Special Pythagorean Triangles with Sum of their Two Legs as Undecic

Mita Darbari, Prashans Darbari

Abstract: ***Some Special Pythagorean Triangles, where the sum of two legs is undecic, are found. An application of such few triangles is realized in cryptography. Various interesting results are seen.***

Keywords: Encryption, Mathematica, Pythagorean triangles, Undecic. Subject Classification Code: 11-04, 11D41, 11T71, 11Z05

I. INTRODUCTION

Even after more than two thousand and five hundred years, Pythagoras theorem remains one of the most important theorems in the world today. It is still fascinating the young and old alike towards its beauty and mystic. Darbari and Darbari [2] have obtained special Pythagorean triangles with two consecutive sides and sum of legs to be a square. Darbari and Rana [3] gave Pythagorean triangles with sum of its two legs a decic. It is natural to ponder on the existence of Pythagorean triangles with its sum of two legs to be undecic. Cryptography is in use since the dawn of civilization. In the modern world, with advanced computer technologies, new methods are sought after again and again to make our messages secure. In this direction, an effort is made to find special Pythagorean triangles with sum of its two legs as the eleventh power and to find their application in cryptography.

II. PROPOSED METHODOLOGY

A. Method of Analysis

In the Pythagorean mathematics, primitive solutions of the Pythagorean Equation

$$X^2 + Y^2 = Z^2 \qquad (1)$$

is given by [1] as

$$X = m^2 - n^2,\ Y = 2mn,\ Z = m^2 + n^2 \qquad (2)$$

where $m, n \in I$ such that $m > n > 0$ and $(m, n) = 1$ with one of them is odd and other is even.

Sum of two legs is a power of eleven, that is, undecic:

If X and Y are two legs of a right angled triangle and Z is the hypotenuse, then

$$X + Y = \beta^{11} \qquad (3)$$

$$\Rightarrow m^2 - n^2 + 2mn = \beta^{11} \qquad (4)$$

B. Algorithm

Solving equation (4) using the software *Mathematica*, by the command

Reduce [m^2+2 m n- n^2 – β^{11}==0, {m, n, β}]

the eleven solutions are given by

$\beta = (m^2-n^2+2mn)^{1/11}$, $\beta = -(-1)^{1/11}(m^2-n^2+2mn)^{1/11}$,
$\beta = (-1)^{2/11}\, m^2-n^2+2mn)^{1/11}$, $\beta = -(-1)^{3/11}(m^2-n^2+2mn)^{1/11}$,
$\beta = (-1)^{4/11}(m^2-n^2+2mn)^{1/11}$, $\beta = -(-1)^{5/11}(m^2-n^2+2mn)^{1/11}$,
$\beta = (-1)^{6/11}(m^2-n^2+2mn)^{1/11}$, $\beta = -(-1)^{7/11}(m^2-n^2+2mn)^{1/11}$,
$\beta = (-1)^{8/11}(m^2-n^2+2mn)^{1/11}$, $\beta = -(-1)^{9/11}(m^2-n^2+2mn)^{1/11}$,
$\beta = (-1)^{10/11}(m^2-n^2+2mn)^{1/11}$ (5)

Seeking the integral solutions of (5), using *Mathematica*, by the following command:

FindInstance[m^2-n^2 +2mn - β^{11}==0&&n<m&&0<m<10^{15}&&0<n<10^{15}&&0< β<10^{11} &&GCD[m,n]==1, {m, n, β}, Integers, 10000] (6)

we get only 71 solutions. They are as follows:

Table I- Values of m, n, β

S.N.	M	n	β
1	36122	10977	7
2	5847127	7102	17
3	25267076	7265951	23
4	115030594	82540623	31
5	549402765	318409768	41
6	1197770154	567573781	47
7	1425293413	1034011446	49
8	11909181220	4661084743	71
9	17078425399	658360154	73
10	26971463650	383385639	79
11	37381583095	32955248764	89
12	72796132771	14097411644	97
13	83381410906	75470616881	103
14	142813090797	93492240238	113
15	211287880148	64596368831	119
16	211645039728	63927374635	119
17	349313598982	24636557097	127
18	508018933237	64096768282	137
19	846326669600	137825933643	151
20	992455663399	699334063528	161
21	1357860339189	14896011230	161
22	1248231429998	701832818903	167
23	2955446295876	690970326995	191
24	2633020812647	2472463785016	193
25	4106585130110	318875919109	199
26	5061183535139	4066047018592	217
27	5774468358473	1718841553668	217
28	5896783404748	4583474883339	223
29	7969657439067	3828187912336	233
30	9457594503362	3664931303255	239
31	11256711620671	1554263686392	241
32	15080053797729	3600940004896	257
33	14678424856392	10869238132213	263

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34	17233562360626	13358880510285	271
35	27715170621699	1758655436960	281
36	23340710700666	22158661563559	287
37	28411930956804	5470803488291	287
38	34188944592533	83153468716	289
39	36486477291096	31004606634557	311
40	41569375875373	16445165114912	313
41	49496110896825	46110337885814	329
42	63331442069667	7353977136824	329
43	66847435016921	16149839177458	337
44	62834792307128	49972890081859	343
45	95395395736893	8244421365092	353
46	80041396333160	73133589792581	359
47	112264514978846	17718642669939	367
48	114439162037064	101027270008649	383
49	147791405074654	42767590614533	391
50	148047093054154	42305491499025	391
51	151261231332279	99793571473438	401
52	176533581734797	82971167181200	409
53	267255905686074	49272041839013	431
54	266824533787439	61701554227944	433
55	243921617875078	196119243958901	439
56	311074816938915	101285681589164	449
57	304619112260881	241362910974654	457
58	323736745415068	320398682001699	463
59	540177758592270	11962319408059	479
60	539372951902166	74180232373759	487
61	481348171252751	401424239786408	497
62	640472881308809	37640620834706	497
63	533121113858202	316455422593483	503
64	624133701673136	225704799908527	511
65	657242633817796	163688590122035	511
66	691784251495945	256595202535372	521
67	673184695507984	484614859627801	527
68	673646183056068	481808558061533	527
69	691219173524177	472766760127354	529
70	978471634124603	318331260856614	553
71	997752097909913	282217100434092	553

III. RESULT AND OBSERVATIONS

Using equation (2), X, Y and Z are obtained which satisfy equations (1) and (3) which are as follows:

Table II-: Values of X, Y, Z satisfying $X^2 + Y^2 = Z^2$ and $X + Y = \beta^{11}$

S.N.	X
1	1184304355
2	34188843715725
3	585631085655375
4	6419083110764707
5	200458617831831401
6	1112513344934547755
7	962281642682177653
8	120102886349373312351
9	291239176016833405485
10	727312866875079884179
11	311334333591266251329
12	5100539931352673355705
13	1256645672276533572675
14	11654779918272259798565
15	40469877431285024195343
16	40706913613743056930759
17	121413030488163770368915
18	253974840823063062066645
19	697272843691662924908551
20	495900111402164448426417
21	1065512397145520780156595
22	1843562809591901843064821
23	819721431616623209026353
24	8257222815019591708877351
25	16762359579052956117258219
26	9082840418961213254086857
27	30390068536390039106637505
28	13763812516311955529154583
29	48860417003920510796853593
30	76014372332444722655708019
31	124297820904112682068512577
32	97315818689993360954980295
33	214441253623540431141586625
34	118535983149512767684330651
35	53782493723837330056097075
36	765037813643938365635605001
37	777308129886700485139793735
38	1168877017851931958608667433
39	369977392550048160574294967
40	1457569555011324155863121385
41	323701733966864602492638029
42	1450921381146350477358712503
43	3956790574894657027664683913
44	4207762262883718605540978477
45	1058103171012491259112944039
46	9032311044153237721725124985
47	12289371025370554845333227995
48	2889812522344920485574934895
49	20013232606968172954722411627
50	20128187150811257010000705091
51	12921203196733043702967793997
52	24279890896705613022407191209
53	21034997815784644904636378283
54	67388250036740262400123711585
55	68997985017098233429628719307
56	34536748760448468867172476445
57	86508752438597963342359158329
58	2150164903514712998162558023
59	70554641680272116695263525537
60	184074087552590069792288601515
61	285420474368631394433763701475
62	291648913792148300787949605419
63	338600220862462917698216764767
64	408788695355185637500646892045
65	405173925171613908086333756391
66	218326072094103865883555648655
67	221659693324675861377417310535
68	254275536366131238243704486013
69	412724352653636304713440264641
70	856071947147909329397062362613
71	915862757106206246568227803105

S.N.	Y
1	793022388
2	83052591908
3	367178672258552
4	18989393785640124
5	349870413884417040
6	1359645870149464548
7	2947539405900810396
8	11101940577232825292O
9	22487509551526302892
10	20680943652441044700
11	246383874017572408916O
12	2052474099528130771048
13	12585693034967922208372
14	26703831587848861779372

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15	27296859671120661733976
16	27059823488662628998560
17	17211768852157208150508
18	65124743693121634377668
19	233291526809181570705600
20	1388116103912399300823344
21	1752099566317638111304388
22	40453405722661906184940
23	13020097208926211643794704
24	4084251386955202479945240
25	2618982215526357200543980
26	41158020447197699636608576
27	19850792329768873784057928
28	54055517256305381037387144
29	61018692547389941715261024
30	69322868297727638278086620
31	34991796200391546417218064
32	319086590339838109668310992
33	108604737992452417307362368
34	460442200684295656144076820
35	1034397818137995899965260588
36	97482871000250019405190080
37	310872181975132744881563928
38	5685858669216500365395256
39	2262497751782256717164008944
40	1367230499988697084271724352
41	4564564794972641915970281100
42	6280072338561094046778381904
43	931475954044849490798235216
44	2159150649897697097903533636
45	11707429291709440916744571920
46	1572959677489293888653478312
47	3978349650848373495090220788
48	23122952245363998095477133072
49	12641364617158834397204693164
50	12526410073315750341926399700
51	30189797000236047635601010404
52	29294394646427753216888432800
53	95675446565784498455968338556
54	32926976881663087019699990832
55	26336488333375100908848009924
56	147047511347602307130453420348
57	63014849717764849246227834120
58	207449653093014717741672401064
59	386449647435426598980044816816
60	337418134759011360174302595132
61	80021621816446020625027323988
62	12923557770820241335066207860
63	281739944504606888319306461344
64	48215593760513078174661450308
65	215166240195455897931189469720
66	652470613434371249117727726368
67	649136992203799253623866064488
68	653570898409864937163036075316
69	355017040246765455495554133080
70	622956218006632740767585348484
71	563165408048335823596419907992

S.N.	Z
1	1425293413
2	34188944592533
3	691219173524177
4	20044992001220965
5	403228178547459049
6	1756793338691819677
7	3100640983600199485
8	163554308312228064449
9	292106052201583212917
10	727606835971462760821
11	2483431175785513306721
12	5498013961473446921177
13	12648273697073897908997
14	29136377887712072391853
15	48815259163586200808465
16	48880332069190339697209
17	122626950379351252503733
18	262191632231455864529693
19	735264819660792203411449
20	1474036376223333928039985
21	2050651008524183168403413
22	1844006591893030415290621
23	13045875768047913341266865
24	9212102800594746160337401
25	16965723282628175139965981
26	42148317133762997241411785
27	36298901109621687355145953
28	55780296528710874848732425
29	78170462388231434838807385
30	102877815247601508202898069
31	129129292117786380720467905
32	333596493839499650691535033
33	240374791461261240149528257
34	475455360125657611665893101
35	1035795058300531303249590037
36	771223551535836302703688201
37	837167511501694432687995097
38	1168890846850650964190044745
39	2292548657677280044067467465
40	1998456466324761384192656873
41	4576028253854732409097523221
42	6445500867413473700800504265
43	4064952534352517261298297865
44	4729396873799233185571662005
45	11755147082911506928318227161
46	9168252011443608595237461913
47	12917271621500520954321755437
48	23302831093145859918644545297
49	23671366220912755251904027805
50	23707696372759410093152606341
51	32838717011581400819646873685
52	38048320063526892929674071209
53	97960513517806497671390633885
54	75002413625028090603944645857
55	73853453231069128579646347645
56	151048858348765973499633315877
57	107026331028560993879323596121
58	207460795760366390441040331225
59	392837482256463291706186610465
60	384362156530229891356036744093
61	296425888118681160041507281637
62	291935107963389150836257900381
63	440485534265959337033351384225
64	411622328028829844324995104917
65	458761834243893056438718838841
66	688029196438250401052059839857
67	685938666567343093382645930713
68	701292355328761390088302568645
69	544406548581973459227262621409
70	1058741530425032586679197452605
71	1075155740661058988437899532033

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We observe that

(1)The values 119, 161, 217, 287, 329, 391, 497, 511, 527, 553 of β are repeated once.

(2)Except for repeated values, which are multiple of 7, rest other values of β are prime numbers.

(3)$X + Y + Z = 0 \pmod 2$

(4) $(Y + Z - X)^2 = 2(Y + Z)(Z - X)$

(5)$(X + 2Y + Z)^2 = (Z - X)^2 + 4(X + Y)(Y + Z)$

(6)$X + 2Y + Z \pm 2\{(X + Y)(Z - Y)\}^{1/2} = 0(\text{mod}16)$ or $= 0(\text{mod}4)$

IV. APPLICATION IN CRYPTOGRAPHY

In this modern world where super computers solve the problems in seconds, sending the coded messages over internet has become unsafe. If the receiver has pre knowledge that which Pythagorean Triangles are used for encrypting the message, only he can decipher it. Here is a very simple procedure for encryption. For simplicity sake, we take the values of β only and arrange them in ascending order. We leave the first and second digit numbers. We are left with 59 values of β out of which ten are repeated. We take only first thirty four. In this paper, we are not using full stop or comma but a gap between two words is assigned a value. First three distinct values of β are assigned for first three gaps. If there are more gaps, then these values can be repeated. The next twenty six distinct values of β are assigned for twenty six English alphabets. The five repeated values of β can be used for consecutive repeated alphabets in the same word in the ascending order. To end the message, we can again use the value of next gap. Let us take a look at the following table:

Table III- Codes for Encryption

S.No.	β	Code
1	103	Gap 1
2	113	Gap 2
3	119	A
4	119	For repeat alphabet
5	127	Gap 3
6	137	B
7	151	C
8	161	D
9	161	For repeat alphabet
10	167	E
11	191	F
12	193	G
13	199	H
14	217	I
15	217	For repeat alphabet
16	223	J
17	233	K
18	239	L
19	241	M
20	257	N
21	263	O
22	271	P
23	281	Q
24	287	R
25	287	For repeat alphabet
26	289	S
27	311	T
28	313	U
29	329	V
30	329	For repeat alphabet
31	343	W
32	337	X
33	353	Y
34	359	Z

For example, let us encrypt the message *THE THREE COMMITTEE MEMBERS WERES NOT VERY HAPPY*, word by word:

THE- 311 199 167

GAP 1- 103

THREE- 311 199 287 167 119

GAP 2- 113

COMMITTEE-151 263 241 161 217 311 217 167 287

GAP 3-127

MEMBERS-241 167 241 137 167 287 289

GAP 4 (Repeat GAP 1)-103

WERE- 343 167 287 167

GAP 5 (Repeat GAP 2)-113

NOT- 257 263 311

GAP 6 (Repeat GAP 3) - 127

VERY- 329 167 287 353

GAP 7 (Repeat GAP 1)-103

HAPPY-199 119 271 329 353

GAP 8 to end (Repeat GAP 2) - 113

Hence, the encrypted message is a long string of numbers-
311199167103311199287167119113151263241161217311
217167287127241167241137167287289103343167287167
113257263311127329167287353103199119271329353113

V. CONCLUSION

Thus, the ancient Pythagorean equation is explored to find special Pythagorean triangles with sum of their two legs as the eleventh power. These triangles can be applied in cryptography very effectively. In conclusion, one may attempt to find other patterns of Pythagorean Triangle which satisfy the conditions presented in the above problem and explore their applications.

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