Matrix Maxima Method to Solve Multi-objective Transportation Problem with a Pareto Optimality Criteria

Khilendra Singh, Sanjeev Rajan

Abstract: ***In this paper we proposed a new method (Matrix Maxima Method) using Geometric mean approach to solve multiobjective transportation problem with a Pareto Optimality Criteria. Fuzzy membership function is used to convert objectives into membership values and then we take Geomertic mean of membership values. We used a different criteria to find Pareto Optimal Solution. This is an easy and fast method to find the Pareto Optimal solution. The method is illustrated by numerical examples. The result is compared with some other available methods in the literature.***

Keywords: Multiobjective transportation problem (MOTP), Fuzzy membership function, Matrix Maxima method, Geometric mean.

I. INTRODUCTION

The transportation problem is to transport the different measures of solitary homogenous items that are first put away at a different starting points to various goals so that the complete transportation cost/ time/ distance is minimum. All things considered, circumstances, all the transportation problems are not having single objective. The transportation problems which are described by multiple objective functions are considered here. A unique kind of linear programming problem where limitations are of correspondence type and every one of the goals are clashing with one another are called the MOTP. Hitchcock was the first person to study the transportation problem in 1947. Li et al. (2000) gave a fuzzy compromised approach to solve MOTP. Wahed et al. (2000) gave a FPA to find compromised solution of MOTP by defining the Fuzzy membership function. Ammar et al. (2005) introduced the concept of alpha-fuzzy efficient in which the ordinary solution is extended based on alpha-level of fuzzy numbers. Lau et al. (2009) gave an algorithm called the fuzzy logic non dominated sorting genetic algorithm to solve the MOTP. Lohgaonkar et al. (2010) used fuzzy programming technique with linear and non linear membership function to find the compromised solution of MOTP. Yeola, M.C. et al. (2016) proposed a parallel method to solve MOTP.

II. RESEARCH METHODOLOGY

A. Mathematical Description:

To $\min f^{K}(x)=\sum_{i=1}^{P}\sum_{j=1}^{q}C_{ij}x_{ij}$

$K=1,2,3,\ldots,m$

subject to the constraints

$$\sum_{j=1}^{q}x_{ij}=a_i,\quad i=1,2,3,\ldots,p$$

$$\sum_{i=1}^{P}x_{ij}=b_j,\quad j=1,2,3,\ldots,q$$

$$x_{ij}\ge 0,\ i=1,2,3,\ldots,P,\ j=1,2,3,\ldots,q.$$

and $\sum_{i=1}^{P}a_i=\sum_{j=1}^{q}b_j$

where

$a_i\,(i=1,2,3,\ldots,p)$ denotes the no. of units available at different sources.

$b_j\,(j=1,2,3,\ldots,q)$ denotes the no. of demands at different destinations.

x_{ij} is the no. of units transported from the i^{th} origin to j^{th} destination.

We have a fuzzy membership function to convert objectives (cost, time) into membership values to minimize a set of objectives. The membership function is defined as

$$\mathrm{M}_r\left(x_{ij}{}^{r}\right)=\begin{cases}1, & x_{ij}{}^{r}\le L_r\\ \dfrac{U_r-x_{ij}{}^{r}}{U_r-L_r}, & L_r\le x_{ij}{}^{r}\le U_r\\ 0, & x_{ij}{}^{r}\ge U_r\end{cases}$$

where

L_r is the lowest crisp value of $x_{ij}{}^{r}$ & U_r is the highest crisp value of $x_{ij}{}^{r}$.

B. Proposed Algorithm:

Step 1: Calculate the membership value for each cell & each objective using the membership function.

Step 2: Make a new Matrix in which each cell is the Geometric mean of the membership values of corresponding calls.

Step 3: Find the maximum membership value in the table and allocate as much as possible (min of a_i & b_j)

Step 4: After making the allocation we remove the row or coloumn or both which are satisfied.

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Step 5: Again we search the maximum membership value in remaining matrix and allocate as much as possible.
Step 6: Repeat these steps until all rows or columns are satisfied. Now a solution is obtained.
Step 7: We find the matrix for the average of objectives (time & cost)
Step 8: We put the above solution (Values of X_{iJ}) at this matrix of average of objectives.
Step 9: Now we use *uv* method to check for optimality.
Step 10: If solution is not optimal then we improve the solution until optimality condition is satisfied.
Step 11: Now we get a Pareto Optimal Solution.

III. RESULT AND DISCUSSION

In section II we gave the mathematical description and developed our proposed algorithm. After developing the algorithm we applied it and the results are shown by two numerical examples given in section III. In numerical example 1, table 1 & 2 gives the data for time and cost. From table 3 to 10 our proposed algorithm is applied and a pareto optimal solution is obtained. In table 11 our solution is compared with the solution obtained by other methods present in the literature. We observe that the cost obtained by our method is minimum and the time is also less than the time obtained by new row maxima method and nearly equal to the time obtained by Product approach. In numerical example 2, table 12 & 13 gives the data for time and cost. Our algorithm is applied from table 14 to 20 and we obtained a pareto optimal solution. Table 21 gives the comparison between our method and other existing methods. Again we see that our method gives the minimum cost and time is also lesser than the time obtained by one of other method.

A. Numerical example 1:

Consider the following transportation problem in which a single homogeneous commodity is to be transported from three origins (O_1, O_2, O_3) to four different destinations (D_1, D_2, D_3, D_4). Cost and time for each unit transported is given in the table. Find the minimum time & cost.

Table 1: Data for time

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand (b_j)	6	10	15	4	**35**

Table 2: Data for cost

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	1	2	3	4	14
O_2	4	3	2	0	16
O_3	0	2	2	1	5
Demand (b_j)	6	10	15	4	**35**

Now Using the membership function we calculate the membership values.

Table 3: Membership values for Time

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	0.375	0.625	1	0.50	14
O_2	0.125	0	0.875	0.25	16
O_3	0.625	0.75	0.375	0.875	5
Demand (b_j)	6	10	15	4	**35**

Table 4: Membership values for Cost

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	0.75	0.50	0.25	0	14
O_2	0	0.25	0.50	1	16
O_3	1	0.50	0.50	0.75	5
Demand (b_j)	6	10	15	4	**35**

Table 5: Geometric Mean between Membership values

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	0.5303	0.5590	0.5	0	14
O_2	0	0	0.6614	0.5	16
O_3	0.7905	0.6123	0.4330	0.81	5
Demand (b_j)	6	10	15	4	**35**

Table 6: Applying Matrix Maxima Method

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	4(0.5303)	10(0.5590)			14
O_2	1(0)		15(0.6614)		16
O_3	1(0.7905)			4(0.81)	5
Demand (b_j)	6	10	15	4	**35**

The solution is
$X_{11} = 4$, $X_{12} = 10$, $X_{21} = 1$, $X_{23} = 15$, $X_{31} = 1$, $X_{34} = 4$.
Min. time = 114 units
Min. Cost = 62 units.
Now we will proceed for Pareto optimal solution

Table 7: Matrix for average of time & cost

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	3.5	3	2	4.5	14
O_2	6	6	2	3.5	16
O_3	2	2.5	4	1.5	5
Demand (b_j)	6	10	15	4	**35**

Table 8: To put our solution on table 7

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)

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O_1	4(3.5)	10(3)			14
O_2	1(6)		15(2)		16
O_3	1(2)			4(1.5)	5
Demand (b_j)	6	10	15	4	**35**

Table 9: To test for optimality of this solution by *uv* method

Destina-tions / Origins	D_1	D_2	D_3	D_4	(u_i)
O_1	3.5	3	–0.5||2.5	3||1.5	3.5
O_2	6	5.5||0.5	2	5.5||–2	6
O_3	2	1.5 ||1	–2||6	1.5	2
(v_j)	0	–0.5	–1	–0.5	

Since $D_{24}<0$, the given solution is not optimum. Now we will improve the solution.

Table 10: Improved Solution

Destina-tions / Origins	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	4 (3.5)	10(3)			14
O_2			15(2)	1(3.5)	16
O_3	2(2)			3(1.5)	5
Demand (b_j)	6	10	15	4	**35**

Now again we apply the test for optimality & it is found that it is an optimal solution.

Now the improved solution (Pareto optimal solution) is given as

$X_{11} = 4$, $X_{12} = 10$, $X_{23} = 15$, $X_{24} = 1$, $X_{31} = 2$, $X_{34} = 3$.

Now values of objectives are as follows:

Min. time = 115 units

Min. Cost = 57 units.

Table 11: Comparison between different methods

Method	Minimum cost	Minimum time
New row Maxima method [10]	83	162
Product Approach [11]	62	114
Our method (Matrix Maxima method)	57	115

B. Numerical example 1

Now we consider one more example with following characteristics:

Table 12: Data for time

Destina-tions / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	13	15	16	17
O_2	7	11	2	12
O_3	19	20	9	16
Demand (b_j)	14	8	23	**45**

Table 13: Data for cost

Destina-tions / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	14	15	10	17
O_2	21	13	19	12
O_3	17	26	9	16
Demand (b_j)	14	8	23	**45**

Table 14: Membership Values for time

Destina-tions / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	0.389	0.278	0.222	17
O_2	0.722	0.50	1	12
O_3	0.056	0	0.611	16
Demand (b_j)	14	8	23	**45**

Table 15: Membership Values for cost

Destina-tions / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	0.706	0.647	0.941	17
O_2	0.294	0.765	0.412	12
O_3	0.529	0	1	16
Demand (b_j)	14	8	23	**45**

Table 16: Geometric Mean between membership values

Destina-tions / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	0.524	0.424	0.457	17
O_2	0.461	0.618	0.642	12
O_3	0.172	0	0.782	16
Demand (b_j)	14	8	23	**45**

Table 17: Applying Proposed method (Matrix maxima method)

Destina-tions / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	14(0.524)	3(0.424)		17
O_2		5(0.618)	7(0.642)	12
O_3			16(0.782)	16
Demand (b_j)	14	8	23	**45**

The solution is

$X_{11} = 14$, $X_{12} = 3$, $X_{22} = 5$, $X_{23} = 7$, $X_{33} = 16$.

Min. time = 440 units

Min. Cost = 583 units.

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Table 18: Matrix for average of time and cost

Destinations / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	13.5	15	13	17
O_2	14	12	10.5	12
O_3	18	23	9	16
Demand (b_j)	14	8	23	**45**

Table 19: Solution for average of time & cost

Destinations / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	14(13.5)	3(15)		17
O_2		5(12)	7(10.5)	12
O_3			16(9)	16
Demand (b_j)	14	8	23	**45**

Now we apply *uv* method to test for optimality for table 8 & improve the solution.

Table 20: Optimal solution for table 8

Destinations / Origins	D_1	D_2	D_3	Supply (a_i)
O_1	14(13.5)		3(13)	17
O_2		8(12)	4(10.5)	12
O_3			16(9)	16
Demand (b_j)	14	8	23	**45**

This is an optimal solution with respect to the average of time & cost.

Now the Pareto optimal solution for our problem is

$X_{11} = 14$, $X_{13} = 3$, $X_{22} = 8$, $X_{23} = 4$, $X_{33} = 16$.

Now we calculate the values of objectives

Min. time = 470 units

Min. Cost = 550 units.

Table 21: Comparison between different approaches

Method	Minimum cost	Minimum time
New row Maxima method [10]	652	656
Product Approach [11]	583	440
Our method (Matrix Maxima method)	550	470

IV. CONCLUSION:

In this paper we solved the MOTP by new method named as Matrix maxima method. We also gave the criteria to get Pareto optimal solution. We took the Geometric mean between membership values to solve the problem. Our method gives the better values for some of the objectives as compared to other methods.

REFERENCES:

1. Hitchcock F.L. (1941). The distribution of a product from several sources to numerous localties.Journal of Mathematics and Physics, 20, 224-230.
2. Lushu Li, K.K. Lai (2000). A fuzzy approach to multiobjective transportation problem. Computers and Operations Research 27,43-57.
3. Waiel F. Abd El-Wahed (2001). A multiobjective transportation problem under fuzziness.Elseveir, Fuzzy sets and systems,vol. 117, page 27-33.
4. E.E. Ammar, E.A. Youness. Study on multi-objective programming problem with fuzzy numbers. Applied Mathematics and Computation. vol 166,241-253.
5. Lau et al (2009). A fuzzy guided multiobjective evolutionary algorithm model for solving transportation problem.Expert systems with applications.
6. Lohgaonkar M.H. and Bajaj V.H.(2010) .Fuzzy approach to solve multiobjective capacitated transportation problem. International journal of Bioinformatics Research vol 2 p.p. 10-14.
7. Yeola M.C. and Jahav V.A. (2016). Solving multiobjective transportation problem using fuzzy programming technique-parallel method. . International journal of recent scientific research vol 7, pp. 8455-8457, Jan 2016.
8. K. Bharathi and C. Vijayalakshmi. Optimization of Multi-objective Transportation Problem Using Evolutionary Algorithms, Global Journal of Pure and Applied Mathematics. ISSN 0973-1768 Volume 12, Number 2 (2016), pp. 1387-13.
9. V.Vidhya & K. Ganeshan. Different approach for the solution of multi objective fuzzy transportation problem.International Journal of pure and applied Mathematics vol 119,no.9 2018,373-383.
10. Maulik Mukesh bhai Patel & Dr. Achyut C. Patel, solving Multi-objective Transportation Problem by row maxima method. Research Review international journal of multidisciplinary ,vol 3,issue 1,Jan 2018.
11. M. Afwat A.E., A.A.M. Salama, N.Farouk. A new efficient approach to solve multiobjective transportation problem in the fuzzy environment (Product Approach). International Journal of Applied Engineering Research. (ISSN 0973-4562), Vol. 13, Pp 13660-13664.

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