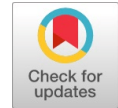


Mathematical Modelling of Two-Wheeler Suspension system in a wavy road



Suresh N, L. Balamurugan

Abstract: Two wheelers like motorbikes and scooters are one of the major transports in India. In major cities and towns, it is most common private transport as it is fast and easy approach to the destination. But the prolonged drive in the two-wheeler leads to the potential health hazard and musco-skeletal disorder due to continuous exposure to the vibration caused during the ride and force transmitted to the vehicle body due to road irregularities. It is a challenge of automobile engineers to design a promising suspension system to overcome the risk of ride comfort during continuous driving. In this research, two-wheeler suspension system is modelled with a condition of bump and valley in a wavy road. The road surface is assumed to be wavy and the response of new suspension spring with different materials (stainless steel, tungsten and polymeric) along with viscous damper is analyzed and compared. By this analysis, it will be proposed to industry to modify the suspension system to improve its efficiency and reduce force transmitted to the human body to improve the ride comfort.

Keywords: Suspension system, ride comfort, mathematical modelling, wavy road.

I. INTRODUCTION

Due to developing economy, and the increase in usage of private owned vehicles in India, there is reasonable increase in number of cars, bikes and scooters in recent years. [1] Debashish Dutta says, in past 50 years, Indian Automotive industry grows about 170 times whereas the road infrastructure has expanded only nine times. This leads to a greater number of vehicles rapidly occupying roads of major cities and towns. Meanwhile, the road conditions are not promising either due to poor maintenance or due to damage during bad weather conditions. Among the increasing segment of all automobiles, two-wheeler segments are bit more usage in major cities. This is because to avoid traffics and approach the destination much faster when compared to usage of cars. But the continuous usage of two-wheelers had potential health hazard due to sitting posture, driving operations like clutching and shifting gears frequently and braking frequently during peak hours driving. During continuous ride in traffic hours, irrespective of skillful driving, the road movements and neighboring vehicles approach may lead to discomfort during driving. This discomfort may be caused due to the continuous force and

vibration transmitted to the body because of road irregularities. It cannot be completely avoided. But it may be reduced by using improvised suspension system. It may reduce the risk to a certain limit. Generally, ergonomics aspects in design of two-wheeler are not much focused. In 2014, [2] Tanelli, Savaresi, M Corno, compiled a book titled "Modelling, Simulation and Control of Two-wheeled Vehicles" which consolidates the research publications related to two-wheeler dynamics. Valentin Keppler of Biomotion solutions modelled a motor bike as Biomechanical rider model and simulate the behavior of vibration response of rider. The result of the multi body simulation proposes the considering human factor into account during the engineering design helps to accelerate the development process. In this research, the response of newly proposed material for suspension system is analyzed. It is desired that the less amount of displacement (amplitude) transmitted through the seating position transfers less transmitted force. The less the force transmitted, the amount of human body withstands more time during riding before reaching driving discomfort. To analyze the maximum displacement (amplitude) transferred in the vehicle, it is proposed to form a mathematical model of rear suspension system of two-wheeler. The model is similar to a quarter car model but the structure of vehicle differs. [3] S.Natsiavas in 2005 develops a Design Optimization of a quarter-car model with semi-active suspension system using random road excitation. Later many of them developed a quarter car model for analysis. This model differs from quarter car model. Here tyre stiffness is not considered as in case of two-wheelers, the load transmitted to the tyres does not have reasonable deflection because of weight of the vehicle is comparatively low with cars. In 2017 [4] Yousuf Abdul Hameed develops a new active suspension system for car model. This active control system designed based on PID Controller and hydraulic actuator. Whereas, implementing active suspension system in two-wheeler is a costlier solution but model can be made. It may be investigated for further research. In this research, models are restricted to passive and semi active control system only.

II. MATERIAL SELECTION

Springs are most commonly used suspension device in any automobiles. A viscous damping device will associate the springs. The stainless-steel springs are widely used for automotive suspension system. In this research, a comparative study of two different material with stainless-steel is made. The range of stiffness or spring constant for stainless-steel spring ranges from [5] 4.9 N/mm to 29.4 N/mm approximately.

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Whereas for Tungsten springs it is higher. The spring constant for the Tungsten Spring starts in the range of ^[6]10 N/mm. Meanwhile, polymer spring has different combinations of spring constant and depends on reinforcing materials. It ranges from ^[7] 4.83 N/mm for glass fiber followed by 5.75 N/mm & 6.36 N/mm for Glass-Carbon fiber and Carbon fiber respectively. From the above information, it can be observed that Tungsten based springs have relatively higher stiffness when compared to stainless-steel and polymer-based springs are relatively lesser stiffness when compared to stainless-steel springs and hence these two materials are selected for analysis.

III. MATHEMATICAL MODELLING

Mathematical modelling is nothing but deriving a governing differential equation for the spring mass damper system provided in the two-wheeler suspension. It may be noted that in this research, the road waviness is also modelled as a sinusoidal wave and the impact of the road waviness on the vehicle body is analyzed. The physical structure of the vehicle suspension is as shown in Fig 1a and Fig 1b. These Fig. 1a and 1b are distinguishing suspension system for Motorbikes and Scooters respectively. With that as input, a model of suspension system is developed as shown in Fig.2



Fig 1a



Fig 1b

Based on the above reference, a spring mass system in is modelled for mathematical analysis a wavy road which is approximated to a sinusoidal wave form.

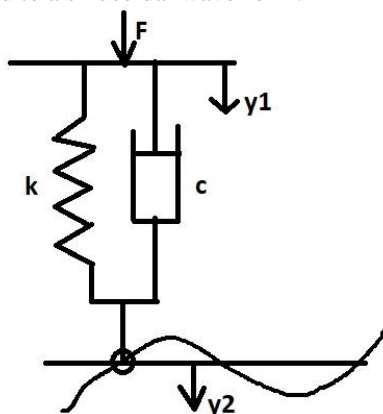


Fig 2

The general governing differential equation for the above motion may be represented as follows.

$$m\ddot{y}_1 + c(\dot{y}_1 - \dot{y}_2) + k(y_1 - y_2) = F \dots (1)$$

where m is the mass of the vehicle, which is supported by the spring and damper, c is the damping coefficient of the viscous damper, y_2 is the amplitude caused due to the road waviness and F is the harmonic force or disturbance due to the vehicle

velocity and y_1 is the amplitude caused and transmitted to the vehicle body which affects the vehicle rider. The term y_2 may be approximated to the wave form as follows.

$$y_2 = A \cos \omega t \dots (2)$$

Based on the above equation, a MATLAB model is developed for analyzing the road response of this spring mass damper model for various parameters, viz, different materials, different road profile, different speed of the vehicle. The parameters for analysis are tabulated in the following tables.

Table- I: Material Properties

Material	Stiffness (or) Spring Constant (N/mm), k	Damping Coefficient of fluid (N/m/s), c
Stainless Steel	4.9	400
Stainless Steel	5.9	400
Tungsten	10	400
Tungsten	12	400
Polymer – Glass fiber	4.83	400
Polymer – Carbon-Glass fiber	5.75	400
Polymer – Carbon fiber	6.83	400

The road profile may be approximated to a curve with different amplitudes and different wave lengths. In this approximation, different heights and lengths of roads are considered for analysis. It is based on the real road profile taken from city sub urban areas which is shown in figure 3a, 3b, 3c & 3d.



Fig 3a



Fig 3b



Fig 3c



Fig 3d

Figure 4 represents the pictorial representation of the road profile when vehicle travels in a wavy road. The length (L) and amplitude (A) differ for different profiles of the road and based on real road profiles the data are approximated and tabulated in Table 2.

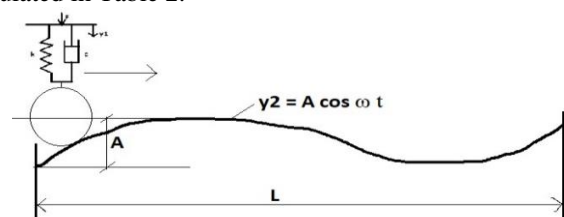


Fig 4

Table- II: Road Profile Data

Length of wavy road (L), m	Height/ Amplitude of the road (A), m	Velocity of the Vehicle, (v), km/hr
1.0	0.10	10, 15, 20 & 25
1.0	0.15	10, 15, 20 & 25
1.0	0.20	10, 15, 20 & 25
1.0	0.25	10, 15, 20 & 25
1.2	0.10	10, 15, 20 & 25
1.2	0.15	10, 15, 20 & 25
1.2	0.20	10, 15, 20 & 25

The term ω in the equation $y_2 = A \cos \omega t$ is the circular natural frequency and it depends on the velocity of the vehicle and the length with which it travels the road. Table 3 shows the various frequencies in which the road profile data varies according to the speed and its corresponding amplitude y_2 . The relationship between velocity and wavelength viz. the length of the road profile gives the frequency of the movement. From the frequency, circular frequency may be determined using the relation as follows.

$$f = L \times v \dots (3)$$

$$\omega = 2 \pi f \dots (4)$$

From the above two expressions, the amplitude of road profiles for various combinations of speed and length of road profile, Table 3 is tabulated as shown.

IV. RESULTS AND DISCUSSION

The response of the road profile for various length and height of the bump to the suspension system and maximum deflection it transferred to the vehicle body which is termed as y_1 is found for various materials given in Table 1. Since, viscous damping fluid's effect is not considered in the analysis, it is taken as $c = 400$ N/m/s for all the analysis. The results are given in the upcoming figures from 5a, 5b, 5c and 5d to 6, 7 and up to 8 a, b, c and d correspondingly.

$\sigma = 4.9$ N/mm (Stainless Steel)
$s = 5.9$ N/mm (Stainless Steel)
$s = 10$ N/mm (Tungsten)
$s = 12$ N/mm (Tungsten)
$s = 4.83$ (Polymer Glass fiber)
$s = 5.74$ (Polymer Carbon-Glass fiber)
$s = 6.36$ (Polymer Carbon fiber)

Fig 5

Table- III: Amplitude of Road Profile

Speed, v (m/s)	Circular frequency ω (rad/sec)		Amplitude of road profile $y_2 = A \cos \omega t$	
	L=1m	L=1.2m	L=1m	L=1.2m
2.78	17.46	20.96	0.1 Cos 17.46 t	0.1 Cos 20.96 t
			0.15 Cos 17.46 t	0.15 Cos 20.96 t
			0.20 Cos 17.46 t	0.20 Cos 20.96 t
			0.25 Cos 17.46 t	0.25 Cos 20.63 t
4.17	26.20	31.44	0.1 Cos 26.20 t	0.1 Cos 31.44 t
			0.15 Cos 26.20 t	0.15 Cos 31.44 t
			0.20 Cos 26.20 t	0.20 Cos 31.44 t
			0.25 Cos 26.20 t	0.25 Cos 31.44 t
			0.1 Cos 34.93 t	0.1 Cos 41.92 t

Speed, v (m/s)	Circular frequency ω (rad/sec)		Amplitude of road profile $y_2 = A \cos \omega t$	
5.56	34.93	41.92	0.15 Cos 34.93 t	0.15 Cos 41.92 t
			0.20 Cos 34.93 t	0.20 Cos 41.92 t
			0.25 Cos 34.93 t	0.25 Cos 41.92 t
6.94	43.60	52.32	0.1 Cos 43.60 t	0.1 Cos 52.32 t
			0.15 Cos 43.60 t	0.15 Cos 52.32 t
			0.20 Cos 43.60 t	0.20 Cos 52.32 t
			0.25 Cos 43.60 t	0.25 Cos 52.32 t

The graph (x axis – time & y-axis amplitude) plotted is the output response of the displacement (amplitude), y_1 of the vehicle body that exerts through the spring and damper provided in the system. There are seven different types of material with various spring constant is compared to determine the optimum one which exerts least amplitude for the given time for the maximum load. For the velocity of 10 km/hr (2.78 m/sec), and for the length of bump of about 1m and the response of bump height of 0.1 m to 0.25 m.

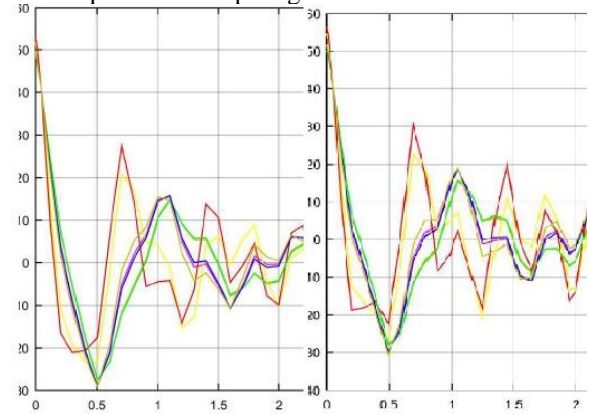


Fig 5a

Fig 5b

From the graph (Fig 5a to 5d) it may be observed that the maximum amplitude exerts ranges from 60 mm to 50 mm for the height of road ranges from 0.1 to 0.25 mm. The curve stabilized after about 2 seconds and the maximum amplitude reduction found to be with Tungsten ($s = 12$ N/mm) and gradual reduction of amplitude with respect to time is found with Polymer ($s = 5.74$ N/mm).

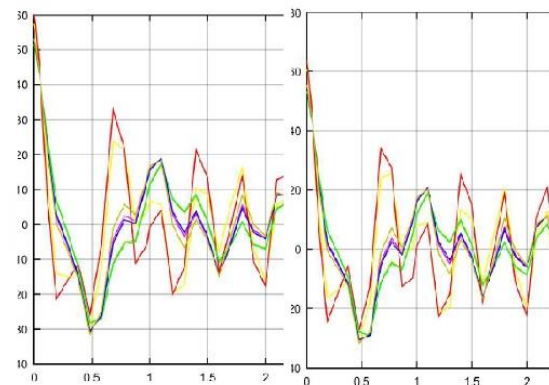


Fig 5c

Fig 5d

From the graph (Fig 6a to 6d) it may be observed that the maximum amplitude exerts ranges from 70 mm to 75 mm for the height of road ranges from 0.1 to 0.25 mm. The curve stabilized after about 2 seconds and the maximum amplitude reduction found to be with Tungsten ($s=12$ N/mm) and gradual reduction of amplitude with Polymer ($s=5.74$ N/mm). The curve for the material stainless steel ($s=5.79$ N/mm) almost aligns with the polymer ($s=5.74$ N/mm).

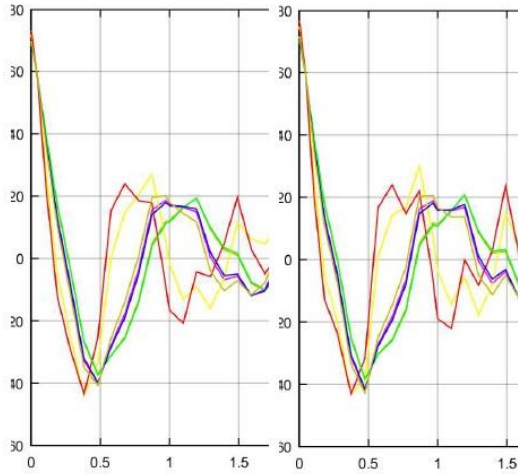


Fig 6a

Fig 6b

From the graph (Fig 7a to 7d) it may be observed that the maximum amplitude exerts ranges from 280 mm and 300mm and above. The steadiness of the amplitude reaches after about 3 seconds. The maximum amplitude reduction is found to be with Tungsten ($s=12$ N/mm) and polymer ($s=5.74$ N/mm) and Stainless steel ($s=5.79$ N/mm) both having similar curve structure. The higher the speed also, the material response remains same.

From the graph (Fig 8a to 8d), it may be observed that the amplitude exerts ranges from 300 mm to 400mm and above. The curve stabilized after about 3.5 seconds only and the material of the analysis have almost similar response in its result. From this analysis, it may be noted that, the amplitude exerts in the vehicle body is reduced in faster rate within short time span for Tungsten ($s=12$ N/mm) material and much gradual, even it takes a bit more time than Tungsten in Polymer with Carbon-Glass fiber ($s=5.74$ N/mm)

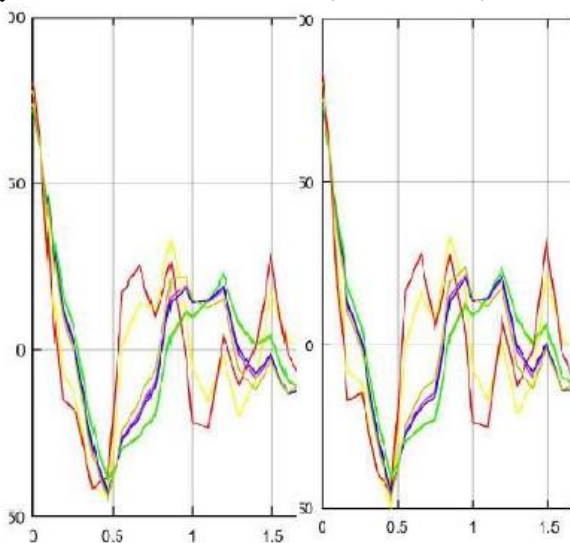


Fig 6c

Fig 6d

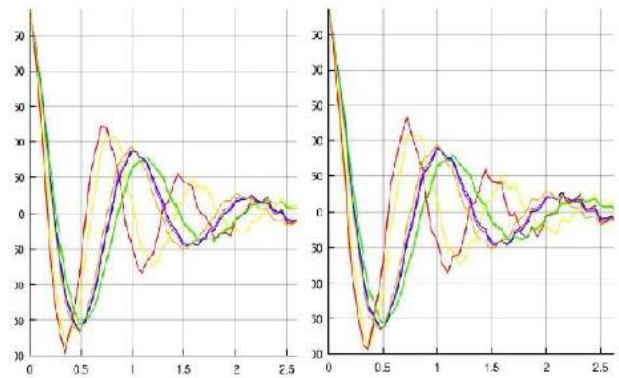


Fig 7a

Fig 7b

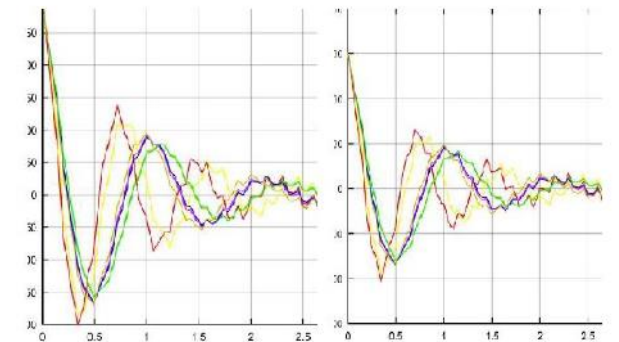


Fig 7c

Fig 7d

The comparative analysis of materials for its best damping with the aid of mathematical modelling and MATLAB analysis is done. The few results shown gives the output for Tungsten material and polymer with carbon-glass fiber is much better in certain aspects when compared with other materials which is taken for analysis.

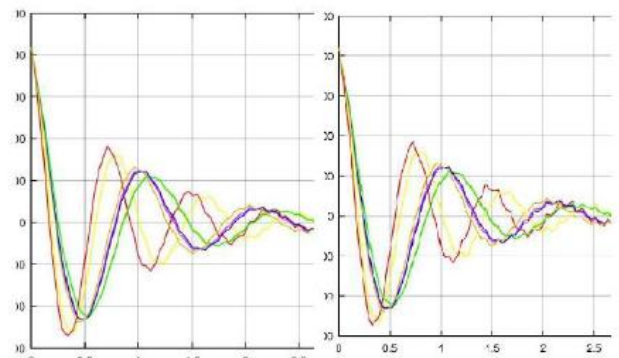


Fig 8a

Fig 8b

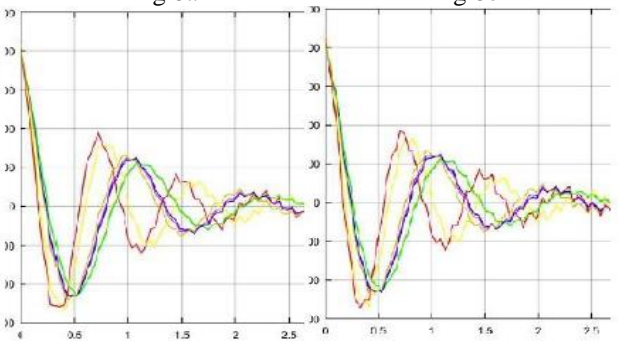


Fig 8c

Fig 8d

V. CONCLUSION

The ride comfort in any vehicle is important aspect for which the amplitude of vibration must be lesser when the vehicle travel past any irregular surface. The surface of road taken for analysis is irregular one but approximated to a sinusoidal form for the study and analysis. Stainless steel is mainly used material for spring in any spring mass damper system. For this research analysis, two more materials are considered, one is Tungsten, which is having little high spring constant when compared to Stainless steel and another one is Polymers which is having little lower spring constant than stainless steel. Based on the analysis and it is profound that the Tungsten having improved damping properties when compared with steel. It may be noted that the Tungsten based alloys having higher stiffness so that irrespective of any high amplitudes, it absorbed the energy and reduce the amplitude within lesser time when compared to steel. Although tungsten has good amplitude reduction factor, the gradual reduction of amplitude occurs in Polymer springs reinforced with Carbon & glass fiber. The polymer springs may be having good damping characteristics but even then, the problem of manufacturing homogenous composite spring is biggest challenge. Hence, Tungsten based springs are more optimum choice for the alternative to stainless steel. The research may be further developed to analyzed for semi active and active suspension system for new nano-based materials.

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