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Abstract: Based on the requirements in industry, there is a need to design a low-cost robotic platform which is to be operated remotely through the gripper and control data. This should facilitate in avoiding human involvement in continuous and heavy environments.

Kinematic analysis is one of the most important tasks in design of a manipulator. It involves many tasks like mathematical modeling of the manipulator using forward or inverse kinematics. Forward kinematics involves defining the target point or reach point using the joint angles (if it is rotary joint) and coordinates of the joint if it is prismatic joint. Inverse kinematics involves finding out the different possible ways of the joints to reach the given target point. Hence inverse kinematics yields more than one solution for a given target position of an end effectors, thereby it becomes more tedious task to calculate all the possible solutions manually. In this paper Matlab software is used for solving the forward kinematic analysis to calculate the possible joint link parameters to reach the target point using generalized programming.

Keywords: Forward Kinematics, Inverse Kinematics

I. INTRODUCTION

Robotics, automation and remote handling technology plays a vital role in almost all facts of pick and place task. The recent advancements in this fascinating area have been due to various necessities unique to industry such as starting from reducing the manpower during operation, technologies requirement to facilitate remote pick and drop at inaccessible areas of industrial plants or to facilitate remote repair/refurbishments of operating plants. Remote handling/robotic tool design is essential in the areas of pick and place tasks. Advancements in this technology, by way of mathematical modeling, control/automation, advance control and various modules coupled with experimental works are facilitating applications in pick and place of objects or material.

With the tenacious need for increased quality, productivity and automation, the world is tuning more and more towards various autonomous and semi-autonomous robots which finds a wide array of application in various fields such as inspection, surveillances, quality check, fault detection, surgical, rehabilitation, agriculture, planetary space exploration etc. A common attribute of such described applications is that robot needs to operate in inhuman, unstructured environment where human intervention is risky. Motion control, trajectories planning for robots in unstructured environments face significant challenges due to various uncertainties in internal as well as external environment. So a complete study and analysis of mathematical modeling and control is essentially needed. Thus mathemtical modeling of a robotic manipulator involves tedious calculations which makes the design complicated. In this paper the complete kinematic analysis is carried out using the generalized algorithm in Matlab.

II. PROBLEM STATEMENT

Kinematics is the study dealing with motion of system without accounting forces and inertia. It defines the position, velocity, acceleration and higher derivatives of the variables. The kinematic studies of robot manipulator are divided into two types: the first one is called forward kinematics and the second one is known as inverse kinematics. Forward kinematics determines the position and orientation (pose) of end-effector when all the joint angles are provided. On the other hand, inverse kinematics calculates the solutions of each joint variable corresponding to a specified end-effector pose in Cartesian space. Hence, forward kinematics is defined as transformation from joint space to Cartesian space where as inverse deals with transformation from Cartesian space to joint space. In industrial serial robots, inverse kinematics is a multi-solution problem.

As it becomes tedious task for calculating each and every particular position of a manipulator having any number of axis, a generalized program is required to solve same problem for “n” set of parameters. Hence in this paper a typical 5 axis articulated arm robot is designed and manufactured using rotary joints equipped with servo motors for the kinematic study of linkages, whose analytical solutions of kinematic study are compared with the results obtained using generalized algorithm written in Matlab and the same is applied for different positions of the end effectors for the consistency of algorithm.
III. METHODOLOGY

There are three revolute joints at the first three axes. The first revolute joint swings robot back and forth about vertical base, while second joint itches the arm up and down about horizontal shoulder axis and third joint pitches the forearm up and down about horizontal elbow axis.

![3D view of a 5 Axis Manipulator](image)

Given a set of rigid bodies connected by joints, the post of this kinematic model is specified by the orientation of the joints. Consider a robot having n links numbered from zero, starting from the base of the robot to the end-effector and the base is taken as link 0, all the joints are numbered from 1 to n.

By considering, di as the distance between Zi-1 and Zi in the direction of Xi-1, αi: as the angle between Zi-1 and Zi in the direction of Xi-1, ai (link length) as the distance between Xi-1 and Xi in the direction of Zi
θi as the angle between Xi-1 and Xi in the direction of Zi.

The resultant transformation between any two joints is given by the following link transformation matrix:

\[
\begin{bmatrix}
\cos \theta & -\sin \theta \cos \alpha & \sin \theta \sin \alpha & a_i \cos \theta \\
\sin \theta & \cos \theta \cos \alpha & -\cos \theta \sin \alpha & a_i \sin \theta \\
0 & \sin \alpha & \cos \alpha & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Joint-1 represents the shoulder and its axis of motion is z1. This joint is assigned with a rotational angle θ1 (angular motion) around z1 axis in x1y1 plane. Similarly joint 2 is the elbow and its axis is perpendicular to joint 1. It has a rotational angular motion of θ2 around z2 axis. Joint 3 facilitates the tool pitching motion denoted as θ3 and joint 4 facilitates tool roll motion along z4 axis which is perpendicular to joint 3. Joint 5 is at a vertical offset with joint 4 and provides an angular motion of θ5 which is also identical to gripper rotation. The gripper sliding motion while picking is not considered as a degree of freedom of the manipulator.

IV. SOLUTION

All the parameters for the above manipulator are listed in Table 1 below, where θi is angular rotation about z axis, ai is the link length, di is the link offset along z axis and αi is link twist along the rotation about x-axis.

![Link Coordinate frame assignment of the 5 axis manipulator](image)

<table>
<thead>
<tr>
<th>S.No</th>
<th>ai</th>
<th>q_i</th>
<th>C0_i</th>
<th>S0_i</th>
<th>Cα_i</th>
<th>Sα_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-90</td>
<td>L_1</td>
<td>θ_1</td>
<td>θ_1</td>
<td>S_1</td>
</tr>
<tr>
<td>2</td>
<td>L_2</td>
<td>0</td>
<td>0</td>
<td>θ_2</td>
<td>θ_2</td>
<td>C_2</td>
</tr>
<tr>
<td>3</td>
<td>L_3</td>
<td>0</td>
<td>θ_3</td>
<td>θ_3</td>
<td>C_3</td>
<td>S_3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-90</td>
<td>θ_4-90</td>
<td>θ_4</td>
<td>θ_4</td>
<td>C_4</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>L_5</td>
<td>θ_5</td>
<td>θ_5</td>
<td>C_5</td>
</tr>
</tbody>
</table>
Substituting the link parameter values in eq(1) and finding out each and every individual transformation matrix and finally multiplying them gives the Overall transformation matrix from link 5 to link 0

\[ T_0 = [ C1, 0, -S1, 0 ] \\
| S1, 0, C1, 0 | \\
| 0, 0, 1, L_1 | \\
| 0, 0, 0, 1 | \\
\]

\[ T_1 = [ C2, -S2, 0, L_2C2 ] \\
| S2, C2, 0, L_2S2 | \\
| 0, 0, 1, 0 | \\
| 0, 0, 0, 1 | \\
\]

\[ T_2 = [ C3, -S3, 0, L_3C3 ] \\
| S3, C3, 0, L_3S3 | \\
| 0, 0, 1, 0 | \\
| 0, 0, 0, 1 | \\
\]

\[ T_3 = [ S4, 0, C4, 0 ] \\
| -C4, 0, S4, 0 | \\
| 0, -1, 0, 0 | \\
| 0, 0, 0, 1 | \\
\]

\[ T_4 = [ C5, -S5, 0, 0 ] \\
| S5, C5, 0, 0 | \\
| 0, 0, 1, L_5 | \\
| 0, 0, 0, 1 | \\
\]

\[ T_5 = T_1 \times T_2 \times T_3 \times T_4 \times T_5 \]

\[ \theta_j \]

\[ \text{Position-2} \]

In starting position the joint-link parameters of 5-DOF manipulator are mentioned below:

\[ \theta_1 = 20; \quad L_1 = 8 \text{cm} \]
\[ \theta_2 = 75; \quad L_2 = 9 \text{cm} \]
\[ \theta_3 = 55; \quad L_3 = \text{cm} \]
\[ \theta_5 = 45; \quad L_5 = 18 \]

Substituting the above values in the eq (2)

Gives the solution for the rest position as

\[ T_3 = [-0.8869, 0.33, -0.108, -10.97] \\
| -0.3137, -0.94, -0.038, 3.88 | \\
| -0.1064, 0, 0.99, 2.41 | (4) \\
| 0, 0, 0, 1 | \\
\]

\[ \text{Position-3} \]

Final position

\[ \theta_1 = 160; \quad L_1 = 8 \]
\[ \theta_2 = 75; \quad L_2 = 9 \]
\[ \theta_3 = 55; \quad L_3 = 9 \]
\[ \theta_5 = 45; \quad L_5 = 18 \]

Substituting the above values in the eq (2)

Gives the solution for the rest position as

\[ T_3 = [-0.0994, 0.33, 0.8843, 18.65] \\
| 0.0365, 0.94, -0.324, 6.843 | (5) \\
| -0.99, 0, -0.096, -9.35 | \\
| 0, 0, 0, 1 | \\
\]

V. RESULTS

The above analytical results are validated with the generalized Matlab Code as follows

Position-1

First Position (Rest position)

% syms o1 o2 o3 o4 o5 a1 a2 a3 a4 a5;
\[ a_1 = 90; \]
\[ a_2 = 90; \]
\[ a_3 = 90; \]
\[ a_4 = 90; \]
\[ a_5 = 0; \]
\[ \alpha_1 = a_1 \times (\phi / 180); \]
\[ \alpha_2 = a_2 \times (\phi / 180); \]
\[ \alpha_3 = a_3 \times (\phi / 180); \]
\[ \alpha_4 = a_4 \times (\phi / 180); \]
\[ \alpha_5 = a_5; \]
\[ L_1 = 8 \]
\[ L_2 = 9 \]
\[ L_3 = 9 \]
\[ L_4 = 18 \]

\[ T_5 \] represents the overall homogeneous transformation matrix from link 5 to link 0

\[ T_3 \]

\[ T_0 = \begin{bmatrix} 0.0911 & 0.3420 & -0.8837 & -18.7618 \\ 0.0332 & -0.9397 & -0.3216 & -6.8287 \\ -0.9962 & 0.0872 & -9.1565 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \]

**Position-3**

Final position

\[ \% \text{ sym} \ 0 \ 2 \ 3 \ 4 \ 5 \ a1 \ a2 \ a3 \ a4 \ a5; \]

\[ a1=160; \]
\[ a2=75; \]
\[ a3=55; \]
\[ a4=45; \]
\[ a5=0; \]

\[ \text{Overall transformation matrix result} \]

\[ T_0 = \begin{bmatrix} 0.0911 & 0.3420 & -0.8837 & -18.7618 \\ 0.0332 & -0.9397 & -0.3216 & -6.8287 \\ -0.9962 & 0.0872 & -9.1565 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \]

**Position-2**

Starting position

\[ \% \text{ sym} \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ a1 \ a2 \ a3 \ a4 \ a5; \]

\[ a1=20; \]
\[ a2=75; \]
\[ a3=55; \]
\[ a4=45; \]
\[ a5=0; \]

\[ \text{Overall transformation matrix result} \]

\[ T_0 = \begin{bmatrix} 0.0911 & 0.3420 & -0.8837 & -18.7618 \\ 0.0332 & -0.9397 & -0.3216 & -6.8287 \\ -0.9962 & 0.0872 & -9.1565 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \]

**A. ABBREVIATIONS**

**D-H CONVENTIONS- DENAVIT & HARTENBERG CONVENTIONS**

**VI. CONCLUSION**

Form the above it is evident that equation-3 is equal to equation-6, equation-4 is equal to equation -7 and equation 5 is equal to equation 8. Hence it can be concluded that using the above Matlab code which was written for forward kinematics of a 5- axis robot, it makes it easy to calculate the final position and orientation of the end effector in homogeneous coordinates by giving the input as joint angles to the final transformation matrix. Hence for any given joint link parameters for any axis robot we can find out the infinite number of positions and orientations of the end effector within the reach of the manipulator’s end effector.
REFERENCES


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G Krishna Teja, Assistant Professor, Mechanical Engineering Department, Anurag Group of Institutions, Hyderabad, India. gkrishnatejamech@csvr.ac.in. Has done M.E Robotics from University college of Engineering, Osmania University is an academician from 7+ years by sharing the knowledge with several graduates and postgraduates to become qualified engineers in the field of Robotics. Major area of research interests in mathematical modeling and optimal design of robotics systems and sub systems for obtaining consistency and accuracy for accomplishing the robotic tasks.

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