



Feature Extraction and Analysis in Multimodal Biometric Authentication using Lu Factorization with Kronecker Algebra

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Abstract: Pattern recognition is one of the current and advanced technologies that focus on analysis and construction of pattern is a complex work. For recognition of patterns Vector logic gives good strategies. This paper focuses on pattern recognition in multimodal authentication system by using vector logic. A framework has been proposed to provide more security in biometric aspect. Initially, features are extracted through PCA from the normalized biometric imaginaries, and then using LU factorization key components are extracted. By using convolution kernel methods such as Khatri Rao an application of Kronecker product weights are computed for different key sizes. In the same way verification process is implemented and verified with MSE. This framework gives better result for chosen threshold value.

Keywords: Khatri-Rao Product, Kronecker Product, LU, MSE, PCA.

I. INTRODUCTION

The mathematical study of characteristics of biometric patterns such as fingerprint, palm, iris, vein, face and so on has been accepted throughout the World wide and used in various domain such as Government offices, educational institutions, airports, health care organizations, government agencies to distribute the various schemes to the beneficiaries and also in private organization to aid the identification of a person. But, successful implementation of biometric systems in these areas does not imply ideal security system and also proper identification is a crucial part of admission control that makes important to any security system. Identification of person generally based on the user knows such as PIN or password and user has smart card.

Biometric is scientific procedure that mainly focuses on methods of verification and or validation depends on the behavioral or physiological characteristics. These characteristics are not duplicate and convenient. Generally, biometric systems are unimodal i.e., the person can be recognized or authorized, but these are not up to the mark due to lack of uniqueness, noisy sensor data. To overcome the inherent problems of unimodal system, the multimodal

authentication[7, 8] focuses on two or more traits as combination either in fusion based or non fusion based from the same person in order to identify. These types of systems provide accuracy due to availability of computing technologies.

As described in [1], the authentication in biometric is layered model and consists of acquisition, extraction of features and storage in the database. After enrolment, verification process takes place and focuses on the acquisition, feature Extraction, comparison, and decision steps. In these methods, feature extraction plays a major role. The extracted features can be fusioned by using mathematical methods such as PCA, LDA and its variants. In the next step, the features can be obtained using statistical techniques such as LU, SVD to produce key components. After this, we can use Kronecker calculus or variants it such as Khatri Rao product[4,5,6] and so on can be used to calculate weights thus to make system more complex. In the decision strategy, the error rate will be computed using Man Square Error (MSE). Finally, acceptance or rejection depends on the threshold value.

In the process of fusioning the patterns, extraction of features, generating keys and computation for matching is done with vector logic. The advantage of vector logic is the system is complex and less error prone, in turn produces effective outcome.

II. VECTOR LOGIC

Vector logic is a mathematical representation of matrix algebra [2] stimulated in the areas of image processing. In this, the image is represented as matrix to compute eigen values and eigen vectors as feature vectors. In matrix algebra, Kronecker product is one of the key product[3], which computes in the areas of image segmentation, analysis and security models. The data elements can be represented as monadic, dyadic and so on. In monadic, the data is represented as one dimensional, where as in dyadic representation data is stored in the form of rows and columns [16].

A. Convolution Kernel Products

In the present research the convolution kernel products [16,17] such as Kronecker Product and application of it, Khatri Rao are used in the areas of image processing for exactness and effectiveness.

Manuscript published on 30 September 2019.

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B. Kronecker Product

The Kronecker product of two matrices denoted by $U \otimes V$, and The Kroecker product has several properties [5, 6] and are

1. $U \otimes (V \otimes W) = (U \otimes V) \otimes W$ associativity,
2. $U \otimes (V + W) = (U \otimes V) + (U \otimes W)$ distributivity
 $(U + V) \otimes W = (U \otimes W) + (V \otimes W)$,
3. $a \otimes U = U \otimes a = aU$, for scalar a .
4. $aU \otimes bV = ab U \otimes V$, for scalars a and b .
5. For conforming matrices, $(U \otimes V)(W \otimes X) = UW \otimes VX$.
6. $(U \otimes V)^T = U^T \otimes V^T$, $(U \otimes V)^H = U^H \otimes V^H$
7. $(U \oplus V)(W \oplus X) = UW \oplus VX$

C. Khatri Rao Product

The KhatriRao product is a column-wise Kronecker product. Originally introduced by Khatri and Rao (1968) [8].

Given matrices $X \in R^{I \times K}$ and $Y \in R^{J \times K}$, Khatri-Rao product is denoted by $X \otimes Y$. The result is a matrix of size $(IJ) \times K(IJ) \times K$ and defined by

$$X \otimes Y = [x_1 \otimes y_1, x_2 \otimes y_2, \dots, x_k \otimes y_k]$$

D. LU Factorization

The matrices can be factorized into various ways with the use of Kronecker Product [3,4], some of the factorization methods are Cholesky, LU, Schur, QR, SVD and many more. Among these methods LU factorization is chosen because it generates triangular system. The computation of LU factorization can be described for the matrices X and Y : $X = P_X^T L_X U_X$ and $Y = P_Y^T L_Y U_Y$. Then

$$X \otimes Y = (P_X^T L_X U_X) \otimes (P_Y^T L_Y U_Y) = (P_X^T \otimes P_Y^T) (L_X \otimes L_Y) (U_X \otimes U_Y)$$

Let A, B are 3×3 matrices, for simple notation, the lower triangular system of LU factorization can be represented as

$$L_A \otimes L_B = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}b_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{11}b_{21} & a_{11}b_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{11}b_{31} & a_{11}b_{32} & a_{11}b_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21}b_{11} & 0 & 0 & a_{22}b_{11} & 0 & 0 & 0 & 0 & 0 \\ a_{21}b_{21} & a_{21}b_{22} & 0 & a_{22}b_{21} & a_{22}b_{22} & 0 & 0 & 0 & 0 \\ a_{21}b_{31} & a_{21}b_{32} & a_{21}b_{33} & a_{22}b_{31} & a_{22}b_{32} & a_{22}b_{33} & 0 & 0 & 0 \\ a_{31}b_{11} & 0 & 0 & a_{32}b_{11} & 0 & 0 & a_{33}b_{11} & 0 & 0 \\ a_{31}b_{21} & a_{31}b_{22} & 0 & a_{32}b_{21} & a_{32}b_{22} & 0 & a_{33}b_{21} & a_{33}b_{22} & 0 \\ a_{31}b_{31} & a_{31}b_{32} & a_{31}b_{33} & a_{32}b_{31} & a_{32}b_{32} & a_{32}b_{33} & a_{33}b_{31} & a_{33}b_{32} & a_{33}b_{33} \end{pmatrix}$$

E. Principal Component Analysis (PCA)

In principal component analysis we find the directions in the data with variation, i.e., the eigen vectors resultant to the largest eigen values of the covariance matrix, and project the data onto these directions. i.e. we might discard important non second order information by PCA. If we denote the

matrix of eigen vectors sorted according to eigen value by \tilde{U} , then PCA transformation of the data as $Y = \tilde{U}^T X$. The eigen vectors are called as principle components [11]. By selecting the first d rows of Y , we have projected the data from n down to d dimensions.

III. PROPOSED FRAME WORK

We showed the efficiency of multimodal biometric authentication system by conducting study into three levels such as fusion, encoding and decoding. In first stage, normalized patterns are fused through Principal Component Analysis (PCA), in stage two, the generated keys will be passed as inputs to the convolution kernel product for computation to increase the complexity, finally encoding and decoding process with Khatri Rao product is done. An overview of the computational process of the proposed work is showed in Fig. 1. Experiments performed on different data sets of Yale and AT&T, FERET.

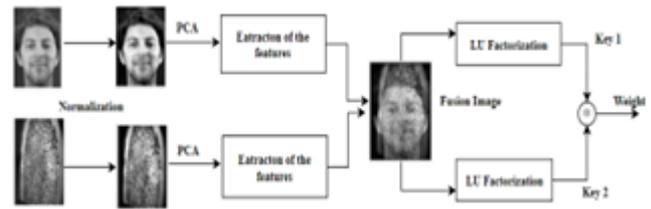


Fig 1. Proposed frame work

In this paper, the associative memory is computed, the keys which are generated from the fusion patterns by using LU factorization is represented by,

$$\text{key1} = LU(X) = P_X^T L_X U_X$$

$$\text{key2} = LU(Y) = P_Y^T L_Y U_Y$$

Where X and Y are fusion patterns of two pair biometric images (face and finger print). The fusion is done by using PCA. The importance of this factorization is here to extract the best features by eliminating noise and lighting effect using Gaussian-distribution models.

IV. DECISION STRATEGY

In this paper, Mean Square Error (MSE) is considered as decision strategy for recognition of chosen patterns. The Mean Square Error (MSE) of an estimator \hat{X} of a parameter X is the function of X is defined by $E(\hat{X} - X)^2$ and it is denoted as $MSE_{\hat{X}}$.

$$MSE(\hat{X}) = E[(\hat{X} - X)^2]$$

The Mean Square Error is equal to the sum of the variance and the squared bias of the estimator or of the predictions. In the case of the MSE of an estimator,

$$MSE(\hat{X}) = Var(\hat{X}) + (Bias(\hat{X}, X))^2$$

IV. EXPERIMENTAL ANALYSIS AND RESULTS

By considering standard datasets Yale and AT&T, FERET experiments made using the proposed framework. Mean square Error (MSE) is considered for verification of the biometric patterns through the framework. Based on the error rate of MSE, the acceptance rate will be determined.

From the observations of datasets for both similar and dissimilar patterns of different poses using the proposed framework, the threshold of MSE is restricted as 0.10. In this paper, we are presenting three kinds of patterns with various key sizes 8x8, 16X16...64X64. The experimental results obtained presented in the Table 1 and executed on Core i7 7th generation processor with 16 GB of RAM. From the observations of MSE the False Acceptance Rate (FAR) and False Non Acceptance Rate (FNAR) have been measured for the chosen key sizes of the patterns.

Table I: MSE of various sizes

Key Size	Mean Square Error			
	Similar	Similar with Different Poses	Dissimilar	Dissimilar
8X8	0.000000	0.04075	0.156252	0.148052
16X16	0.000000	0.006621	0.100864	0.212347
24X24	0.000000	0.008528	0.151144	0.204937
32X32	0.000000	0.093009	0.143208	0.218194
40X40	0.000000	0.014700	0.153279	0.151572
48X48	0.000000	0.086422	0.136317	0.132662
56X56	0.000000	0.076474	0.139456	0.12681
64X64	0.000000	0.091097	0.154058	0.134501

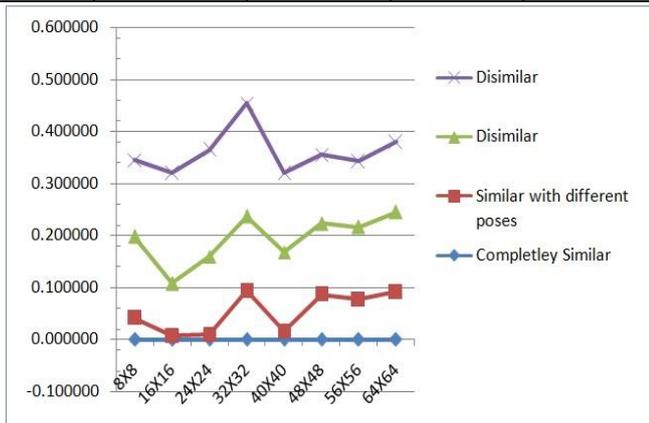


Figure 2. Graphical Representation of results

V. CONCLUSION

The Convolution kernel products such as Kronecker Product and its variants like Khatri Rao playing an important part in many image analysis algorithms and removal of noisy with the use of LU factorization since Gaussian transformation eliminates noise. In future, these products can applied with deep and machine learning algorithms to obtain good results.

ACKNOWLEDGMENT

I gratefully acknowledge the funding agency, the University Grant Commission (UGC) of the Government of India, for providing financial support through UGC-SERO (MRP-6521/16).

REFERENCES

1. Vaclav Matyas, ZdenekRiha, "Biometric Authentication —Security and Usability", <http://www.fi.muni.cz/usr/matyas/>

2. A. K. Jain, A. Ross, S. Prabhakar, S. An Introduction to Biometric Recognition. IEEE Trans. Circuits Syst. Video Technol., 14, 4-20, 2004.

3. H. V. Henderson, F. Pukelsheim and S. R. Searle. On the history of the Kronecker product. Linear and Multilinear Algebra. 14:113-120, 1983.

4. John W. Brewer, Kronecker Products and Matrix Calculus in System Analysis, IEEE Transactions on Circuits and Systems, Vol. 25, No. 9, 1978.

5. Lester Lipsky and Appie van deLiefvoort, Transformations of the Kronecker Product of Identical Servers to Reduced Product Space, 1995.

6. Wolfgang Hackbusch, Boris N. Khoromskij, Hierarchical Tensor-Product Approximations, 1-26.

7. V. Ghattis, A.G. Bors and I. Pitas, " Multimodal decision level fusion for person authentication," IEEE Trans. Systems, Man and Cybernetics, vol. 29, no. 6, pp. 674-680, Nov. 1999.

8. <https://stat.ethz.ch/R-manual/R-devel/library/Matrix/html/KhatriRao.html>

9. A.K. Jain and Ross, "Learning user specific parameters in multibiometric system," Proc. Int. Conf. Image Processing (ICIP), pp. 57-60,2002.

10. A. K.Jain, L.Hong and Y. Kulkarni, "A multimodal biometric system using fingerprint, face and speech," Proc. Second Int. Conf. AVBPA, pp.182-187, 1999.

11. S.Ribaric, I. Fratic and K. Kris, "A biometric verification system based on the fusion of palmprint and face features," Proc. Fourth Int. Symposium Image and Signal Processing, pp. 12-17, 2005.

12. B.Ducet. al., "Fusion of audio and video information for multimodal person authentication," Pattern Recognition Letters, vol. 18, pp.835-845, 1997.

13. A. Haq, A.M. Mirza and S. Qamar, An Optimized Image Fusion Algorithm For Night-Time Surveillance And Navigation, Proceedings of the International Conference on Emerging Technologies, Islamabad, 2005, pp138-143

14. G. Pajares, J.M. de La Cruz, A Wavelet Based Image Fusion Tutorial, Pattern Recognition, 37, 2004,pp.1855-1872,

15. C. Pohl, and J.L. VanGenderen, Multisensory Image Fusion in Remote Sensing : Concepts, Methods and Applications, International Journal of Remote Sensing Vol. 19, No. 5, 2008, pp. 823-854

16. Y. Suresh, K. Pavan Kumar, PESN Krishna Prasad, "Pattern Recognition Using Context Dependent Memory Model (CDMM) In Multimodal Authentication System", International Journal in Foundations of Computer Science & Technology (IJFCST), Vol.5, No.1, PP.47-57, January 2015.

17. Y. Suresh, K. Pavan Kumar, PESN Krishna Prasad, "Feature Selection in Multimodal Authentication Using LU Factorization with CSEAM Model", International Journal of Computer Science and Application (IJCSA), Vol.5, No.1, February 2015.

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