c
1	0	90.00	20.00	90.00	20.00
2	0.2	89.23	23.07	89.23	23.07
3	0.4	88.51	25.97	88.51	25.97
4	0.6	87.82	28.71	87.82	28.71
5	0.8	87.17	31.30	87.17	31.30
6	1	86.56	33.75	86.56	33.75
7	1.2	85.98	36.07	85.98	36.07
8	1.4	85.44	38.26	85.44	38.26
9	1.6	84.92	40.33	84.92	40.33
10	1.8	84.43	42.28	84.43	42.28
11	2	83.97	44.13	83.97	44.13

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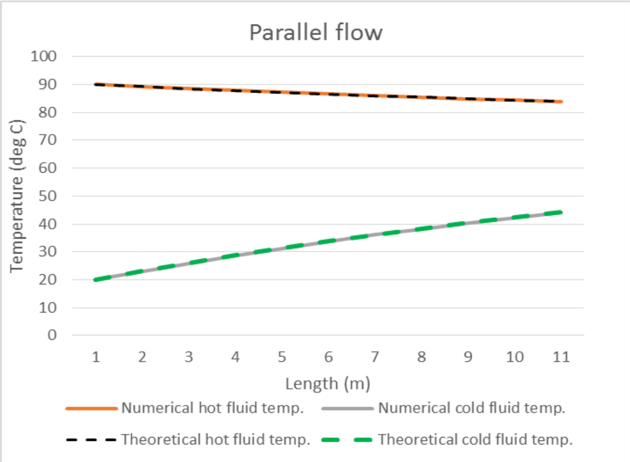

Fig 5.Temperature Distribution of Parallel flow heat exchanger

Heat given up by the hot fluid = $m_h * c_{ph} * (t_{h1} - t_{h2})$

= $3.3333^{e^{-02}} * 4180 * (90 - 83.9669)$

= 840.599 W

Heat picked up by the cold fluid = $m_c * c_{pc} * (t_{c2} - t_{c1})$

= $1.6667^{e^{-02}} * 2090 * (44.12937 - 20)$

=840.523 W

Heat given up by the hot fluid in numerical analysis = Heat picked up by the cold fluid in numerical analysis [3], [10], [11].

% of correctness=840.523/840.599= 100%

NTU method:

$C_{min} = m_c * c_{pc} = 34.834W/K$

$C_{max} = m_h * c_{ph} == 139.33\ W/K$

Effectiveness (€)=$m_h * c_{ph} * (t_{h1} - t_{h2})/C_{min} * (t_{h1} - t_{c1})$

$= \frac{139.33*(90-83.9669)}{34.834*(90-20)}$

=0.3447

Therefore the rate of heat transfer in the heat exchanger

Q= €*$C_{min} * (t_{h1} - t_{c1})$

= 0.3447*34.893*(90-20)

= 840.528 W

% of correctness=840.528/840.599= 99.99%

V. CONCLUSION

The theoretical and numerical modelling with finite and boundary elements which represents an efficiency way to obtain temperature distribution in steady state heat exchanger. The results obtained by theoretical and numerical coincides with each with large percentage.

On the basis of the boundary element method, the numerical computations of the temperature field, has led to close values to the ones determined analytically even if a small number of boundary elements and respectively internal points of the analysis domain was used. Different simulation program could be realized using the present methods, which makes it possible to effectuate a lot of different numerical experiments of practical problems.

Temperature distribution would be used to develop principal component analysis or models based on the proper orthogonal decomposition method. By measuring only in a few spatial locations this allows to build the temperature distributions. Temperature distributions bin fluids can be calculated from heat convection equation, which can be solved using a broad variety of analytical or numerical methods. Existing numerical procedure are generally simpler and more straightforward than analytical and they can be used to solve more complex engineering problems. In most of finite differences schemes and finite element method systems of linear algebraic equation have to be solved for every time step. The process of calculation is sequential and only neighboring fluid is taken into account

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