Time Dependent Probabilities of $M/M/1$ Queue with Working Vacation Subject to Disasters and Repair

B. Janani, M. Lakshmi Priya

Abstract*: A single server Markovian queueing model with working vacation subject to disaster and repair is considered. Whenever the server finds nobody in the system, the server is allowed to take a working vacation where, the server provides service at a slower rate than usual. Also disaster can occur either during busy state or during working vacation state. Whenever the system met with disaster all customers are flushed out and the system transits to repair state. Customers are allowed to join the queue even during repair time. After repair, if the server finds customer then the server moves to busy state otherwise the server moves to working vacation state. Using generating function and Laplace transform techniques explicit time dependent probabilities for various states have been obtained.

Keywords: ***Disaster, Repair, Vacation, Working vacation, Generating functions, Laplace transform.***

I. INTRODUCTION

In recent years queueing models with disaster have been paid more attention by researchers due to their applications in communication systems, networks etc. During disaster all the customers who are waiting for service are flushed out. This type of situation can be seen to prevail in computer where arrival of virus can be consider as disaster since they remove all data present in computer.

Kumar and Arivudainambi (2000) derived transient solution of single server queue with catastrophes Yang and Kim (2002) dealt with *M/G/1* stochastic clearing system and derived length of the system. Reader may refer to Economu and Fakinos (2003), Gani and Swift (2007), Shin (2007) for better understanding of queue with disasters.

Queueing system with disaster and impatient customer was studied by Yechialli (2007). Also he derived average time of a served customer temporarily, the rate of lost customer due to disasters and the proportion of customers served. Also reader can refer papers related to queue with disaster and impatient by Sudhesh, Dimou and Economu (2010, 2013). Finite source discrete time queue with disaster was studied by Jolai and Asadzadeh (2008). Later on Park and Yang (2009) extended it to queue. Batch Markovian arrival process with geometric catastrophes was studied by Economu and Gomez-Corral (2007). Single server Markovian queueing model with balking and catastrophes was studied by Boudali and Economu (2012, 2013).

Single server queue with working breakdown and disasters was studied by Kim and Lee (2014). Steady state expressions for length of the queue with disasters and multiple working vacations were presented by Yi and Kim (2007). Queue in a multi-phase stochastic environment with disasters was studied by Jiang et al (2015) and they derived the distribution for the transient queue. Stochastic clearing system in three-phase environment was studied by Zhang et al (2015). Queueing system with disaster and repair under multiple adapted vacation policy was studied by Mytalas and Zazanis (2015).

Also, most of the authors have given importance to steady state analysis. As probability of number of customers entering into the system and service time of a customer depends upon time, steady state probabilities couldn't define the behaviour of queuing system appropriately. Hence transient solution contributes significantly to analyze the cost minimization and profit maximization of queuing system. For instance chauffeur while operating a cab met with disaster by means of sudden breakdown of the cab. At such case he was forced to be delay in driving or completely stop his driving. This leads to less efficiency in rendering service to passengers at designated areas in time scheduled earlier.

To the best of our knowledge only there are few papers available on the topic "vacation queue with disasters and repair". Kalidass et al (2014) were the first to study queue with repairable server and multiple vacations and they derived explicit expressions for time dependent probabilities. Suranga Sampath (2018) derived steady state expressions for queue with repairable server under working vacations and system disaster using probability generating function method. In this paper the time dependent probabilities of an queue with working vacation subject to disasters and repair are obtained using generating function and Laplace transform techniques This paper is organized as follows. Section 2 gives description of the model. Section 3 provides transient probabilities in closed form. Section 4 deals with conclusion and future scope.

II. MODEL DESCRIPTION

A single server Markovian queueing model with working vacation subject to disaster and repair is considered. Arrivals are joining the system according to Poisson process with parameter λ and service takes place exponentially with parameter μ .

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Whenever the system becomes empty the server is allowed to take working vacation where the service is provided at a slower rate μ_v . In addition the system can met with a disaster either during busy state or during working vacation state. Whenever the system met with a disaster all customers are flushed out and system goes to repair state. Vacation, disaster and repair times of the system are also exponentially distributed with parameter γ , η and ϑ respectively. Customers are allowed to join the system during repair time according to Poisson process with parameters λ . After repair, the server moves to busy state if the server finds customers in the system, otherwise the server moves to working vacation state. Number of customers in the system and system states are represented by $\chi(t)$ and $\mathcal{J}(t)$ respectively. Mathematically,

$$\mathcal{J}(t)=\begin{cases}1 & ;server\ is\ in\ \ busy\ \ \ state\\ 2; & server\ \ is\ in\ working\ vacation\\ 0 & ;server\ is\ in\ disaster\ \ \ state\end{cases}$$

Hence $(\mathcal{J}(t),\chi(t))$ is a Markov process with continuous time parameter whose state space $\Omega=\{(0,0)\cup(j,n);\ j=0,1\ \ n=1,2,....\}$

.Let $P_{j,n}(t)$ denotes the transient probability with n number of customers in state j at time t. Mathematically,

$$P_{0,0}(t) = P\{\mathcal{J}(t) = 0, \chi(t) = 0\},$$
$$P_{2,0}(t) = P\{\mathcal{J}(t) = 2, \chi(t) = 0\},$$

and

$$P_{j,n}(t) = P\{\mathcal{J}(t) = j, \chi(t) = n\}; j = 0,1,2 \text{ and } n = 1,2 ...$$

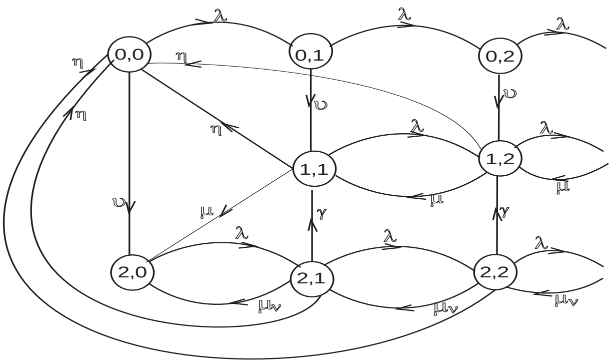

Fig.1 State transition diagram of *M/M/1* queue with working vacation subject to disaster and repair.

Governing equations of the above system are,

$$P_{0,0}^{1}(t) = -(\lambda+\vartheta)P_{0,0}(t) + \eta\sum_{n=1}^{\infty}\left(P_{1,n}(t)+P_{2,n}(t)\right) \quad (1)$$

$$P_{0,n}^{1}(t) = -(\lambda+\vartheta)P_{0,n}(t) + \lambda P_{0,n-1}(t); n = 1,2.., \quad (2)$$

$$P_{1,1}^{1}(t) = -(\lambda+\mu+\eta)P_{1,1}(t) + \mu P_{1,2}(t) + \vartheta P_{0,1}(t) + \gamma P_{2,1}(t), \quad (3)$$

$$P_{1,n}^{1}(t) = -(\lambda+\mu+\eta)P_{1,n}(t) + \lambda P_{1,n-1}(t) + \mu P_{1,n+1}(t) + \vartheta P_{0,n}(t) + \gamma P_{2,n}(t); n = 2,3..., \quad (4)$$

$$P_{2,0}^{1}(t) = -\lambda P_{2,0}(t) + \mu_v P_{2,1}(t) + \mu P_{1,1}(t) + \vartheta P_{0,0}(t), \quad (5)$$

and

$$P_{2,n}^{1}(t) = -(\lambda+\mu_v+\eta+\gamma)P_{2,n}(t) + \lambda P_{2,n-1}(t) + \mu_v P_{2,n+1}(t); n = 1,2.. \quad (6)$$

subject to the condition $P_{2,0}(0)=1$

III. TRANSIENT PROBABILITIES

In this section explicit expression for time dependent probabilities of an $M/M/1$ queue with working vacation subject to disaster and repair are derived using generating function and Laplace transform techniques.

Evaluation of $P_{0,n}(t)$

Taking Laplace Transform for equation (2) gives,

$$\hat{P}_{0,n}(s)=\frac{\lambda}{s+\lambda+\vartheta}\hat{P}_{0,n-1}(s),$$

and hence recursively we get,

$$\hat{P}_{0,n}(s)=\left(\frac{\lambda}{s+\lambda+\vartheta}\right)^{n}\hat{P}_{0,0}(s), n=1,2..... \quad (7)$$

On inverting equation (7) leads to,

$$P_{0,n}(t)=\lambda^{n}e^{-(\lambda+\vartheta)t}\frac{t^{n-1}}{(n-1)!}*P_{0,0}(t). \quad (8)$$

Therefore $P_{0,n}(t)$ is expressed in terms of $P_{0,0}(t)$.

Evaluation of $P_{0,0}(t)$

By normalization condition,

$$P_{0,0}(t)+P_{2,0}(t)+\sum_{n=1}^{\infty}\left(P_{0,n}(t)+P_{1,n}(t)+P_{2,n}(t)\right)=1. \quad (9)$$

Therefore

$$\sum_{n=1}^{\infty}\left(P_{1,n}(t)+P_{2,n}(t)\right)=1-P_{0,0}(t)-P_{2,0}(t)-\sum_{n=1}^{\infty}P_{0,n}(t). \quad (10)$$

Taking Laplace transform for equation (1) gives,

$$\hat{P}_{0,0}(s)(s+\lambda+\vartheta)=\eta\sum_{n=1}^{\infty}\left(\hat{P}_{1,n}(s)+\hat{P}_{2,n}(s)\right). \quad (11)$$

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Similarly taking Laplace transform for equation (10) gives,

$$\sum_{n=1}^{\infty}\left(\hat{P}_{1,n}(s)+\hat{P}_{2,n}(s)\right)=\frac{1}{s}-\hat{P}_{0,0}(s)-\hat{P}_{2,0}(s)-\sum_{n=1}^{\infty}\hat{P}_{0,n}(s). \tag{12}$$

Substituting equation (12) in equation (11) yields,

$$\hat{P}_{0,0}(s)(s+\lambda+\vartheta)+\eta\hat{P}_{0,0}(s)+\eta\sum_{n=1}^{\infty}\left(\frac{\lambda}{s+\lambda+\vartheta}\right)^{n}\hat{P}_{0,0}(s)=\eta\left(\frac{1}{s}-\hat{P}_{2,0}(s)\right),$$

$$\hat{P}_{0,0}(s)=\frac{\eta}{s+\lambda+\vartheta+\eta}\left\{\sum_{j=0}^{\infty}\left(\frac{-\eta}{s+\lambda+\vartheta+\eta}\sum_{n=1}^{\infty}\left(\frac{\lambda}{s+\lambda+\vartheta}\right)^{n}\right)^{j}\right\}\left(\frac{1}{s}-\hat{P}_{2,0}(s)\right),$$

$$=\hat{\pi}(s)\left(\frac{1}{s}-\hat{P}_{2,0}(s)\right), \tag{13}$$

where

$$\hat{\pi}(s)=\frac{\eta}{s+\lambda+\vartheta+\eta}\sum_{j=0}^{\infty}\left(\frac{-\eta}{s+\lambda+\vartheta+\eta}\sum_{n=1}^{\infty}\left(\frac{\lambda}{s+\lambda+\vartheta}\right)^{n}\right)^{j}.$$

Inverting equation (13) gives,

$$P_{0,0}(t)=\pi(t)*\left(1-P_{2,0}(t)\right),$$

where

$$\pi(t)=\eta e^{-(\lambda+\vartheta+\eta)t}*\sum_{j=0}^{\infty}\left(-\eta e^{-(\lambda+\vartheta+\eta)t}*\sum_{n=1}^{\infty}\left(\frac{\lambda^{n}e^{-(\lambda+\vartheta)t}t^{n-1}}{(n-1)!}\right)\right)^{*j}$$

Hence $P_{0,0}(t)$ is expressed in terms of $P_{2,0}(t)$.

Evaluation of $P_{2,n}(t)$

Define

$$P_2(z,t)=\sum_{n=1}^{\infty}P_{2,n}(t)z^{n},$$

then

$$P_2'(z,t)=\sum_{n=1}^{\infty}P_{2,n}'(t)z^{n}. \tag{14}$$

By substituting equation (6) in equation (14) yields,

$$P_2^{1}(z,t)-\left(-(\lambda+\mu_v+\eta+\gamma)+\lambda z+\frac{\mu_v}{z}\right)P_2(z,t)=\lambda zP_{2,0}(t)-\mu_vP_{2,1}(t) \tag{15}$$

Integrating equation (15) with respect to 't'yields,

$$P_2(z,t)=\lambda\int_0^t ze^{\left(-(\lambda+\mu_v+\eta+\gamma)+\lambda z+\frac{\mu_v}{z}\right)(t-y)}P_{2,0}(y)\,dy$$

$$-\mu_v\int_0^t P_{2,1}(y)e^{\left(-(\lambda+\mu_v+\eta+\gamma)+\lambda z+\frac{\mu_v}{z}\right)(t-y)}dy$$

. (16)

It is well known that if $\alpha=2\sqrt{\lambda\mu_\vartheta}$ and $\beta=\sqrt{\dfrac{\lambda}{\mu_\vartheta}}$, then

$$e^{-\left(\frac{\mu_\vartheta}{2}+\lambda z\right)t}=\sum_{n-\infty}^{\infty}(\beta z)^{n}I_n(\alpha(t))$$

where $I_n(t)$ is the modified Bessel function of the first kind. Equating coefficient of z^{n} in equation (16) gives,

$$P_{2,n}(t)=\lambda\int_0^t e^{-(\lambda+\mu_v+\eta+\gamma)(t-y)}P_{2,0}(y)\beta^{n-1}I_{n-1}(\alpha(t-y))\,dy$$

$$-\mu_v\int_0^t e^{-(\lambda+\mu_v+\eta+\gamma)(t-y)}P_{2,1}(y)\beta^{n}I_n(\alpha(t-y))\,dy;n=1,2...$$

. (17)

Similarly equating coefficients of z^{-n} in equation (16) and multiplying by β^{2n} gives,

$$0=\lambda\int_0^t e^{-(\lambda+\mu_v+\eta+\gamma)(t-y)}P_{2,0}(y)\beta^{n-1}I_{n+1}(\alpha(t-y))\,dy$$

$$-\mu_v\int_0^t e^{-(\lambda+\mu_v+\eta+\gamma)(t-y)}P_{2,1}(y)\beta^{n}I_n(\alpha(t-y))\,dy$$

. (18)

Subtracting equation (18) from equation (17) gives,

$$P_{2,n}(t)=\lambda\int_0^t e^{-(\lambda+\mu_v+\eta+\gamma)(t-y)}P_{2,0}(y)\beta^{n-1}\left(I_{n-1}\alpha(t-y)\right)-I_{n+1}(\alpha(t-y))\,dy,$$

$$=\lambda\beta^{n-1}\left[P_{2,0}(t)*\frac{2nI_n(\alpha(t))}{\alpha(t)}e^{-(\lambda+\mu_v+\eta+\gamma)t}\right],$$

$$P_{2,n}(t)=\lambda\beta^{n-1}\left[\varepsilon_n(t)*P_{2,0}(t)\right] \tag{19}$$

where $\varepsilon_n(t)=\dfrac{2nI_n(\alpha(t))}{\alpha(t)}e^{-(\lambda+\mu_v+\eta+\gamma)t}$

Substituting $n=1$ in equation (19) yields,

$$P_{2,1}(t)=\lambda\left[\varepsilon_1(t)*P_{2,0}(t)\right], \tag{20}$$

where

$$\varepsilon_1(t)=e^{-(\lambda+\mu_v+\eta+\gamma)t}\frac{2I_1(\alpha(t))}{\alpha(t)}$$

Evaluation of $P_{1,n}(t)$

Define

$$P_1(z,t)=\sum_{n=1}^{\infty}P_{1,n}(t)z^{n},$$

then

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$$P_1'(z,t)=\sum_{n=1}^{\infty}P_{1,n}'(t)z^n \quad . \qquad (21)$$

Substituting equations (3) and (4) in equation (21) gives,

$$P_1'(z,t)-P_1(z,t)\left(-(\lambda+\mu+\eta)+\lambda z+\frac{\mu}{z}\right)=-\mu P_{1,1}(t)+\gamma P_2(z,t)+\vartheta\sum_{n=1}^{\infty}P_{0,n}(t)z^n \qquad (22)$$

By following the same procedure as in the evaluation of $P_{2,n}(t)$ yields,

$$P_{1,n}(t)=\gamma\int_0^t\sum_{m=1}^{\infty}P_{2,m}(y)e^{-(\lambda+\mu+\eta)(t-y)}\beta_1^{n-m}\left(I_{n-m}(\alpha_1(t-y))-I_{n+m}(\alpha_1(t-y))dy\right)$$
$$+\vartheta\int_0^t\sum_{m=1}^{\infty}P_{0,m}(y)e^{-(\lambda+\mu+\eta)(t-y)}\beta_1^{n-m}\left(I_{n-m}(\alpha_1(t-y))-I_{n+m}(\alpha_1(t-y))dy\right)$$
$$n=1,2,3\ldots, \qquad (23)$$

where $\alpha_1=2\sqrt{\lambda\mu}$ and $\beta_1=\sqrt{\frac{\lambda}{\mu}}$.

Substituting $n=1$ in equation (23) gives,

$$P_{1,1}(t)=\gamma\int_0^t\sum_{m=1}^{\infty}P_{2,m}(y)e^{-(\lambda+\mu+\eta)(t-y)}\beta_1^{1-m}\left(I_{m-1}(\alpha_1(t-y))-I_{m+1}(\alpha_1(t-y))dy\right)$$
$$+\vartheta\int_0^t\sum_{m=1}^{\infty}P_{0,m}(y)e^{-(\lambda+\mu+\eta)(t-y)}\beta_1^{1-m}\left(I_{m-1}(\alpha_1(t-y))-I_{m+1}(\alpha_1(t-y))dy\right) \qquad (24)$$

Taking Laplace transform for equation (24) yields,

$$\hat{P}_{1,1}(s)=\left(\lambda\beta_1^{m-1}\gamma\hat{\varepsilon}_m(s)\hat{P}_{2,0}(s)+\vartheta\left(\frac{\lambda}{\lambda+\vartheta+s}\right)^m\left(\frac{1}{s}-\hat{P}_{2,0}(s)\right)\hat{\pi}(s)\left(\frac{1}{(2\mu)^{1-m}}\left(\frac{1}{P+\sqrt{P^2-\alpha^2}}\right)^m\right)\right)$$

Inverting we get,

$$P_{1,1}(t)=\theta_1(t)*P_{2,0}(t)+\theta_2(t) \qquad (25)$$

where

$$\theta_1(t)=\left(\lambda\beta_1^{m-1}\gamma\varepsilon_m(t)-\vartheta\pi(t)*\lambda^m e^{-(\lambda+\vartheta)t}\frac{t^{m-1}}{(m-1)!}\right)*\frac{2}{(2\mu)^{1-m}}m\alpha_1^{-m+1}\frac{I_m(\alpha_1(t-y))}{(\alpha_1(t-y))}$$

$$\theta_2(t)=\vartheta\lambda^m e^{-(\lambda+\vartheta)t}\frac{t^{m-1}}{(m-1)!}*\pi(t)*\frac{2m\alpha_1^{-m+1}}{(2\mu)^{1-m}}\frac{I_m(\alpha_1(t-y))}{(\alpha_1(t-y))}$$

Hence all time dependent probabilities are expressed in terms of $P_{2,0}(t)$.

Evaluation of $P_{2,0}(t)$

Applying Laplace transform for equation (5) yields,

$$\hat{P}_{2,0}(s)(s+\lambda)=\mu_v\hat{P}_{2,1}(s)+\mu\hat{P}_{1,1}(s)+\vartheta\hat{P}_{0,0}(s)+1 \qquad (26)$$

By using equations (12), (16) and (20) in equation (21) gives,

$$\hat{P}_{2,0}(s)=\frac{1}{s+\lambda}\left\{\mu_v\lambda\hat{\varepsilon}_1(s)\hat{P}_{2,0}(s)+\mu\left(\hat{\theta}_1(s)\hat{P}_{2,0}(s)+\hat{\theta}_2(s)\right)+\vartheta\hat{\pi}(s)\left(\frac{1}{s}-\hat{P}_{2,0}(s)\right)+1\right\}$$

which on further simplification yields,

$$\hat{P}_{2,0}(s)=\sum_{j=0}^{\infty}\left(\frac{\mu_v\lambda\hat{\varepsilon}_1(s)+\mu\hat{\theta}_1(s)-\vartheta\hat{\pi}(s)}{s+\lambda}\right)^j\left(\frac{\mu\hat{\theta}_2(s)}{s+\lambda}+\frac{\vartheta\hat{\pi}(s)}{s(s+\lambda)}+\frac{1}{s+\lambda}\right) \qquad (27)$$

Inverting equation (26) gives,

$$P_{2,0}(t)=\sum_{j=0}^{\infty}\left(\left(\mu_v\lambda\varepsilon_1(t)+\mu\theta_1(t)-\vartheta\pi(t)\right)*e^{-\lambda t}\right)^{*j}*\left(\mu\theta_2(t)e^{-\lambda t}+\vartheta\pi(t)\left(1*e^{-\lambda t}\right)+e^{-\lambda t}\right)$$

where

$$\pi(t)=\sum_{j=0}^{\infty}\left(-\eta e^{-(\lambda+\vartheta+\eta)t}*\sum_{n=1}^{\infty}\lambda^n e^{-(\lambda+\vartheta)t}\frac{t^{n-1}}{(n-1)!}\right)^{*j}*\eta e^{-(\lambda+\vartheta+\eta)t}$$

$$\varepsilon_1(t)=e^{-(\lambda+\mu_v+\eta+\gamma)t}\frac{2I_1(\alpha(t))}{\alpha(t)}$$

$$\theta_1(t)=\left(\lambda\beta_1^{m-1}\gamma\varepsilon_m(t)-\vartheta\pi(t)*\lambda^m e^{-(\lambda+\vartheta)t}\frac{t^{m-1}}{(m-1)!}\right)*\frac{2}{(2\mu)^{1-m}}m\alpha_1^{-m+1}\frac{I_m(\alpha_1(t-y))}{(\alpha_1(t-y))}$$

$$\theta_2(t)=\vartheta\lambda^m e^{-(\lambda+\vartheta)t}\frac{t^{m-1}}{(m-1)!}*\pi(t)\frac{2m\alpha_1^{-m+1}}{(2\mu)^{1-m}}\frac{I_m(\alpha_1(t-y))}{(\alpha_1(t-y))}$$

Hence $P_{2,0}(t)$ is explicitly determined.

IV. CONCLUSION AND FUTURE SCOPE

Time dependent probabilities of an $M/M/1$ queue with working vacation subject to disaster and repair are derived explicitly using Laplace transform and probability generating function techniques. This model can be further extended for a single server queue with finite capacity, multi-server queue and by including impatience during working vacation.

V. REMARK

Case : i :

When $\eta=0$ and $\vartheta=0$, the model reduces to $M/M/1$ queue with working vacation wherein,

$$P_{1,n}(t)=\gamma\int_0^t\sum_{m=1}^{\infty}P_{2,m}(y)e^{-(\lambda+\mu+\eta)(t-y)}\beta_1^{n-m}\left(I_{n-m}(\alpha_1(t-y))-I_{n+m}(\alpha_1(t-y))dy\right)$$

and

$$P_{2,n}(t)=\lambda\int_0^t e^{-(\lambda+\mu_v+\eta+\gamma)(t-y)}P_{2,0}(y)\beta^{n-1}\left(I_{n-1}\alpha(t-y)\right)-I_{n+1}\left(\alpha(t-y)\right)dy,$$

which coincides with the results obtained by Sudesh (2012).

Case : ii :

Let $\pi_{n,j}$ denote the steady state probability for the system to be in state j with 'n'customers. Mathematically, let

$$\pi_{j,n}=\lim_{t\to\infty}P_{j,n}(t)$$

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Using the initial value theorem of Laplace transform, which states

$$\lim_{t\to\infty} P_{j,n}(t) = \lim_{s\to 0} s\hat{P}_{j,n}(s)$$

it is observed that $\pi_{j,n} = \lim_{s\to 0} s\hat{P}_{j,n}(s)$. Therefore from the equation (13), we get

$$\lim_{s\to 0} s\hat{P}_{0,0}(s) = \frac{\eta\nu}{(\lambda+\nu)(\eta+\nu)}(1-\pi_{2,0})$$

Similarly by taking $\lim_{s\to 0} s\hat{P}_{j,n}(s)$ for the equation (19),(24) and (26), our results are seen to coincide with the results obtained by Suranga Sampath (2018).

REFERENCES

1. Kumar B K, Arivudainambi D. Transient solution of an $M/M/1$ queue with catastrophes. Computers and Mathematics with Applications, 2000, 40(10): 1233–1240.
2. Yang W S, Kim J D, Chae K C. Analysis of $M/G/1$ stochastic clearing systems. Stochastic Analysis and Applications, 2002, 20: 1083–1100.
3. Economou A, Fakinos D. A continuous-time Markov chain under the influence of a regulating point process and applications in stochastic models with catastrophes. European Journal of Operational Research, 2003, 149(3): 625–640.
4. Gani J, Swift R J. Death and birth-death and immigration processes with catastrophes. Journal of StatisticalTheory and Practice, 2007, 1(1): 39–48.
5. Shin Y W. Multi-server retrial queue with negative customers and disasters. Queueing Systems, 2007, 55(4):223–237.
6. Yechiali U. Queues with system disasters and impatient customers when system is down. Queueing Systems, 2007, 56(3–4): 195–202.
7. Jolai F, Asadzadeh S M, Taghizadeh M R. Performance estimation of an email contact center by a finite source discrete time $Geo/Geo/1$ queue with disasters. Computers & Industrial Engineering, 2008, 55(3):543–556.
8. Park H M, Yang W S, Chae K C. Analysis of the $GI/Geo/1$ queue with disasters. Stochastic Analysis andApplications, 2009, 28(1): 44–53.
9. Boudali O, Economou A. Optimal and equilibrium balking strategies in the single server Markovian queuewith catastrophes. European Journal of Operational Research, 2012, 218(3): 708–715.
10. Boudali O, Economou A. The effect of catastrophes on the strategic customer behaviour in queueing systems. Naval Research Logistics, 2013, 60(7): 571–587.
11. Sudhesh R. Transient analysis of a queue with system disasters and customer impatience. Queueing Systems, 2010, 66(1): 95–105.
12. Dimou S, Economou A. The single server queue with catastrophes and geometric reneging. Methodology and Computing in Applied Probability, 2013, 15(3): 595–621.
13. Kim B K, Lee D H. The $M/G/1$ queue with disasters and working breakdowns. Applied Mathematical Modelling, 2014, 38(5): 1788–1798.
14. Yi X W, Kim J D, Choi D W, et al. The $Geo/G/1$ queue with disasters and multiple working vacations. Stochastic Models, 2007, 23(4): 537–549.
15. Economou A, Gomez-Corral A. The batch Markovian arrival process subject to renewal generated geometric catastrophes. Stochastic Models, 2007, 23(2): 211–233.
16. Mytalas G C, Zazanis M A. An $M^X/G/1$ queueing system with disasters and repairs under a multiple adapted vacation policy. Naval Research Logistics, 2015, 62(3): 171–189.
17. Jiang T, Liu L, Li J. Analysis of the $M/G/1$ queue in multi-phase random environment with disasters. Journal of Mathematical Analysis and Applications, 2015, 430: 857–873.
18. Zhang X, Liu L, Jiang T. Analysis of an $M/G/1$ Stochastic Clearing Queue in a 3-Phase Environment. Journal of Systems Science and Information, 2015, 3(4): 374–384.
19. Nagarajan, R. and Kurose, J. On defining, computing and guaranteeing quality-of-service in high-speed networks. Proceeding of INFOCOM, Florence, Italy,1992, pp.2016–2025.
20. Bonomi, F., Mitra, D. and Seery, J. Adaptive algorithms for feedback based below control in high-speed wide-area ATM network. IEEE Jl. on Selected Areas in Communication, 1995, 13(7):1267–1283.
21. SurangaSampath M.I.G. "Steady State Expression for a repairable Server Queue with working Vocations and System disasters APORS -2018, Triennial Conference, Nepal 2018.
22. Kalidass, K., Gnanaraj, J., Gopinath, S., and RamnathKasturi. Transient analysis of an $M/M/1$ queue with a repairable server and multiple vacations. International Journal of Mathematics in Operational Research,2014, 6(2):193-216.
23. Sudhesh.R. and Francis Raj .L. Computational analysis of Stationary and Transient distribution of single server queue with working vacation. Global trends in Computing and Communication systems, Communication in Computer and Information Science, 2012, 269:480-489.

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