International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-8 Issue-11, September 2019

Cycle With Parallel Chords Are Odd Even Graceful

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Abstract— ***If C_n is a cycle of length n, then the graph cycle with parallel chords is obtained from C_n by adding an "edge between the non adjacent vertices of" C_(n.) Crown, C_n K_1 "is the graph obtained by attaching a pendant edge at each vertex of the cycle" C_(n.) In this paper we prove that the graphs n-cycle with parallel chords for n≥6 and the crowns, C_n K_(1,) for n≡0,3(mod 4). the graph P_(a,b) obtained by identifying the end points of a internally disjoint paths each of length b, are odd even graceful for odd values of a and b.***

Keywords— ***Cycles; Cycles with parallel chords; vertex labeling; odd even graceful labeling.***

I. INTRODUCTION

Much interest towards the concept of graph labeling originates from the paper by Rosa in 1967 and he introduced graceful labeling as a tool to decompose the complete graph K_{2m+1} into copies of a given tree on m edges. A labeling (valuation) of a graph is an assignment f of labels from a set of positive integers to the vertices of G that induce a label for each edge uv defined by the labels $f(u)$ and $f(v)$. If G is any simple graph with m edges, then an injective function $f: V(G) \to \{0, 1, 2, \ldots, m\}$ is said to be graceful, when each edge uv is assigned the label $|f(u) - f(v)|$, the resulting edge labels are distinct. In 2012, Sridevi, Navaneethakrishnan, A. Nagarajan and K. Nagarajan [7] defined a graph G is odd-even graceful if there is an injection f from $V(G)$ to $\{1, 3, 5, \ldots, 2m+1\}$ such that when each edge uv is assigned the label $|f(u) - f(v)|$, the resulting edge labels are $\{2, 4, 6, \ldots, 2m\}$. They have verified the odd even gracefulness of some known standard graphs. In 1977, Bodendiek[1] conjectured that any cycle with a chord is graceful and later it is verified by Delorme[2] in 1984. In analogous to this the graph, cycle with parallel chords has been defined and many authors[5], [6], [9] have verified its gracefulness. In 1991, Gnanajothi defined a graph to have odd graceful labeling if there is an injection f from $V(G)$ to $\{0,1,2,3,\ldots,2m-1\}$ such that when each edge uv is assigned the label $|f(u) - f(v)|$, the resulting edge labels are $\{1,3,5,\ldots,2m-1\}$. For detailed survey refer to the dynamic survey by Gallian[4].

Definition 1.

Crown, $C_n \odot K_1$ is "the graph obtained by attaching" a pendant edge at each vertex of the cycle C_n.

Definition 2.

Let $C_n: v_0, v_1, v_2, v_3, \ldots, v_{\frac{n}{2}}, v'_{\frac{n}{2}-1}, v'_{\frac{n}{2}-2}, \ldots, v'_3, v'_2, v'_1 v_0$ be a cycle of length n. Then the graph cycle with parallel chord is obtained from the cycle C_n by adding a an edge between the vertices $(v_1, v'_1), (v_2, v'_2), \ldots, (v_a, v'_a)$ where $a = \left\lfloor \frac{n}{2} \right\rfloor - 1$. Refer Figure.1.

Definition 3.

Let $C_n: v_1 v_2 v_3 \ldots v_n v_1$ be a cycle of length n. The graph $C_{n,k}$, a cycle with a $C_k -$ chord, is obtained from C_n by adding a cycle C_k of length k between two non-adjacent vertices v_2 and v_n.

Definition 4.

The graph $C^+_{n,k}$, a cycle with parallel $C_k -$ chord, is the graph obtained from a cycle C_n by adding a cycle C_k of length k between every pair of non-adjacent vertices $v_2, v_n, v_3, v_{n-1}, \ldots, v_a, v_b$, where $a = \frac{n}{2}$, $b = \frac{n}{2} + 2$, if n is even and $a = \left\lfloor \frac{n}{2} \right\rfloor$, $b = \left\lfloor \frac{n}{2} \right\rfloor + 3$, if n is odd.

Definition 5

. $P_{a,b}$ is the graph obtained by identifying the end points of a internally disjoint paths each of length b.

In the next section, we prove that the graphs n −cycle with parallel chords for $n \geq 6$ and the crowns, $C_n \odot K_1$, for $n \equiv 0,3 (\text{mod } 4)$ and admits odd even graceful labeling.

II. MAIN RESULTS

In this section we prove that every n −cycle with parallel chords is odd even graceful for all $n \geq 6$.

Theorem 2.1. Cycle with parallel chords admits odd even graceful labeling for all $n \geq 6$.

Proof:

Manuscript published on 30 September 2019.
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Consider a $n-$cycle $C_n: v_0, v_1, v_2, v_3, \dots, v_{\frac{n}{2}}, v'_{\frac{n}{2}-1}, v'_{\frac{n}{2}-2}, \dots, v'_3, v'_2, v'_1 v_0$ with the vertices arranged in the order as illustrated in figure.1.

1. Let C_n^+ denotes the graph C_n with parallel chords it is observed that C_n^+ has $p = n$ vertices and $q = n + \left\lfloor\frac{n}{2}\right\rfloor - 1$ edges.

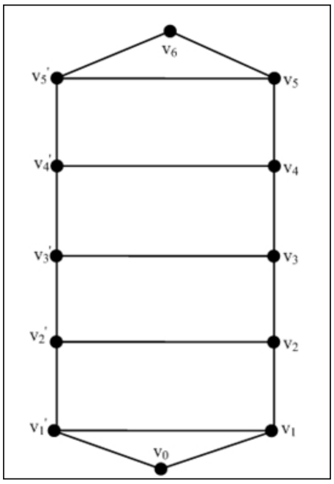

Fig.1 The cycle C_{12}^+ with parallel chords

Now, we label the vertices of the given graph G as follows,

Case 1.

When $n = 6$ and 7 then Figure.2(a) and figure. 2(b) provides the odd – even graceful labeling of the graph G.

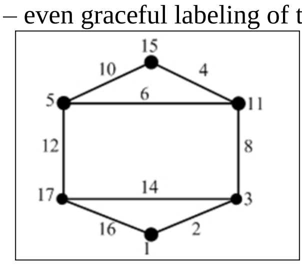

Fig. 2(a). Odd Even Gracefulness of C_6^+

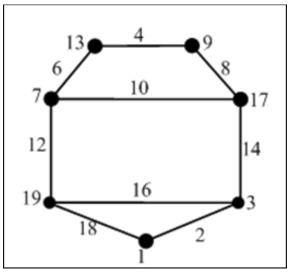

Fig. 2(b). Odd Even Gracefulness of C_7^+

Case 2.1

When $n = 4k + 3$, for $k \geq 2$.

Let $f(v_0) = 1, f(v_1) = 3$ $f(v'_1) = 2q + 1$

For $1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 1, i -$even, define

$$f(v_i) = 2q - 3i + 5,$$
$$f(v'_i) = 3i + 1$$

For $2 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 1, i -$odd, define

$$f(v_i) = 3i, \quad f(v'_i) = 2q - 3i + 4$$

From the above vertex labeling, if U and V be set of all values realized by the vertices as defined below,

Let $U_1 = \left\{f(v'_i), f(v_{i+1}) : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 1, i - odd\right\}$ and $V = \left\{f(v_i), f(v'_{i+1}) : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 1, i - odd\right\}$.

It is observed that the elements in the set U along with the $f(v_0)$ forms a monotonically decreasing sequence and the elements in the set V forms a monotonically increasing sequence. Further, it is noted that, $min\{U\} < max\{V\}$. Hence all the vertex labels are distinct.

Let $A = \left\{(v_i, v'_i) : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$,

$B = \left\{(v'_i, v'_{i+1}) : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 1\right\}$.

$C = \left\{(v_i, v_{i+1}) : 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 1\right\}$ and $D_1 = (v_0, v'_1)$ and , $D_2 = (v_0, v_1)$ denote the edges of G.

Let A', B', C', D'_1 and D'_2 denotes the edge values realized by the sets A, B, C, D_1 and D_2 respectively.

If $M = 2q$, then

$$A' = \{M-2, M-8, M-14, \dots, 10, 4\}$$
$$B' = \{M-6, M-12, M-18, \dots, 12, 6\}$$
$$C' = \{M-4, M-10, M-16, \dots, 14, 8\}$$
$$D'_1 = M \text{ and } D'_2 = 2$$

Observe that the elements of A', B', C', D'_1 and D'_2 are all distinct and further $A' \cup B' \cup C' \cup D'_1 \cup D'_2 = \{2,4,6, \dots, 2q\}$. Hence G is odd even graceful.

Case 2.2 When $n = 4k + 1$, for $k \geq 2$.

Let $f(v_0) = 1$, $f(v'_1) = 2q + 1$

For $2 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor - 1, i -$even, define

$$f(v_i) = 2q - 3i + 5,$$
$$f(v'_i) = 3i + 1$$

For $2 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor, i -$odd, define

$$f(v_i) = 3i$$
$$f(v'_i) = 2q - 3i + 4$$

If $N = \frac{n-1}{2}$, then $f(v_N) = f(v_{N-1}) + 6$

$$f(v'_N) = f(v'_{N-1}) - 8.$$

From the vertex labeling all the vertices of G realises odd integers from 1 to $2q + 1$ and its corresponding edge labels are distinct from 2 to $2q$. Hence G is odd even graceful..

Case 3.1

When n is even of the form, $n = 4k$, for $k \geq 2$.

Define $f(v_0) = 1$, $f(v'_1) = 2q + 1$

$$f(v_{2i-1}) = 6i - 3, \text{ for } 1 \leq i \leq \frac{n}{4}$$
$$f(v'_{2i-1}) = 2q - 6i + 7, \text{ for } 2 \leq i \leq \frac{n-4}{4}$$
$$f(v_{2i}) = 2q - 6i + 5, \text{ for } 1 \leq i \leq \frac{n-4}{4}$$
$$f(v'_{2i}) = 6i + 1, \text{ for } 1 \leq i \leq \frac{n-4}{4}$$

If $Q = \frac{n}{2}$, then $f\left(v'_{Q-1}\right) = f\left(v'_{Q-3}\right) - 8$,

$$f\left(v_Q\right) = f\left(v_{Q-2}\right) - 2$$

Retrieval Number: K24940981119/19©BEIESP
DOI: 10.35940/ijitee.K2494.0981119
Journal Website: www.ijitee.org

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From the vertex labeling all the vertices of G realises odd integers from 1 to $2q+1$ and its corresponding edge labels are distinct from 2 to $2q$. Hence G is odd even graceful. Refer the Appendix.

Case 3.2

When n is even of the form, $n = 4k+2$, for $k \geq 2$.

Define $f(v_0) = 1,\ f(v_1') = 2q+1$

$f(v_{2i-1}) = 6i-3$, for $1 \leq i \leq \frac{n-2}{4}$

$f(v_{2i-1}') = 2q-6i+7$, for $2 \leq i \leq \frac{n-2}{4}$

$f(v_{2i}) = 2q-6i+5$, for $1 \leq i < \frac{n-2}{4}$

$f(v_{2i}') = 6i+1$, for $1 \leq i \leq \frac{n-2}{4}$

If $Q = \frac{n}{2}$, then $f(v_{Q-1}') = f(v_{Q-3}') + 4$,

$f(v_{Q-1}) = f(v_{Q-3}) - 10$

$f(v_Q) = f(v_{Q-1}) + 4$

From the vertex labeling all the vertices of G realises odd integers from 1 to $2q+1$ and its corresponding edge labels are distinct from 2 to $2q$. Hence G is odd even graceful. Refer the Appendix.

Theorem 2.2. Crowns $C_n \odot K_1$ is odd even graceful for $n \equiv 0, 3(mod\ 4)$.

Proof:

Let G be the given crown graph $C_n \odot K_1$ having $p = 2n$ vertices and $q = p = 2n$ edges with $n \equiv 0, 3(mod\ 4)$.

The vertices of G are arranged in the order as illustrated in figure.6 and its corresponding vertex labeling are defined as below,

$f(a_i) = 2q-2i+3$, for $1 \leq i < \frac{n}{2}, i -$ odd.

$f(a_i) = 2q-2i+1$, for $i \geq \frac{n}{2}, i -$ odd.

$f(a_i) = 2i-1$, for $1 \leq i \leq n, i -$ even.

$f(b_i) = 2i-1$, for $1 \leq i < n, i -$ odd.

$f(b_i) = 2q-2i+3$, for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil, i -$ even.

$f(b_i) = 2q-2i+1$, for $i \geq \left\lceil \frac{n}{2} \right\rceil + 1, i -$ even.

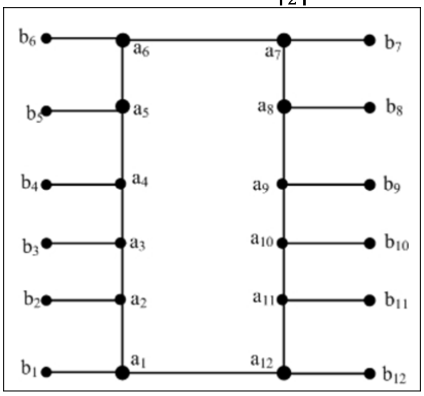

Fig. 3. The graph Crown, $C_{12} \odot K_1$

From the vertex labeling all the vertices of G realizes odd integers from 1 to $2q+1$ and its corresponding edge labels are distinct from 2 to $2q$. Hence G is odd even graceful

Theorem 2.3. The graph $P_{a,b}$ admits odd graceful labeling for all odd values of a, b.

Proof:

Let G be the given $P_{a,b}$ with a, b as odd. Then G have $n = a(b-2)+2$ vertices and $m = a(b-1)$ edges.

For the convenience of the labeling, the vertices are arranged in the order as shown in the figure. 5.

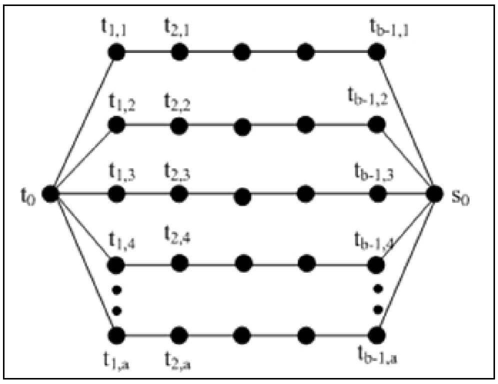

Fig 4. The graph $P_{a,b}(a, b-$ odd$)$

Now we label the vertices of the given graph G as follows,

$f(t_0) = 2m-1, \qquad f(s_0) = 2m-1-a(b-1)$

$f(t_{i,j}) = a(i-1)+2(j-1)$, for $1 \leq i < b, 1 \leq j \leq a, i - odd$

$f(t_{i,\ a+1-j}) = (2m-1)-(4j-2)-a(i-2)$, for $2 \leq i < b, 1 \leq j \leq a, i - even$

From the above labeling we observe that all the vertex labels are distinct and its corresponding edge values results odd values from 1 to $2m+1$ and hence G is odd graceful.

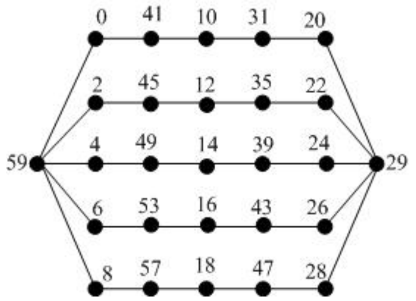

Figure 5. The Odd graceful labeling of $P_{5,7}$

III. VERTEX CORDIAL LABELING OF CYCLE WITH A C_4 − CHORD AND CYCLE WITH PARALLEL C_4 − CHORD

Recall that if $C_n: v_1v_2v_3 \ldots v_nv_1$ be a cycle of length n, then the graph $C_{n,k}$, a cycle with a $C_k -$ chord, is obtained from C_n by adding a cycle C_k of length k between two non-adjacent vertices v_2 and v_n. The graph $C_{n,k}^+$, a cycle with parallel $C_k -$ chord, is the graph obtained from a cycle C_n by adding a cycle C_k of length k between every pair of non-adjacent vertices $v_2, v_n, v_3, v_{n-1}, \ldots, v_a, v_b$, where $a = \frac{n}{2}$, $b = \frac{n}{2}+2$, if n is even and $a = \left\lfloor \frac{n}{2} \right\rfloor$, $b = \left\lfloor \frac{n}{2} \right\rfloor + 3$, if n is odd.

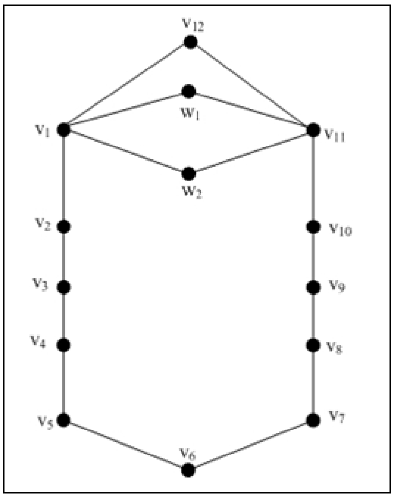

Figure 6(a). The Graph $C_{12,4}$

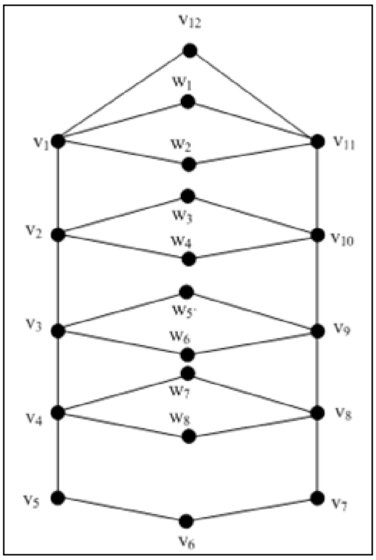

Figure 6(b). The Graph $C_{12,4}^{+}$

Theorem 3.1. The graph $C_{m,4}$ admits vertex cordial labeling for $m \equiv 0(mod\ 4)$.

Proof:

Let G be the graph $C_{m,4}$ with $m = 4k$, for $k \geq 1$. Denote the vertices of the cycle C_m and the chord C_4 as $C_m: v_1v_2v_3 \ldots v_mv_1$ and chord $C_4: v_2w_1v_nw_2v_2$ respectively. Then G has $p = m + 2$ vertices and $q = m + 4$ edges

We label the vertices of G in the order as provided in the figures 6 as follows,

$$f(v_i) = \begin{cases} 1, & i = 4t+1, 4t+2, t \geq 1 \\ 0, & i = 4t+3, 4t, t \geq 1 \end{cases}$$

$$f(w_1) = 1, f(w_2) = 0$$

Let V_0 and V_1 denote the set of all vertices assigned the label 0 and 1 respectively. Let $E_0$and E_1denote the set of all edges assigned the label 0 and 1 respectively.

A particular 0-1 sequence is matched corresponding with the above sequence of vertices of the given graph G. It is evident that in G, $|V_0| = |V_1|$ and $|E_1| = |E_0|$. Hence G is vertex cordial.

Theorem 3.2. The graph $C_{m,4}^{+}$ admits vertex cordial labeling for $m \equiv 0(mod\ 4)$.

Proof:

Let G be the graph $C_{m,4}^{+}$, with $m = 4k$, for $k \geq 1$, then Then $C_{m,4}^{+}$ has $p = 2m - 2$ vertices and $q = 3m - 4$ edges.

For the convenience of the labeling, the vertices of the given graph G are ordered in the way as shown in figure. 12,

Define,

$$f(v_i) = \begin{cases} 1, & i = 4t+1, 4t+2, t \geq 1 \\ 0, & i = 4t+3, 4t, t \geq 1 \end{cases}$$

$$f(w_i) = \begin{cases} 1, & i = \text{odd} \\ 0, & i = \text{even} \end{cases}$$

Clearly from the above definition, it is evident that in G, $|V_0| = |V_1|$ and $|E_1| = |E_0|$. Hence G is vertex cordial.

IV. ACKNOWLEDGMENT

The author thankfully acknowledges the referee for his/her valuable suggestions in improving the presentations of the paper. The author thankfully acknowledges the referee for his/her valuable suggestions in improving the presentations of the paper. Further the authors thanks the management of SASTRA deemed University for providing support in presenting this paper.

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VI. APPENDIX

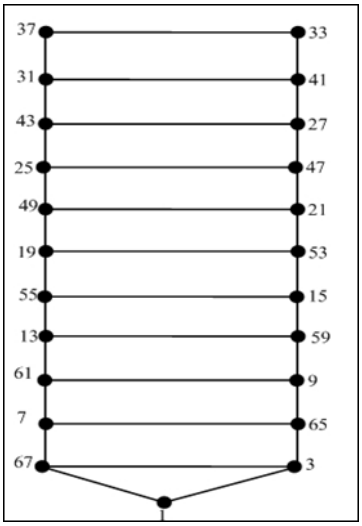

Fig. 7(a). Odd Even Gracefulness of $C_{23}^{+}(n = 4k + 3)$

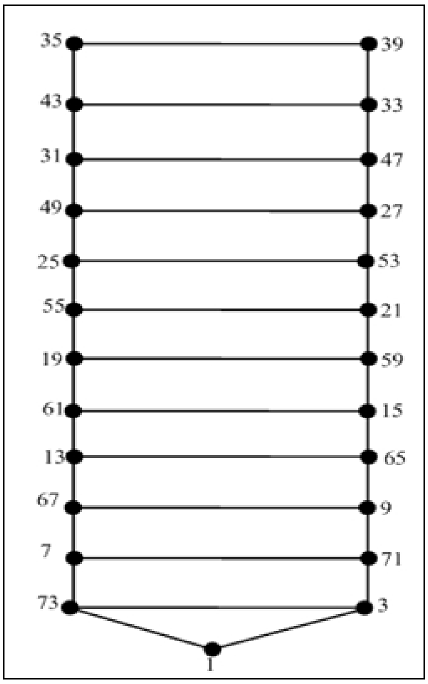

Fig. 7(b). Odd Even Gracefulness of $C_{25}^{+}(n = 4k + 1)$

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