On Third Leap Zagreb Index of Some Generalized Graph Structures

S.Swathi, C.Natarajan, K.Balasubramanian, M.Swathi

Abstract— ***Let G be a connected graph with n vertices. The 2-degree of a vertex v in G is the number of vertices which are at distance two from v in G. The third leap Zagreb index of G is the sum of product of degree and 2-degree of all vertices in G. In this paper, we determine the exact values for the third leap Zagreb index of some generalized graph structures***

Keywords— ***Thorn graphs, generalized graph structures, leap Zagreb indices.***

I. INTRODUCTION

Topological indices of graphs play a vital role in Mathematical Chemistry to study the structural properties of some complicated chemical compounds. In general, topological indices are widely classified into two types: degree based indices and distance based indices. Thorn tree is a well known graph structure in Chemical graph Theory. In 2012 *K.M.Kathirasen* and *C.Parameswaran* [1] introduced the idea of *Generalized Thorn graphs* as a generalization of thorn trees. In 2017, Naji et al. [5] introduced a new topological invariant called *leap Zagreb indices* and studied their properties. Also they discussed the first leap Zagreb index of some graph operations [4]. Basavanagoud and Praveen [2] computed first leap Zagreb index of some nano structures. Shao et al. [6] found some interesting results on leap Zagreb indices of trees and unicyclic graphs. Shiladhar et al. [7] computed leap Zagreb indices of some windmill type graphs.

Venkatakrishnan et al. [8] found eccentric connectivity index of certain generalized thorn graphs. In this sequel, we are interested in computing exact values for the third leap Zagreb index of some generalized thorn graphs. We recall their definitions in the following:

II. BASIC DEFINITIONS

Definition 1

The generalized thorn graphs $G_P, G_C, G_K, G_{A,} G'_C, G'_K$ G_P *and* G'_A G'_A *are constructed using the following graph operations:*

G_P : Attach t_i copies of a path of order $r \geq 2$ to each vertex v_i of G by identifying v_i as the initial vertex of such paths.

G_C : Attach t_i copies of a cycle of order $r \geq 3$ to each vertex v_i by identifying v_i as one of the vertices in C_r .

G_K : Attach t_i copies of a complete graph K_r of order $r \geq 4$ to every vertex v_i of G by identifying v_i as a vertex of K_r .

G_A : Attach t_i copies of a complete bipartite graph $K_{r,s}$ to every vertex v_i of G by identifying v_i as a vertex of $K_{r,s}$.

G'_C : Attach t_i copies of a cycle of order $r \geq 3$ to each vertex v_i of G by an edge.

G'_K : Attach t_i copies of a complete graph K_r of order $r \geq 4$ to each vertex v_i of G by an edge.

G'_A : Attach t_i copies of a complete bipartite graph $K_{r,s}$ to each vertex v_i of G by an edge.

Recently Naji et al. [5] introduced a new set of topological invariants called Leap Zagreb Indices in 2017 and they are defined as follows:

Definition 2

The first leap Zagreb index of G is denoted by $LM_1(G)$ *and defined as* $LM_1(G) = \sum_{v \in V(G)} d_2(v:G)^2$.

Definition 3

The second leap Zagreb index of G is defined as $LM_2(G) = \sum_{uv \in E(G)} d_2(u:G) d_2(v:G)$.

Definition 4

The third leap Zagreb index of a graph G is defined as $LM_3(G) = \sum_{v \in V(G)} \deg(v:G) d_2(v:G)$.

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III. THE THIRD LEAP ZAGREB INDEX OF GENERALIZED THORN GRAPHS & RESULTS

In this section, we compute the exact values of the third leap Zagreb index of generalized thorn graphs G_P, G_C, G_K, G_A, G_C', G_K' and G_A'.

Let $S_i = \sum_{v_j \in N_G(v_i)} t_j$.

2.1 Generalized Thorn Graph G_P

Observation 1

Let $u_1, u_2, \cdots, u_r$ *be the vertices of* P_r. *The degree and 2-degree of any vertex in* G_P *is given as follows:*

(1) $\deg(v_i : G_P) = \deg(v_i : G) + t_i$
(2) $d_2(v_i : G_P) = d_2(v_i : G) + t_i + S_i$
(3) $\deg(u_j : G_P) = 2; 3 \le j \le r-1$
(4) $d_2(u_j : G_P) = 2; 3 \le j \le r-1$; $3 \le j \le r-1$
(5) $\deg(u_2 : G_P) = 2$
(6) $d_2(u_2 : G_P) = \deg(v_i : G) + t_i;\ 1 \le i \le n$
(7) $\deg(u_r : G_P) = 1$
(8) $d_2(u_r : G_P) = 1.$

Theorem **5**

$$LM_3(G_P) = LM_3(G) + \sum_{i=1}^{n} \deg(v_i / G)[3t_i + S_i] + \sum_{i=1}^{n} d_2(v_i / G)t_i + \sum_{i=1}^{n} t_i S_i + 3\sum_{i=1}^{n} t_i^2 + (4r-13)\sum_{i=1}^{n} t_i$$

Proof.

By observation 1 we have

$$LM_3(G_P) = \sum_{v \in V(G_P)} \deg(v : G_P) d_2(v : G_P)$$

$$= \sum_{v_i \in V(G_P)} \deg(v_i : G_P) d_2(v_i : G_P) + \sum_{i=1}^{n} t_i$$

$$+ 2\sum_{i=1}^{n} t_i + 4(r-4)\sum_{i=1}^{n} t_i + 2\sum_{i=1}^{n} t_i[\deg(v_i : G) + t_i]$$

$$= \sum_{i=1}^{n} [\deg(v_i : G) + t_i][d_2(v_i : G) + t_i + S_i] + \sum_{i=1}^{n} t_i$$

$$+ 2\sum_{i=1}^{n} t_i + 4(r-4)\sum_{i=1}^{n} t_i + 2\sum_{i=1}^{n} t_i[\deg(v_i : G) + t_i]$$

$$= \sum_{i=1}^{n} \deg(v_i : G) d_2(v_i : G) + \sum_{i=1}^{n} \deg(v_i : G) t_i$$

$$+ \sum_{i=1}^{n} \deg(v_i : G) S_i + \sum_{i=1}^{n} d_2(v_i : G) t_i + \sum_{i=1}^{n} {t_i}^2 +$$

$$\sum_{i=1}^{n} t_i S_i + (4r-13)\sum_{i=1}^{n} t_i + 2\sum_{i=1}^{n} \deg(v_i : G) t_i + 2\sum_{i=1}^{n} t_i^2$$

Thus the result follows.

Corollary 6

If $t_i = t$ for all i=1,2,…,n in G_P, then

$$LM_3(G_P) = LM_3(G) + 6mt + \sum_{i=1}^{n} \deg(v_i : G) S_i + t\sum_{i=1}^{n} d_2(v_i : G) + t\sum_{i=1}^{n} S_i + 3nt^2 + (4r-13)nt$$

Example 7

(1) If t copies of P_r (r≥2) are attached to each vertex of $G = P_n$, then

$LM_3(G_P) = 2(2n-5) + nt(4r-1) - 2t(t+8) + 5nt^2$.

(2) In particular if we attach t copies of P_2 to every vertex of $G = P_n$ by identifying one of the vertices in P_2 as a vertex in P_n then the resulting graph is a Thorn tree T and $LM_3(T) = 2(2n-5) + nt(4r-1) - 2t(t+8) + 5nt^2$.

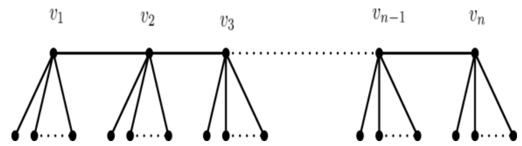

Figure 1: Thorn tree T

(3) If t copies of P_r (r≥2) are attached to each vertex of $G = C_n$, then $LM_3(G_P) = 4n + 12nt + 3nt^2 + (4r-13)nt$.

2.2 Generalized Thorn Graph G_C

Observation 2:

Let $u_1, u_2, \dots, u_r$ be the vertices in a cycle $C_r, r \ge 5$. Then the degree and 2-degree of any vertex in G_C is as follows:

(1) $\deg(v_i : G_C) = \deg(v_i : G) + 2t_i$
(2) $d_2(v_i : G_C) = d_2(v_i : G) + 2t_i + 2S_i$
(3) $\deg(u_j : G_C) = 2$, *where* $u_j \in N(v_i : G_C) \backslash V(G)$
(4) $d_2(u_j : G_C) = \deg(v_i : G) + 2t_i$, *where* $u_j \in N(v_i : G_C) \backslash V(G)$
(5) $\deg(u_j : G_C) = 2$, *where* $u_j \in V(G_C) \backslash (N(v_i : G_C) \cup V(G))$
(6) $d_2(u_j : G_C) = 2(r-3)$, *where* $u_j \in V(G_C) \backslash (N(v_i : G_C) \cup V(G))$

Theorem8

$$LM_3(G_C) = LM_3(G) + 2\sum_{i=1}^{n} \deg(v_i : G)[2t_i + S_i] + 2\sum_{i=1}^{n} d_2(v_i : G)t_i + 4(r-2)\sum_{i=1}^{n} t_i + 4\sum_{i=1}^{n} t_i S_i + 4\sum_{i=1}^{n} t_i^2$$

Proof:

By Observation 2 we have

$$LM_3(G_C) = \sum_{v\in V(G_C)} \deg(v:G_C)d_2(v:G_C)$$

$$= \sum_{v_i\in V(G_C)} \deg(v_i:G_C)d_2(v_i:G_C) + \sum_{u_k\in N(v_i:G_C)\setminus V(G)} \deg(u_j:G_C)d_2(u_j:G_C) + \sum_{u_j\in V(G_C)\setminus(N(v_i:G_C)\cup V(G))} \deg(u_j:G_C)d_2(u_j:G_C)$$

$$= \sum_{i=1}^{n}[\deg(v_i:G)+2t_i][d_2(v_i:G)+2t_i+2S_i] + 2\sum_{i=1}^{n}t_i[\deg(v_i:G)+2t_i] + 4(r-3)\sum_{i=1}^{n}t_i$$

$$= LM_3(G) + 2\sum_{i=1}^{n}\deg(v_i:G)[2t_i+S_i] + 2\sum_{i=1}^{n}d_2(v_i:G)t_i + 4\sum_{i=1}^{n}t_i(r-2) + 4\sum_{i=1}^{n}t_iS_i + 4\sum_{i=1}^{n}t_i^2$$

Corollary 9

If $t_i = t$ for all $i = 1,2,...,n$ in G_C, then

$$LM_3(G_C) = LM_3(G) + 8mt + 2\sum_{i=1}^{n}\deg(v_i:G)S_i + 2t\sum_{i=1}^{n}d_2(v_i:G) + 4(r-2)nt + 4t\sum_{i=1}^{n}S_i + 4nt^2$$

Example 10

(1) If t copies of C_r., $r \geq 5$ are attached to each vertex of $G = P_n$, then $LM_3(G_C) = 2(2n-5) + 4nt(3t+(r+3)) - 2t(4t+14)$

(2) If t copies of C_r., $r \geq 5$ are attached to each vertex of $G = C_n$, then $LM_3(G_C) = 4n + 12nt + 4rnt + 12nt^2$.

2.3 Generalized Thorn graph G_K

Observation 3

Let $u_1, u_2, ..., u_r$ be the vertices in a clique K_r. Then the degree and 2-degree of any vertex G_K are as follows:

(1) $\deg(v_i:G_K) = \deg(v_i:G) + (r-1)t_i$ where $v_i \in V(G)$

(2) $d_2(v_i:G_K) = d_2(v_i:G) + (r-1)S_i$ where $v_i \in V(G)$

(3) $\deg(u_j:G_K) = r-1$ where $u_j \in V(G_K)\setminus V(G)$

(4) $d_2(u_j:G_K) = \deg(v_i:G) + (t_i-1)(r-1)$ where $u_j \in V(G_K)\setminus V(G)$

Theorem 11

$$LM_3(G_K) = LM_3(G) + (r-1)\sum_{i=1}^{n}\deg(v_i:G)[t_i+S_i] + (r-1)\sum_{i=1}^{n}d_2(v_i:G)t_i + (r-1)^2\sum_{i=1}^{n}[t_iS_i+t_i^2] - (r-1)^2\sum_{i=1}^{n}t_i.$$

Proof:

By Observation 3 we have

$$LM_3(G_K) = \sum_{v\in V(G_K)}\deg(v:G_K)d_2(v:G_K)$$

$$= \sum_{v_i\in V(G)}\deg(v_i:G_K)d_2(v_i:G_K) + \sum_{u_j\in V(G_K)\setminus V(G)}\deg(u_j:G_K)d_2(u_j:G_K)$$

$$= \sum_{i=1}^{n}[\deg(v_i:G)+(r-1)t_i][d_2(v_i:G)+(r-1)S_i] + (r-1)\sum_{i=1}^{n}t_i[\deg(v_i:G)+(r-1)(t_i-1)]$$

$$= LM_3(G) + (r-1)\sum_{i=1}^{n}\deg(v_i:G)[t_i+S_i] + (r-1)\sum_{i=1}^{n}d_2(v_i:G)t_i + (r-1)^2\sum_{i=1}^{n}[t_iS_i+t_i^2-t_i]$$

Corollary 12

If $t_i = t$ for all $i = 1,2,...,n$ in G_K, then

$$LM_3(G_K) = LM_3(G) + 2(r-1)mt + (r-1)\sum_{i=1}^{n}\deg(v_i:G)S_i + (r-1)t\sum_{i=1}^{n}d_2(v_i:G) + (r-1)^2t\sum_{i=1}^{n}S_i + (r-1)^2nt^2 - (r-1)^2nt$$

Example 13

(1) If t copies of $K_r\,(r \geq 4)$ are attached to each vertex of $G = P_n$, then

$$LM_3(G_K) = 2(2n-5) - 12rt + 12t + (3n-2)r^2t^2 - (6n-4)rt^2 + (3n-2)t^2 - (r^2-10r+1)nt$$

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(2) If t copies of K_r $(r \geq 4)$ are attached to each vertex of $G = C_n$, then

$$LM_3(G_K) = 4n + rnt(10 - r) + 3nt(t - 3) + 3nrt^2(r - 2).$$

2.4 Generalized Thorn graph G_A

Observation 4:

Let (X,Y) be a partition of the vertex set of t_i-copy of $K_{r,s}$ and let $V(X) = \{x_1, x_2, ..., x_r\}$ and $V(Y) = \{y_1, y_2, ..., y_s\}$. Then the degree and 2-degree of any vertex in G_A is as follows:

(1) $\deg(v_i : G_A) = \deg(v_i : G) + st_i$

(2) $d_2(v_i : G_A) = d_2(v_i : G) + (r-1)t_i + sS_i$

(3) $\deg(x_j : G_A) = s, 2 \leq j \leq r$

(4) $d_2(x_j : G_A) = r - 1, 2 \leq j \leq r$

(5) $\deg(y_k : G_A) = r, 1 \leq k \leq s$

(6) $d_2(y_k : G_A) = (s-1) + s(t_i - 1) + \deg(v_i : G), 1 \leq k \leq s$

Theorem 14

$$LM_3(G_A) = LM_3(G) + \sum_{i=1}^{n}[\deg(v_i : G)(2r-1)t_i + sS_i] + s\sum_{i=1}^{n} t_i d_2(v_i : G) + \sum_{i=1}^{n} t_i^2(3rs - 2s) + s^2\sum_{i=1}^{n} t_i S_i - r\sum_{i=1}^{n} t_i.$$

Proof:

By Observation 4 we have

$$LM_3(G_A) = \sum_{v \in V(G_A)} \deg(v : G_A) d_2(v : G_A)$$
$$= \sum_{v_i \in V(G)} \deg(v_i : G_A) d_2(v_i : G_A) + \sum_{x_j \in V(G_A) \setminus V(G)} \deg(x_j : G_A) d_2(x_j : G_A) + \sum_{y_k \in V(G_A) \setminus V(G)} \deg(y_k : G_A) d_2(y_k : G_A)$$
$$= \sum_{i=1}^{n}[\deg(v_i : G) + st_i][d_2(v_i : G) + (r-1)t_i + sS_i] + s(r-1)\sum_{i=1}^{n} t_i^2 + r\sum_{i=1}^{n} t_i[s - 1 + s(t_i - 1) + \deg(v_i : G)]$$

Thus the result follows.

Corollary 15

If $t_i = t$ for all $i = 1,2,...,n$ in G_A, then

$$LM_3(G_A) = LM_3(G) + 2(2r-1)mt + s\sum_{i=1}^{n}\deg(v_i : G)S_i + st\sum_{i=1}^{n} d_2(v_i : G) + (3rs - 2s)nt^2 + 2ms^2t^2 - rnt.$$

2.5 Generalized Thorn Graph $G_C^{'}$

Observation 5

Let $u_1, u_2, ..., u_r$ be the vertices in the t_i-copy of C_r. Then degree and 2-degree of any vertex in $G_C^{'}$ are as follows:

(1) $\deg(v_i : G_C^{'}) = \deg(v_i : G) + t_i$

(2) $d_2(v_i : G_C^{'}) = d_2(v_i : G) + 2t_i + S_i$

(3) $\deg(u_j : G_C^{'}) = d_2(u_j : G_C^{'}) = 2, 4 \leq j \leq r$

(4) $\deg(u_j : G_C^{'}) = 2,$ *for* $j = 2,3$

(5) $d_2(u_j : G_C^{'}) = 3,$ *for* $j = 2,3$

(6) $\deg(u_1 : G_C^{'}) = 3,$ *for* $u_1 \in N(v_i : G_C^{'}) \setminus V(G)$

(7) $d_2(u_1 : G_C^{'}) = t_i + 1 + \deg(v_i : G)$ *for* $u_1 \in N(v_i : G_C^{'}) \setminus V(G)$

Theorem 16

$$LM_3(G_C^{'}) = LM_3(G) + \sum_{i=1}^{n}\deg(v_i : G)[5t_i + S_i] + \sum_{i=1}^{n} d_2(v_i : G)t_i + 5\sum_{i=1}^{n} t_i^2 + (4r+7)\sum_{i=1}^{n} t_i + \sum_{i=1}^{n} t_i S_i.$$

Proof:

By Observation 5 we have

$$LM_3(G_C^{'}) = \sum_{v \in V(G_C^{'})} \deg(v : G_C^{'}) d_2(v : G_C^{'})$$
$$= \sum_{v_i \in V(G)} \deg(v_i : G_C^{'}) d_2(v_i : G_C^{'}) + \sum_{u_1 \in N(v_i : G_C^{'}) \setminus V(G)} \deg(u_1 : G_C^{'}) d_2(u_1 : G_C^{'})$$
$$+ \deg(u_2 : G_C^{'}) d_2(u_2 : G_C^{'}) + \deg(u_r : G_C^{'}) d_2(u_r : G_C^{'}) + \sum_{j=3}^{r-1}\deg(u_j : G_C^{'}) d_2(u_j : G_C^{'})$$
$$= \sum_{i=1}^{n}[\deg(v_i : G) + t_i][d_2(v_i : G) + 2t_i + S_i]$$

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$$+3\sum_{i=1}^{n} t_i[t_i+1+\deg(v_i:G)]+[12+4(r-2)]\sum_{i=1}^{n} t_i$$

Thus the result follows.

Corollary 17

If $t_i = t$ *for all* $i = 1,2,...,n$ in $G_C^{'}$, then

$$LM_3(G_C^{'}) = LM_3(G)+10mt+5nt^2+2mt^2$$

$$+(4r+7)nt+\sum_{i=1}^{n}\deg(v_i:G)S_i+t\sum_{i=1}^{n} d_2(v_i:G).$$

2.6 Generalized Thorn Graph $G_K^{'}$

Observation 6

Let $u_1,u_2,...,u_r$ be vertices in a clique K_r. Then the degree and 2-degree of any vertex in $G_K^{'}$ are as follows:

(1) $\deg(v_i:G_K^{'}) = \deg(v_i:G)+t_i$

(2) $d_2(v_i:G_K^{'}) = d_2(v_i:G)+(r-1)t_i+S_i$

(3) $\deg(u_j:G_K^{'}) = r-1,\ 2\le j\le r$

(4) $d_2(u_j:G_K^{'}) = 1,\ 2\le j\le r$

(5) $\deg(u_1:G_K^{'}) = r,\ where\ u_1\in N(v_i:G_K^{'})\setminus V(G)$

(6) $d_2(u_1:G_K^{'}) = \deg(v_i:G)+(t_i-1)$ $where\ u_1\in N(v_i:G_K^{'})\setminus V(G)$

Theorem 18

$$LM_3(G_K^{'}) = LM_3(G)+\sum_{i=1}^{n}\deg(v_i:G)[(2r-1)t_i$$

$$+S_i]+\sum_{i=1}^{n} d_2(v_i:G)t_i+(2r-1)\sum_{i=1}^{n} t_i^2-\sum_{i=1}^{n} t_i$$

$$+\sum_{i=1}^{n} t_iS_i.$$

Proof:

$$LM_3(G_K^{'}) = \sum_{v\in V(G_K^{'})}\deg(v:G_K^{'})d_2(v:G_K^{'})$$

$$\sum_{v_i\in V(G)}\deg(v:G_K^{'})d_2(v:G_K^{'})+$$

$$\sum_{u_j\in V(G_K^{'})}\deg(u_j:G_K^{'})d_2(u_j:G_K^{'})$$

$$+\sum_{u_1\in N(v_i:G_K^{'})\setminus V(G)}\deg(u_1:G_K^{'})d_2(u_1:G_K^{'})$$

$$=\sum_{i=1}^{n}[\deg(v_i:G)+t_i][d_2(v_i:G)+(r-1)t_i+S_i]$$

$$+(r-1)\sum_{i=1}^{n} t_i+r\sum_{i=1}^{n} t_i[\deg(v_i:G)+t_i-1]$$

$$=LM_3(G)+(2r-1)\sum_{i=1}^{n}\deg(v_i:G)t_i$$

$$+\sum_{i=1}^{n}\deg(v_i:G)S_i+\sum_{i=1}^{n} d_2(v_i:G)t_i$$

$$+(2r-1)\sum_{i=1}^{n} t_i^2-\sum_{i=1}^{n} t_i+\sum_{i=1}^{n} t_iS_i.$$

Corollary 19

If $t_i = t$ *for all* $i = 1,2,...,n$ in $G_K^{'}$, then

$$LM_3(G_K^{'}) = LM_3(G)+2(2r-1)mt$$

$$+\sum_{i=1}^{n}\deg(v_i:G)S_i+t\sum_{i=1}^{n} d_2(v_i:G)+(2r-1)nt^2$$

$$-nt+t\sum_{i=1}^{n} S_i.$$

2.7 Generalized Thorn Graph $G_A^{'}$

Observation 7

Let (X,Y) be a bipartition of the vertex set of t_i-copy of $K_{r,s}$ and let $V(X)=\{x_1,x_2,...,x_r\}$ and $V(Y)=\{y_1,y_2,...,y_s\}$. Then the degree and 2-degree of any vertex in $G_A^{'}$ are as follows:

(1) $\deg(v_i:G_A^{'}) = \deg(v_i:G)+t_i$

(2) $d_2(v_i:G_A^{'}) = d_2(v_i:G)+st_i+S_i$

(3) $\deg(x_j:G_A^{'}) = s,\ where$ $x_j\in V(G_A^{'})\setminus\{N(v_i:G_A^{'})\cup V(G)\},2\le j\le r$

(4) $d_2(x_j:G_A^{'}) = r-1,\ where$ $x_j\in V(G_A^{'})\setminus\{N(v_i:G_A^{'})\cup V(G)\},2\le j\le r$

(5) $\deg(x_1:G_A^{'}) = s+1,\ where\ \ x_1\in N(v_i:G_A^{'})\setminus V(G)$

(6) $d_2(x_1:G_A^{'}) = (r-1)+\deg(v_i:G)+(t_i-1)$

where $x_1\in N(v_i:G_A^{'})\setminus V(G)$

(7) $\deg(y_k:G_A^{'}) = r, 1\le k\le s$

(8) $d_2(y_k:G_A^{'}) = s,\ 1\le k\le s$

Theorem 20

$$LM_3(G_A^{'}) = LM_3(G)+$$

$$\sum_{i=1}^{n}\deg(v_i:G)[s(s+1)t_i+S_i]$$
$$+\sum_{i=1}^{n}d_2(v_i:G)t_i+\sum_{i=1}^{n}t_iS_i$$
$$+s(s+1)\sum_{i=1}^{n}t_i^2+[3s(r-1)-2]\sum_{i=1}^{n}t_i.$$

Proof:

By virtue of Observation 7, we have

$$LM_3(G_A^{'})=\sum_{v\in V(G_A^{'})}\deg(v:G_A^{'})d_2(v:G_A^{'})$$
$$=\sum_{v_i\in V(G)}\deg(v_i:G_A^{'})d_2(v_i:G_A^{'})$$
$$+\sum_{y_k\in V(G_A^{'})}\deg(y_k:G_A^{'})d_2(y_k:G_A^{'})$$
$$+\sum_{x_j\in V(G_A^{'})\backslash\{N(v_i:G_A^{'})\cup V(G)\}}\deg(x_j:G_A^{'})d_2(x_j:G_A^{'})$$
$$+\sum_{x_1\in N(v_i:G_A^{'})\backslash V(G)}\deg(x_1:G_A^{'})d_2(x_1:G_A^{'})$$
$$=\sum_{i=1}^{n}[\deg(v_i:G)+t_i][d_2(v_i:G)+st_i+S_i]$$
$$+rs\sum_{i=1}^{n}t_i+s(r-1)\sum_{i=1}^{n}t_i$$
$$+(s+1)\sum_{i=1}^{n}t_i[(r-1)+\deg(v_i:G)+(t_i-1)]$$

Thus the result follows.

Corollary 21

If $t_i=t,\ for\ all\ i=1,2,...,n$ in $G_A^{'}$, then

$$LM_3(G_A^{'})=LM_3(G)+2s(s+1)mt$$
$$+\sum_{i=1}^{n}\deg(v_i:G)S_i+t\sum_{i=1}^{n}d_2(v_i:G)$$
$$+t\sum_{i=1}^{n}S_i+s(s+1)nt^2+3s(r-1)nt-2nt.$$

IV. CONCLUSION

The exact values of the third leap Zagreb index of some generalize thorn graphs have been determined. Results on the second leap Zagreb index of these graph structures will be reported in near future.

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