On-grid Adaptive Compressive Sensing Framework for Underdetermined DOA Estimation by Employing Singular Value Decomposition

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Abstract: In the field of Array Signal Processing, the problem of Direction of Arrival (DOA) estimation has attracted colossal attention of researchers in the past few years. The problem refers to estimating the angle of arrival of the incoming signals at the receiver end, from the knowledge of the received signal itself. Generally, an array of antenna/sensors is employed at the receiver for this purpose. In over-determined DOA estimation, the number of signal sources, whose direction needs to be estimated are usually lesser than half the number of antenna array elements, whereas the challenge is to estimate the DOAs in under-determined case, where the signal source number is quiet larger than the number of antenna array elements. This paper tackles such a problem by the application of multiple level nested array. Instead of subspace-based techniques for the estimation, sparse signal representation for Compressive Sensing (CS) framework is used, which eliminates the requirement of prior information about the source number and also the tedious task of computing the inverse of the covariance matrices. In this paper, we propose an adaptive approach for Least Absolute Shrinkage and Selection Operator (LASSO) with reduced number of computations by singular value decomposing of the received signal vector. The outcomes of this paper showcase that the presented algorithm achieves high degree of freedom (DOF), good resolution, minimum root mean square error and less computational complexity with increased speed of estimation.

Index Terms: Nested Arrays, LASSO, On-grid estimation, Singular Value Decomposition, Sparse Signal Representation, Underdetermined-Direction of Arrival estimation.

I. INTRODUCTION

In the recent years, direction of arrival estimation has become the most addressed hot-topic of the researchers, playing a key role in several applications such as RADAR, SONAR, Seismology, wireless communication etc. There are several algorithms designed for this purpose since from the last decade including conventional beamforming, Capon beamforming [2], subspace-based estimation like MUSIC [1], ESPRIT [3] and its improved versions like I-MUSIC, R-MUSIC [4] etc. These algorithms provide good resolution and accuracy but requires the prior information about the source number. They also suffer from yielding sharp spatial spectral peaks at the actual DOAs. These algorithms exhibit better results for the overdetermined case of DOA estimation. In the recent years, after the emerge of sparse signal

representation theory, the insight of solving DOA estimation problem has been and being changed with respect to compressive sensing (CS) framework. The underlying spatial sparsity enables one to link CS framework to angle of arrival estimation problem. The application of sparse signal reconstruction technique for DOA estimation has wider advantages such as smaller number of snapshots, lesser sensitivity over SNR, ability to deal with highly correlated or coherent sources etc, over the aforementioned traditional estimation methods. In [5], a single snapshot l1-norm minimization and its dimension reduction algorithm l1-SVD for multiple snapshot algorithm was proposed. Several applications involving DOA estimation based on CS framework has been investigated in [6]-[9]. Representing the array covariance vectors in sparse domain with error suppression criteria for DOA estimation is proposed in [10]. In [11]-[13], it has been proved that estimating DOA in underdetermined case is very difficult. For such cases, special array structures have been proposed in [14]-[17]. In recent years, nested array structures and co-prime array structures are the two sparse arrays that are proposed in [18] and [19]. By considering the nested or co-prime array structures, a virtual linear array with more number of array elements can be modelled and used for underdetermined-DOA estimation which enhances the degrees of freedom (DOFs). Spatial smoothing based sub-space approaches [20]-[22] were proposed for difference co-array structures for the underdetermined case. In [23], the underdetermined DOA estimation problem is addressed by employing signal reconstruction methods of CS framework. Furthermore in [24] and [25], super-nested arrays and higher order difference co-array structures are proposed to increase the DOFs with limited number of physical antenna array elements.

For both the classes of sparse array structures aforementioned, atleast two uniform linear sub-arrays are required to optimize the virtual array locations in correspondence to the difference co-array. The major problem faced in on-grid DOA estimation is grid mismatch. This problem is where the actual DOAs of the signal sources are not found in the defined values of finite search grid. One simple solution for this problem is to define a thicker search grid having a step size of very less value, but the algorithm's computational complexity increases which leads to reduced speed of estimation. The alternative solution proposed in [5] is to adapt and reframe the search grid only in and about the angle spaces where the actual signal sources are localized. Use of off-grid based DOA estimation methods can also eliminate the problem of grid mismatch [23].

This paper proposes an adaptive SVD-Least Absolute Shrinkage & Selection Operator algorithm using nested array structure for underdetermined-Direction of Arrival estimation in the following sections.

II. SIGNAL MODEL FOR UNDERDETERMINED-DOA ESTIMATION

Assuming the narrowband signal case and the signal is emitted from the point source in the far-field, let us consider a uniform linear array (ULA) shown in Fig 1, which contains 'M' number of array elements, each spaced uniformly at a distance of 'd' between each other. Consider 'D' number of signal sources $\mathbf{x}=[x_1(n), x_2(n)....x_D(n)]^T$ that are impinging on the ULA each with the direction of arrival $\theta_1, \theta_2.... \theta_D$ respectively. Where n=1,2....K are the number of snapshots taken. Initially, let us model the problem for a single snapshot case with K=1.

Let $\mathbf{y}=[y_1(n), y_2(n)....y_M(n)]^T$ be the received signal vector from the array elements, $\mathbf{w}=[w_1(n), w_2(n)....w_M(n)]^T$ be the noise vector corresponding to each array element. Representing the array received signal $\mathbf{y}$ in matrix form gives (1).

$$\mathbf{y} = \mathbf{Ax} + \mathbf{w} \qquad (1)$$

Where $\mathbf{A}=[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2).... \mathbf{a}(\theta_D)]$ is the array manifold matrix, each of its columns $\mathbf{a}(\theta_i)$ referred to as 'atoms' corresponding to i[th] signal source and given by (2).

$$\mathbf{a}(\theta_i) =[1, e^{-j\beta d\sin\theta_i}, e^{-j2\beta d\sin\theta_i}..... e^{-j(M-1)\beta d\sin\theta_i}]^T \qquad (2)$$

where $\beta=2\pi/\lambda$ and λ stands for wavelength of the received signal, note that for a ULA $d \leq \lambda/2$ always. Let us assume the array element noise $\mathbf{w}(n)$ as white in spatial and temporal domains and uncorrelated with the source signals. By assuming the signal sources are also to be temporally uncorrelated with each other, the autocorrelation matrix $\mathbf{R_{xx}}$ of the signal source vector $\mathbf{x}$ is diagonal. The received signal vector's ($\mathbf{y}$'s) autocorrelation matrix is given by (3).

$$\mathbf{R_{yy}} = E[\mathbf{yy}^H]$$

$$\mathbf{R_{yy}} = \mathbf{AR_{xx}A}^H + \sigma_n^2\mathbf{I} \qquad (3)$$

Where $\mathbf{R_{xx}} = diag(\sigma_1^2, \sigma_2^2..... \sigma_D^2)$. Now vectorizing the equation (3), we get equation (4).

$$\mathbf{z} = vec(\mathbf{R_{yy}}) = vec(\sum_{i=1}^{D} \sigma_i^2\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)) + vec(\sigma_n^2\mathbf{I})$$
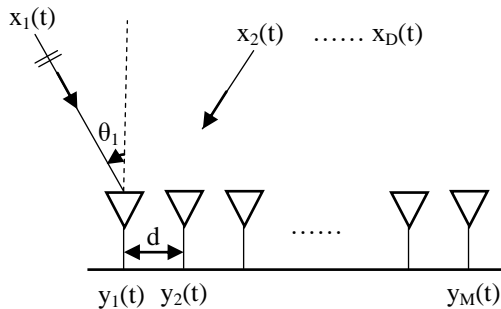

Fig 1. Uniform Linear Array

Where $\mathbf{p} = [\sigma_1^2, \sigma_2^2..... \sigma_D^2]^T$, $\mathbf{i_v}$ is the vectorized form of the identity matrix and σ_n^2 is the noise variance. By comparing equation (4) with equation (1), $\mathbf{z}$ can be treated as the signal vector received by an array with manifold ($\mathbf{A}^*\Theta\mathbf{A}$), where Θ denotes the KR product. The vector $\mathbf{p}$ represents the signal source vector and $\sigma_n^2\mathbf{i_v}$ represents the deterministic noise vector. The unique rows of ($\mathbf{A}^*\Theta\mathbf{A}$) denotes the manifold of virtual longer array with sensors located at positions given by unique values in the set {(d_i-d_j) for $1\leq i,j \leq M$ }, where d_i and d_j denotes the positions of the ith and j[th] array elements in the original array respectively.

This virtual array formed by vectorizing the autocorrelation matrix of received signal vector $\mathbf{y}$ is named as difference virtual co-array of the original physical array [24].

III. NESTED ARRAYS

This section, elaborates the concept of the difference virtual co-array, its properties and the degrees of freedom (DOF) offered by the co-array. Later we discuss the specific nested array structure for the original array which yields a uniform linear difference virtual co-array.

A. Difference Co-array

Consider an array with M sensors, where the ith sensor in the array located at d_i position. The difference virtual co-array of the given array has sensor positions given by the distinct values of the set { (d_i-d_j) for $1\leq i,j \leq M$ }. By cardinality of the distinct position set of difference virtual co-array, the degrees of freedom (DOF) can be obtained. As mentioned in [25], for an array of any geometry with M sensors, the maximum DOF that can be obtained from a difference virtual co-array is given by (5)

$$DOF_{max} = M(M-1)+1 \qquad (5)$$

The DOF obtained is directly proportional to the source number that can be resolved using the smaller number of physical sensors. The key behind increasing the DOF of a difference virtual co-array is to use a non-uniform original physical array [25]. One such non-uniform array structure proposed by [25] can be easily obtained in a systematic method is called the "Nested Arrays" for which it is possible to obtain $O(M^2)$ degrees of freedom for the difference virtual co-array obtained from an $O(M)$ original physical array which is a nested array.
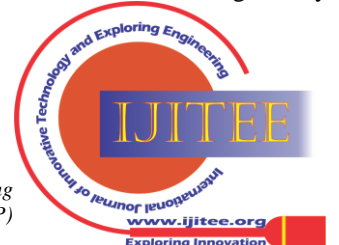
B. Level-2 Nested Array

Consider a level-2 nested array with M number of sensor elements, which is a combination of two ULA's: inner and outer ULA. The inner ULA consists of M_1 array elements each element spaced at a distance of d_1 with its adjacent element. The array locations of inner ULA are given by (6). The outer ULA consists of M_2 array elements with an element spacing of d_2 and such that $d_2 = (M_1+1)d_1$. The array locations of outer ULA are given by (7).

$$Loc_{inner} = \{m\ d_1, \quad m = 1, 2....M_1\} \qquad (6)$$
$$Loc_{outer} = \{m\ d_2, \quad m = 1, 2....M_2\}$$
$$Loc_{outer} = \{m(M_1+1)d_1, \quad m = 1, 2....M_2\} \qquad (7)$$

The difference virtual co-array for this original level-2 nested array will be a filled ULA with number of virtual array elements given in (8) whose locations are given by the set in (9).
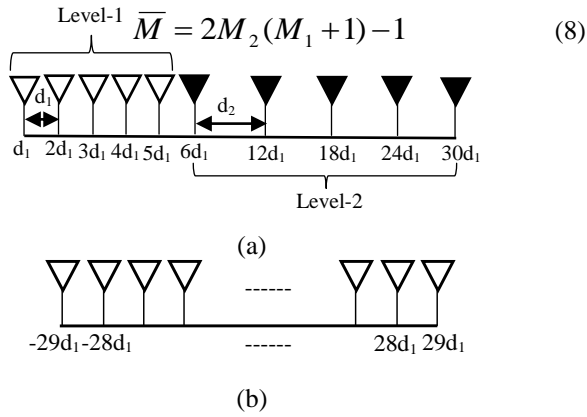
$$\text{Level-1} \quad \overline{M} = 2M_2(M_1+1)-1 \qquad (8)$$

Fig 2. (a) 2-Level nested array with M=10 (b) Difference co-array

$$L_{cr} = \{md_1, m = -M_2(M_1+1)+1...M_2(M_1+1)-1\} \quad (9)$$

Thus, we can achieve a DOF of $2M_2(M_1+1)-1$ by considering this level-2 nested array as the original array having only $M=M_1+M_2$ physical array elements. A level-2 nested array with M=10 and its difference virtual co-array is shown in Fig 2. Further the DOF can be increased by increasing the number of levels of the nested array, but the resultant difference virtual co-array will no longer be a filled ULA. The number of nested levels and the number of sensor elements in each level can be estimated for a given M number of elements by maximizing the degree of freedom. This can be casted as an N-level nested array optimization problem.

IV. COMPRESSIVE SENSING FRAMEWORK

The compressive sensing (CS) framework can be applied to DOA estimation by considering the problem of estimating the direction of arrival as a problem of sparse signal reconstruction. For the sparse signal reconstruction, firstly, the approach should be discrete and secondly, the signal to be estimated must be preferably sparse in nature. In DOA estimation, the search range of angle of arrival (AOA) is actually continuous. Thus, it is necessary to discretize the continuous AOA range into a finite set. This technique is defined as DOA estimation by on-grid approach. The signal sources become sparse in spatial domain by assuming them as far-field signal sources.

A. Least Absolute Shrinkage & Selection Operator (LASSO) for Multiple Snapshot

Let us consider a finite grid of length N and the possible DOAs in the search grid is given by the set $\mathbf{\Omega} = \{\theta_n\}_{n=1 \text{ to } N}$ and manifold of the physical array is $\mathbf{A}=[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2)\dots \mathbf{a}(\theta_N)]$. Considering a multiple snapshot signal model for underdetermined DOA estimation with K number of snapshots and re-representing the equation (4) as:

$$\mathbf{Z} = \tilde{\mathbf{A}}\mathbf{P} + \mathbf{N_w} \qquad (10)$$

Where, $\mathbf{Z} \in \mathbb{C}^{MsxK}$ represents the received signal from the virtual array, $\tilde{\mathbf{A}}=(\mathbf{A}^*\Theta\mathbf{A}) \in \mathbb{C}^{MsxN}$ array manifold of the virtual difference co-array, $\mathbf{P} \in \mathbb{C}^{NxK}$ is the received signal power as a function of the source location and $\mathbf{N_w} \in \mathbb{C}^{MsxK}$ is the sensor noise matrix with $M_s=M^2$ number of virtual elements of the difference virtual co-array.

The methodology of DOA estimation involves estimating the signal power $\mathbf{P}$ as a function of all the possible DOA points, the angle points for which the signal power shows peaks are the actual signal source locations. As the sources are assumed to be point sources the spatial spectrum is sparse. Therefore, one such sparse solution for the problem in (10) is given by ℓ_1 minimization as in (11).

$$\min_{\mathbf{P}} \left\|\mathbf{P}\right\|_{2,1} \quad st \quad \left\|\mathbf{Z} - \tilde{\mathbf{A}}\mathbf{P}\right\|_2^2 \leq \varepsilon \qquad (11)$$

Re-representing (11) using LASSO minimization [26] gives (12).

$$\min_{\mathbf{P}} \left\{\left\|\mathbf{Z} - \tilde{\mathbf{A}}\mathbf{P}\right\|_2^2 + \tau\left\|\mathbf{P}\right\|_{2,1}\right\} \qquad (12)$$

$$\hat{\mathbf{P}} = \arg\min \left\{\left\|\mathbf{Z} - \tilde{\mathbf{A}}\mathbf{P}\right\|_2^2 + \tau\left\|\mathbf{P}\right\|_{2,1}\right\} \qquad (13)$$

Where, $\left\|\mathbf{P}\right\|_{2,1} = \left\|\tilde{\mathbf{p}}\right\|_1$ & $\tilde{\mathbf{p}}=[\left\|\mathbf{p}_1\right\|_2, \left\|\mathbf{p}_2\right\|_2 \dots \left\|\mathbf{p}_N\right\|_2]^T$ $\mathbf{p}_1$ is the first row of $\mathbf{P}$ matrix and so on, τ is the non-negative regularization parameter. As the regularization parameter τ increases, the estimated coefficients $\mathbf{P}$ are shrinked to zero by LASSO algorithm. Thus the regularization parameter optimizes between the accurate fitness of the solution to the measurements (first term: ℓ_2-norm) and the sparsity of the solution (second term: ℓ_1-penalty). For large coefficients, the ℓ_1- norm based LASSO produces biased estimates which degrades the estimation accuracy. An adaptable algorithm for LASSO (A-LASSO), where the coefficients of ℓ_1-norm are penalized by adaptable weights iteratively was proposed by Zou in [27].

B. Adaptive-LASSO for Multiple Snapshot

The regularization parameter in (13) penalizes the coefficients of ℓ_1-norm equally, which results in biased estimation and reduced estimation accuracy. This deficiency is overcome by applying Adaptive-LASSO algorithm, which converts (12) to (14).

$$\min_{\mathbf{P}} \left\{\left\|\mathbf{Z} - \tilde{\mathbf{A}}\mathbf{P}\right\|_2^2 + \tau\sum_{n=1}^{N}\hat{\mathbf{w}}_n^{(i)}\left|\tilde{\mathbf{p}}_n\right|\right\} \qquad (14)$$

$$\hat{\mathbf{P}}^{(i)} = \arg\min\left\{\left\|\mathbf{Z} - \tilde{\mathbf{A}}\mathbf{P}\right\|_2^2 + \tau\sum_{n=1}^{N}\hat{\mathbf{w}}_n^{(i)}\left|\tilde{\mathbf{p}}_n\right|\right\} \qquad (15)$$

Where, $\hat{\mathbf{w}}_n^{(i)}$ is the nth element of the weight vector $\hat{\mathbf{w}} \in \mathbb{C}^{Nx1}$ at i^{th} iteration. The weight vector is updated at the beginning of each iteration using (16).

$$\hat{\mathbf{w}}^{(i)} = \frac{1}{\left|\tilde{\hat{\mathbf{p}}}^{(i-1)}\right|^\gamma} \qquad (16)$$

Where, $\tilde{\hat{\mathbf{p}}}=[\left\|\hat{\mathbf{p}}_1\right\|_2, \left\|\hat{\mathbf{p}}_2\right\|_2 \dots \left\|\hat{\mathbf{p}}_N\right\|_2]^T$, $\hat{\mathbf{p}}_1$ is the first row of $\hat{\mathbf{P}}$ matrix and so on. γ is any positive weight factor. For the first iteration, initialize $\tilde{\hat{\mathbf{p}}}^{(0)} = \{1,1,...1\}$ and estimate first iteration weight vector as shown in (16). The array manifold also needs to be updated during every iteration.

C. SVD-A-LASSO for Dimensionality Reduction

The DOA estimation accuracy depends on the number of snapshots considered.

It has been proved in sparse signal reconstruction that multiple measurement variable (MMV) achieves good accuracy than the single measurement variable (SMV) but also increases the computational time and complexity. In order to optimize both the parameters: Accuracy and complexity, this paper proposes a multiple snapshot LASSO along with Singular Value Decomposition for dimension reduction.

In singular value decomposition, the technique is to decompose the data matrix $\mathbf{Z}$ into signal and noise subspaces, by retaining the signal subspace and carrying out the estimation process can reduce the dimension and complexity of the LASSO algorithm. Without noise on the array sensors, the set of vectors $\mathbf{Z}=\{\mathbf{z}(n)\}_{n=1 \text{ to } K}$ would lie in a D-dimensional subspace, where D is the number of signal sources impinging on the array. Hence, it would be reduction in complexity if only D-dimension basis subspace is used for the further DOA estimation. With additive noise, this turns out to be singular value decomposition of data matrix into signal and noise subspaces and use only the basis for signal subspaces for further estimation steps.

Mathematically, by SVD

$$\mathbf{Z} = \mathbf{ULV}^H \tag{17}$$

$$\mathbf{ZV} = \mathbf{UL} \tag{18}$$

In order to retain only signal subspace of D-dimension, multiplying (18) by a truncating matrix $\mathbf{S_D}$ on both the sides.

$$\mathbf{Z_{sv}} = \mathbf{ZVS_D} = \mathbf{ULS_D} \tag{19}$$

Where $\mathbf{S_D} = [\mathbf{I_D} \quad \mathbf{0}]$, $\mathbf{I_D}$ is DxD identity matrix and $\mathbf{0}$ is (K-D)xD zero matrix. In addition to this, let $\mathbf{P_{sv}} = \mathbf{PVS_D}$ and $\mathbf{N_{wsv}} = \mathbf{N_wVS_D}$. Hence, the signal model in (10) modifies to (20).

$$\mathbf{Z_{SV}} = \tilde{\mathbf{A}}\mathbf{P_{SV}} + \mathbf{N_{WSV}} \tag{20}$$

Where, $\mathbf{Z_{sv}} \in \mathbb{C}^{MxD}$, $\mathbf{P_{sv}} \in \mathbb{C}^{NxD}$ and $\mathbf{N_{wsv}} \in \mathbb{C}^{MxD}$.
Considering the dimensionality reduced model in (20) for Adaptive LASSO algorithm yields SVD-A-LASSO algorithm given in (21) & (22).

$$\hat{\mathbf{P}}_{sv}^{(i)} = \arg\min \left\{ \left\| \mathbf{Z_{SV}} - \tilde{\mathbf{A}}\mathbf{P_{SV}} \right\|_2^2 + \tau \sum_{n=1}^{N} \hat{\mathbf{w}}_n^{(i)} \left| \tilde{\mathbf{p}}_{sv_n} \right| \right\} \tag{21}$$

$$\hat{\mathbf{w}}^{(i)} = \frac{1}{\left| \tilde{\mathbf{p}}_{sv}^{(i-1)} \right|^{\gamma}} \tag{22}$$

V. THE PROPOSED ALGORITHM

The singular value decomposition based adaptive least absolute shrinkage & selection operator algorithm for underdetermined DOA estimation with reduced dimensionality and complexity is summarized in Table 1.

TABLE I
SVD-A-LASSO ALGORITHM

Input Parameters: Array received signal : $\mathbf{Y}$
Regularization parameter: τ
A positive weight factor: γ

1. Compute autocorrelation of $\mathbf{Y}$: $\mathbf{R_{yy}} = E\{\mathbf{YY}^H\}$.
2. Vectorize $\mathbf{R_{yy}}$ for all the K snapshots to get '$\mathbf{Z}$'.
3. Generate on-grid manifold matrix $\mathbf{A}=[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2)\ldots \mathbf{a}(\theta_N)]$ with respect to the sensor locations of nested array.
4. Compute manifold matrix of virtual array (difference co-array): $\tilde{\mathbf{A}}=(\mathbf{A}^*\Theta\mathbf{A})$.
5. Perform SVD on $\mathbf{Z}$ matrix: $\mathbf{Z} = \mathbf{ULV}^H$
6. Reduce the dimension of $\mathbf{Z}$ by truncating: $\mathbf{Z_{sv}} = \mathbf{ZVS_D}$ where, $\mathbf{S_D} = [\mathbf{I_D} \quad \mathbf{0}]$ & $\mathbf{0}$ is (K-D)xD zero matrix.
7. Initialize
   $$\tilde{\mathbf{p}}_{sv}^{(0)} = \{1,1,\ldots1\}$$
8. Estimate the weight vector: $\hat{\mathbf{w}}^{(i)} = \dfrac{1}{\left| \tilde{\mathbf{p}}_{sv}^{(i-1)} \right|^{\gamma}}$
9. Each column of virtual array manifold matrix is updated by dividing by corresponding value in the weight vector.
10. Solve LASSO minimization problem in (21).
11. Update the estimated spatial spectrum:
    $$\tilde{\mathbf{p}}_{sv}^{(i)} = \frac{\tilde{\mathbf{p}}_{sv}^{(i)}}{\hat{\mathbf{w}}^{(i)}}$$
12. Repeat steps from 8 to 11 until the stopping criteria is reached.
13. Plot $\tilde{\mathbf{p}}_{sv}$ v/s on-grid points $\mathbf{\Omega}$. Identify peaks and corresponding θ's are the DOA's.

VI. RESULTS & DISCUSSIONS

In this section, the simulation results for exhibiting the performance of the proposed DOA estimation algorithm: SVD-A-LASSO under various conditions are presented.

The simulation results are based on MATLAB software in conjuction with CVX pacakage for defining and solving convex relaxation problems.

For all the simulations, the regularization parameter τ=2.2 is considered, which optimizes between the sparsity and accuarcy of the solution obtained. A positive weight factor γ=0.5 is considered for the root-absolute value of solutions obtained previously which are used for weight updations.

Case-1: In this case, the number of physical array elements taken is M=6, number of signal sources impinging on the array D=2, number of snapshots K=700, SNR=0dB, actual DOA = [-50°, 10°], angular frequencies of the signal sources w= [0.2π, 0.1π] rad/s.

Fig 3 shows the simulation results for case-1, though MVDR and MUSIC algorithms successively indicates the DOA points, they suffer from having less steeper peaks and less non-peak attenuation, whereas the proposed SVD-A-LASSO algorithm exhibits sharp steeper peaks at actual DOAs and negligible non-peak attenuation.
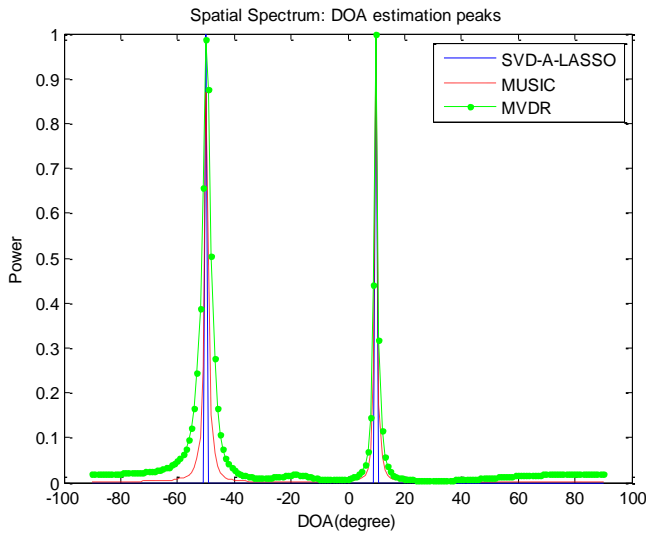
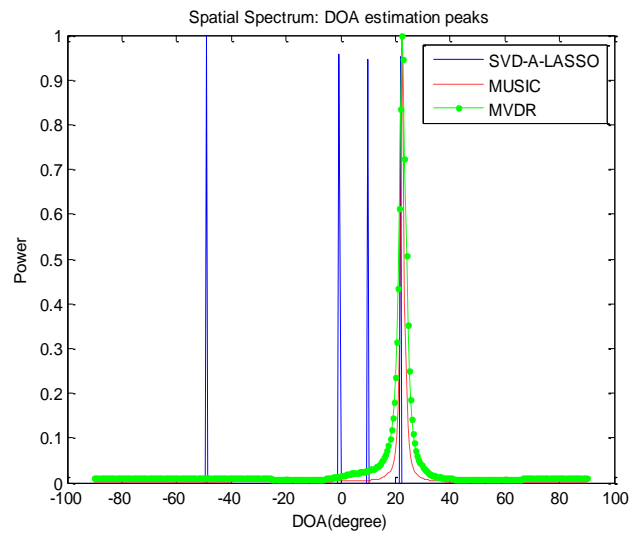**Fig 3. Spatial Spectrum DOA peaks for case-1**

Fig 5. Spatial Spectrum DOA peaks for case-3

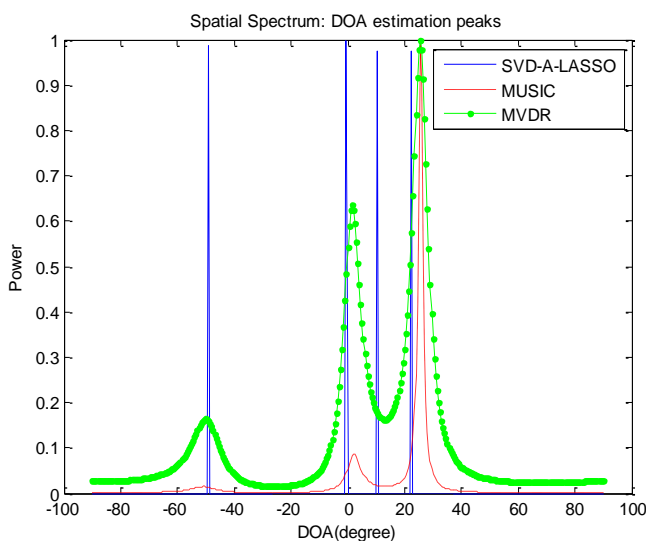**Fig 4. Spatial Spectrum DOA peaks for case-2**

Fig 6. Spatial Spectrum DOA peaks for case-4

Case-2: In this case, the number of physical array elements taken is M=6, number of signal sources impinging on the array D=4, number of snapshots K=700, SNR=0dB, actual DOA = [-50º, 0º, 10º, 22.5º], angular frequencies of the signal sources w= [0.2π, 0.1π, 0.3π, 0.4π] rad/s.

Fig 4 shows the simulation results for case-2, as the number of signal sources that are impinging on the array increases more than half the number of physical array elements, MVDR and MUSIC algorithms fails to indicate all the DOA points, whereas the proposed SVD-A-LASSO algorithm clearly indicates all the actual DOA points with sharper peaks.

Case-3: This case shows the effect of highly correlated signal sources on DOA estimation. The number of physical array elements taken is M=6, number of signal sources impinging on the array D=4, number of snapshots K=700, SNR=0dB, actual DOA = [-50º, 0º, 10º, 22.5º], angular frequencies of the signal sources w= [0.2π, 0.2π, 0.2π, 0.1π] rad/s.

Fig 5 shows the simulation results for case-3, MUSIC and MVDR indicates only the signal arriving from 22.5º direction, whereas the highly correlated signals arriving from the directions -50º, 0º, 10º are not detected. The proposed algorithm shows peaks at all the actual DOA points,
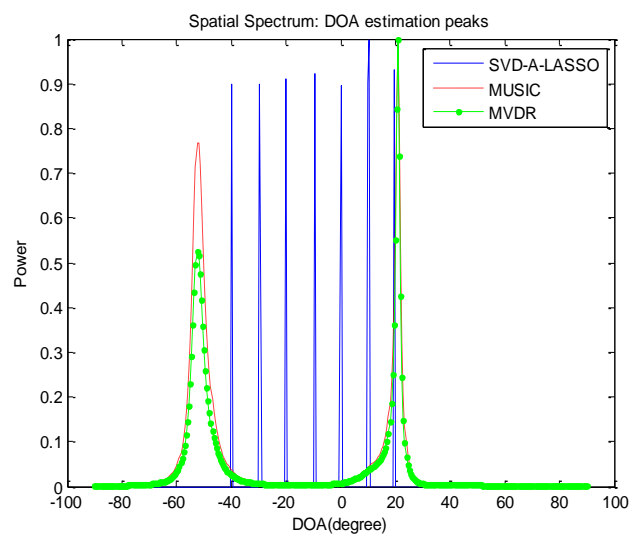
indicating that the proposed algorithm also works well in the highly correlated signal source environment.

Case-4: This case shows the simulation results of under-determined DOA estimation. The number of physical array elements taken is M=6, number of signal sources impinging on the array D=7, number of snapshots K=700, SNR=0dB, actual DOA = [-40º, -30º, -20º, -10º, 0º, 10º, 20º], angular frequencies of the signal sources w= [0.2π, 0.3π, 0.4π, 0.5π, 0.6π, 0.7π, 0.8π] rad/s.

Fig 6 clearly indicates that the proposed algorithm can detect all the signal sources even in the situation where the number of signal sources are more than the number of physical array elements (under-determined DOA estimation).

The performance comparison graph of root-mean square error (RMSE) v/s SNR for the proposed algorithm with respect to different values for number of snapshots N is as shown in Fig 7. For lesser number of snapshots, RMSE is high for all the distinct values of SNR considered. The proposed algorithm results in good accuracy if large number of snapshots are measured.
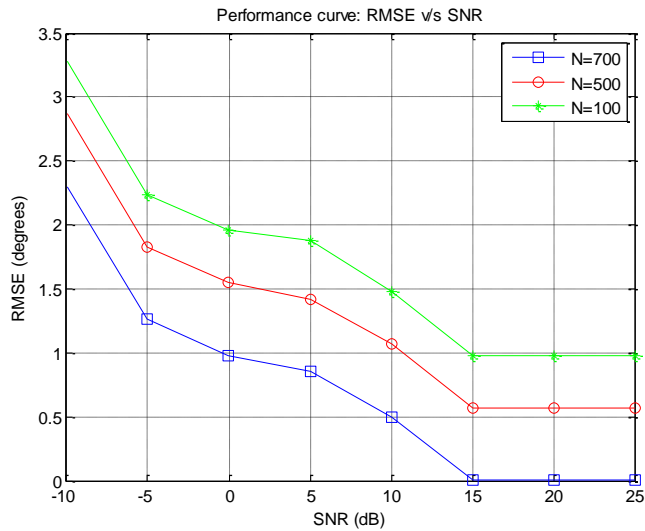
3080

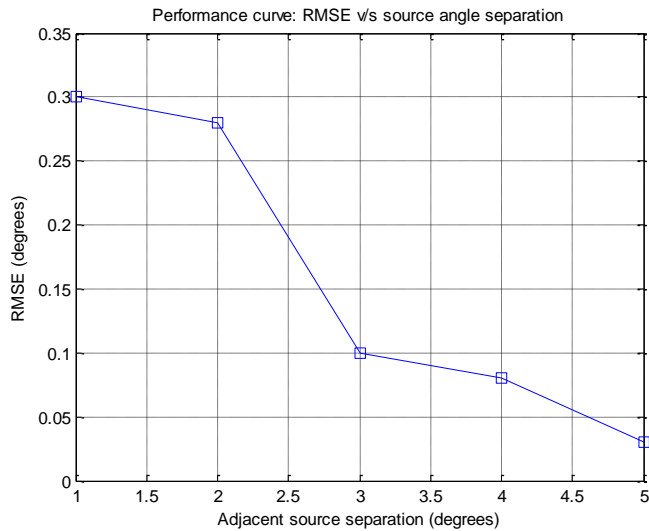**Fig 7. Performance curve of SVD-A-LASSO: RMSE v/s SNR**

Fig 8. SVD-A-LASSO: RMSE v/s source angle separation

The resolution of the algorithm depends on the step size chosen for defining the grid points. The RMSE decreases with increase in the angle separation between the adjacent signal sources as shown in the Fig 8.

The overall performance comparison of the proposed algorithm with that of the subspace-based algorithm MUSIC and with the CS based framework is presented in Fig 9. It can be observed that the proposed algorithm SVD-A-LASSO has lower RMSE for the lower SNR range from -5dB to +5dB when compared with the other algorithms.
The proposed algorithm later meets the performance level of other algorithms in the higher SNR range.

VII. CONCLUSION

In this paper, an adaptive approach for LASSO minimization for underdetermined DOA estimation with reduced dimensionality by employing singular value decomposition is proposed. The simulation results justify that the proposed algorithm performs better compared to subspace and general CS based DOA estimation techniques in the case where the number of sources to be detected are
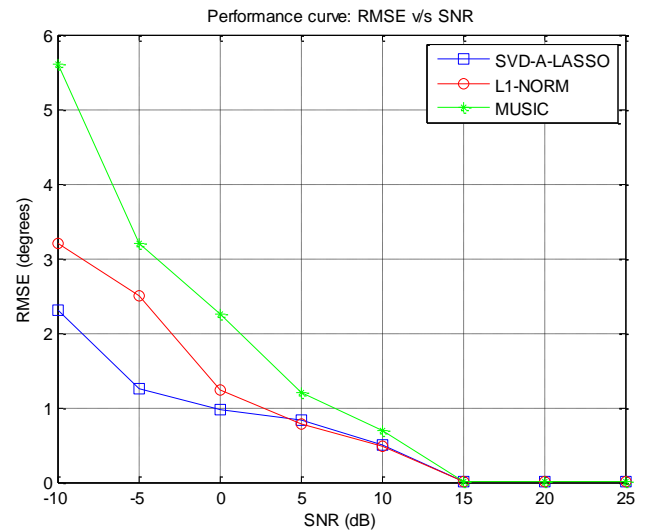

Fig 9. Performance comparison of various DOA estimation algorithms

more than the number of actual physical array elements. The algorithm also yields good results in the presence of highly correlated sources. The performance of the proposed algorithm with respect to RMSE is considerably good in the low SNR ranges. The overall improvement in the degrees of freedom, complexity reduction, accuracy of the estimation, resolution are the major characteristics of the proposed algorithm.

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