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Qualitative Inspection Fourth order Non Linear Difference Equations

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Abstract— In this paper, the authors obtained some new sufficient conditions for the oscillation of all solutions of the fourth order nonlinear difference equation of the form $\Delta^3(a_n\Delta x_n) - p_n\Delta x_n + q_n f(x_{n+1}) = 0$ n = 0,1,2, … ., where a_n, p_n, q_n positive sequences. The established results extend, unify and improve some of the results reported in the literature. Examples are provided to illustrate the main result.

Key words— Oscillation, Non Oscillation, Neutral Delay, Difference Equation, Positive sequence.AMS subject classification: 39A10

I. INTRODUCTION

We are concerned with the oscillatory properties of all solutions of fourth order linear and non- linear difference equations of the form

$$\Delta^3(a_n\Delta x_n) - p_n\Delta x_n + q_n f(x_{n+1}) = 0 \qquad (1.1)$$

Where the following conditions are assumed to hold.

(H1) $\{a_n\}$, $\{p_n\}$, $\{q_n\}$ are real positive sequences where $n\in N=\{0,1,2,\dots,\}$.

(H2) $f: R\rightarrow R$ is continous and $xf(x)>0$ for all $x\neq 0$.

(H3) $\frac{(n+1)(a_{n+2}\Delta x_{n+2} - a_{n+1}\Delta x_{n+1})}{f(x_{n+1})} < \infty$

By a solution of equation (1.1), we mean a real sequence $\{x_n\}$ satisfying (1.1) for n= 0, 1….. A solution $\{x_n\}$ of (1.1) is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise, it is called non oscillatory. Δ Is the forward difference operator defined by $\Delta x_n = x_{n+1} - x_n$. Fourth order Neutral Delay Difference Equation is gaining interest because they are the discrete analogue of differential Equations. In recent years, several papers on oscillation of solutions of Neutral Delay Difference Equations have appeared. [3]S.S. Cheng and Y.Z. Lin (Lin, 1998) have provided a complete characterization of oscillation solutions of first order Neutral Delay Difference Equations with positive coefficient in the Neutral term. [6]John R.Graef, R.Savithri, E.Thandapani (John R. Graef, 2004) have analyzed the non oscillatory solutions of first order Neutral Delay Differential Equations with positive coefficient in the Neutral term. [4]Elmetwally M.Elabbasy, Taher S.Hassan, Samir H.Saker (Elmetwally M. Elabbasy, 2005) have provided oscillatory solutions of first order Neutral Delay Differential Equations with negative coefficient in the Neutral term.

[8]Ozkan Ocalan extensively discuss the problem of Oscillation of neutral differential equation with positive and negative coefficients, J.Math.Anal.Appl.

[9]Tanaka, S. (2002) discussed the various Solutions of Oscillation First order Neutral Delay Differential Equations.

Here some oscillation results in difference equations based on the existence results of differential equations are provided. Examples are provided to illustrate the results.

In recent years, much research is going on the study of oscillatory behaviour of solutions fourth order difference equations.

II. MAIN RESULTS

In this section, we present some sufficient conditions for the oscillation of all solutions of (1.1)-(1.4).We begins with the following lemma.

Lemma 2.1 Let p(n,s,x) be defined on $N\times N\times R^+$, $N=\{0,1,2,..\}$, $R^+=[0,\infty)$, such that for fixed n and s , the function P(n,s,x) is non-decreasing in x. Let $\{r_n\}$ be a given sequences and the sequences $\{x_n\}$ and $\{z_n\}$ be defined on N satisfying, for all $n\in N$,

$$x_n \geq \sum_{s=0}^{n-1} P(n,s,x_s) r_n \qquad (2.1)$$

and

$$z_n \geq r_n + \sum_{s=0}^{n-1} P(n,s,z_s) \qquad (2.2)$$

Then $z_n \leq x_n$ for all $n\in N$

2.2 Theorem

If H_1, H_2, H_3 holds, Then every solution of $\Delta^3(a_n\Delta x_n) - p_n\Delta x_n + q_n f(x_{n+1}) = 0$ is oscillatory.

Proof:

Suppose $\{x_n\}$ be a non-oscillatory solution of $\Delta^3(a_n\Delta x_n) - p_n\Delta x_n + q_n f(x_{n+1}) = 0$,

such that $x_n > 0$ (or $x_n < 0$) for all $n \geq M-1$, $M > 0$ is an integer.

Then (1.1) becomes,

$$\Delta(a_{n+2}\Delta x_{n+2} - 2\Delta(a_{n+1}\Delta x_{n+1}) + \Delta(a_n\Delta x_n) - p_n\Delta x_n + q_n f(x_{n+1}) = 0 \qquad (2.3)$$

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Multiplying Eqn. (2.3) by $\dfrac{n+1}{f(x_{n+1})}$ and summing from M to $(n-1)$, we obtain

$$\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+2}\Delta x_{s+2})-2\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+1}\Delta x_{s+1})+\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_s\Delta x_s)$$
$$-\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)p_s\Delta x_s+\sum_{s=M}^{n-1}(s+1)q_s=0 \tag{2.4}$$

$$\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+2}\Delta x_{s+2})=\left(\left(\frac{s+1}{f(x_{s+1})}\right)(a_{s+2}\Delta x_{s+2}\right)_{s=M}^{n}-\sum_{s=M}^{n-1}\Delta\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+3}\Delta x_{s+3})$$

Using $\Delta\left(\dfrac{f}{g}\right)=\dfrac{g(x)\Delta f(x)-f(x)\Delta g(x)}{g(x+h)g(x)}$

$$\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+2}\Delta x_{s+2})$$
$$=\frac{(n+1)a_{n+2}\Delta x_{n+2}}{f(x_{n+1})}-\frac{(M+1)a_{M+2}\Delta x_{M+2}}{f(x_{M+1})}-\sum_{s=M}^{n-1}\left[\frac{f(x_{s+1})\Delta(s+1)-(s+1)\Delta f(x_{s+1})}{f(x_{s+1})}\right](a_{s+3}\Delta x_{s+3})$$
$$=\frac{(n+1)a_{n+2}\Delta x_{n+2}}{f(x_{n+1})}-\frac{(M+1)a_{M+2}\Delta x_{M+2}}{f(x_{M+1})}-\sum_{s=M}^{n-1}\left[\frac{f(x_{s+1})\Delta(s+1)-(s+1)[f(x_{s+2})-f(x_{s+1})]}{f(x_{s+2})f(x_{s+1})}\right](a_{s+3}\Delta x_{s+3})$$

By using $f(u)-f(v)=g(u,v)(u-v)$

$$=\frac{(n+1)a_{n+2}\Delta x_{n+2}}{f(x_{n+1})}-\frac{(M+1)a_{M+2}\Delta x_{M+2}}{f(x_{M+1})}-\sum_{s=M}^{n-1}\frac{a_{s+3}\Delta x_{s+3}}{f(x_{s+2})}+\sum_{s=M}^{n-1}\left[\frac{(s+1)g(x_{s+2},x_{s+1})\Delta x_{s+1}a_{s+3}\Delta x_{s+3}}{f(x_{s+2})f(x_{s+1})}\right]$$

$$\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+1}\Delta x_{s+1})=\left(\left(\frac{s+1}{f(x_{s+1})}\right)(a_{s+1}\Delta x_{s+1}\right)_{s=M}^{n}-\sum_{s=M}^{n-1}\Delta\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+2}\Delta x_{s+2})$$

Using $\Delta\left(\dfrac{f}{g}\right)=\dfrac{g(x)\Delta f(x)-f(x)\Delta g(x)}{g(x+h)g(x)}$

$$\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+1}\Delta x_{s+1})$$
$$=\frac{(n+1)a_{n+1}\Delta x_{n+1}}{f(x_{n+1})}-\frac{(M+1)a_{M+1}\Delta x_{M+1}}{f(x_{M+1})}-\sum_{s=M}^{n-1}\left[\frac{f(x_{s+1})\Delta(s+1)-(s+1)\Delta f(x_{s+1})}{f(x_{s+1})}\right](a_{s+2}\Delta x_{s+2})$$
$$=\frac{(n+1)a_{n+1}\Delta x_{n+1}}{f(x_{n+1})}-\frac{(M+1)a_{M+1}\Delta x_{M+1}}{f(x_{M+1})}-\sum_{s=M}^{n-1}\left[\frac{f(x_{s+1})\Delta(s+1)-(s+1)[f(x_{s+2})-f(x_{s+1})]}{f(x_{s+2})f(x_{s+1})}\right](a_{s+2}\Delta x_{s+2})$$

By using $f(u)-f(v)=g(u,v)(u-v)$

$$=\frac{(n+1)a_{n+1}\Delta x_{n+1}}{f(x_{n+1})}-\frac{(M+1)a_{M+1}\Delta x_{M+1}}{f(x_{M+1})}-\sum_{s=M}^{n-1}\frac{a_{s+2}\Delta x_{s+2}}{f(x_{s+2})}+\sum_{s=M}^{n-1}\left[\frac{(s+1)g(x_{s+2},x_{s+1})\Delta x_{s+1}a_{s+2}\Delta x_{s+2}}{f(x_{s+2})f(x_{s+1})}\right] \tag{2.5}$$

$$\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_s\Delta x_s)=\left(\left(\frac{s+1}{f(x_{s+1})}\right)(a_s\Delta x_s\right)_{s=M}^{n}-\sum_{s=M}^{n-1}\Delta\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_{s+1}\Delta x_{s+1})$$

Using $\Delta\left(\dfrac{f}{g}\right)=\dfrac{g(x)\Delta f(x)-f(x)\Delta g(x)}{g(x+h)g(x)}$

$$\sum_{s=M}^{n-1}\left(\frac{s+1}{f(x_{s+1})}\right)\Delta(a_s\Delta x_s)$$
$$=\frac{(n+1)a_n\Delta x_n}{f(x_{n+1})}-\frac{(M+1)a_M\Delta x_M}{f(x_{M+1})}-\sum_{s=M}^{n-1}\left[\frac{f(x_{s+1})\Delta(s+1)-(s+1)\Delta f(x_{s+1})}{f(x_{s+1})}\right](a_{s+1}\Delta x_{s+1})$$
$$=\frac{(n+1)a_n\Delta x_n}{f(x_{n+1})}-\frac{(M+1)a_M\Delta x_M}{f(x_{M+1})}-\sum_{s=M}^{n-1}\left[\frac{f(x_{s+1})\Delta(s+1)-(s+1)[f(x_{s+2})-f(x_{s+1})]}{f(x_{s+2})f(x_{s+1})}\right](a_{s+1}\Delta x_{s+1})$$

By using $f(u)-f(v)=g(u,v)(u-v)$

$$=\frac{(n+1)a_n\Delta x_n}{f(x_{n+1})}-\frac{(M+1)a_M\Delta x_M}{f(x_{M+1})}-\sum_{s=M}^{n-1}\frac{a_{s+3}\Delta x_{s+3}}{f(x_{s+2})}+\sum_{s=M}^{n-1}\left[\frac{(s+1)g(x_{s+2},x_{s+1})(\Delta x_{s+1})^2a_{s+1}}{f(x_{s+2})f(x_{s+1})}\right] \tag{2.6}$$

Substituting Eqn. (2.4),(2.5) & (2.6) in (2.3)

$$\left(\frac{(n+1)a_{n+2}\Delta x_{n+2}}{f(x_{n+1})}-2\frac{(n+1)a_{n+1}\Delta x_{n+1}}{f(x_{n+1})}+\frac{(n+1)a_n\Delta x_n}{f(x_{n+1})}\right)-\left(\sum_{s=M}^{n-1}\frac{a_{s+3}\Delta x_{s+3}}{f(x_{s+2})}-2\sum_{s=M}^{n-1}\frac{a_{s+2}\Delta x_{s+2}}{f(x_{s+2})}+\sum_{s=M}^{n-1}\frac{a_{s+1}\Delta x_{s+1}}{f(x_{s+2})}\right)$$
$$+\sum_{s=M}^{n-1}\left[\frac{(s+1)g(x_{s+2},x_{s+1})\Delta x_{s+1}\Delta x_{s+3}a_{s+3}}{f(x_{s+2})f(x_{s+1})}\right]-2\sum_{s=M}^{n-1}\left[\frac{(s+1)g(x_{s+2},x_{s+1})\Delta x_{s+1}\Delta x_{s+2}a_{s+2}}{f(x_{s+2})f(x_{s+1})}\right]+\sum_{s=M}^{n-1}\left[\frac{(s+1)g(x_{s+2},x_{s+1})(\Delta x_{s+1})^2a_{s+1}}{f(x_{s+2})f(x_{s+1})}\right]$$
$$=\sum_{s=M}^{n-1}\frac{(M+1)a_{M+2}\Delta x_{M+2}}{f(x_{M+1})}-2\sum_{s=M}^{n-1}\frac{(M+1)a_{M+1}\Delta x_{M+1}}{f(x_{M+1})}+\sum_{s=M}^{n-1}\frac{(M+1)a_M\Delta x_M}{f(x_{M+1})} \tag{2.7}$$

Using Schwarz's inequality, we have

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2\le\sum_{i=1}^{n}a_i^2\sum_{i=1}^{n}b_i^2$$

$$\sum_{s=M}^{n-1}\left(\frac{a_{s+3}\Delta x_{s+3}}{f(x_{s+2})}\right)\le\left(\sum_{s=M}^{n-1}a_{s+3}^2\right)^{1/2}\left(\sum_{s=M}^{n-1}\frac{(\Delta x_{s+3})^2}{f^2(x_{s+2})}\right)^{1/2} \tag{2.8}$$

$$\sum_{s=M}^{n-1}\left(\frac{a_{s+2}\Delta x_{s+2}}{f(x_{s+2})}\right)\le\left(\sum_{s=M}^{n-1}a_{s+2}^2\right)^{1/2}\left(\sum_{s=M}^{n-1}\frac{(\Delta x_{s+2})^2}{f^2(x_{s+2})}\right)^{1/2} \tag{2.9}$$

$$\sum_{s=M}^{n-1}\left(\frac{a_{s+1}\Delta x_{s+1}}{f(x_{s+2})}\right)\le\left(\sum_{s=M}^{n-1}a_{s+1}^2\right)^{1/2}\left(\sum_{s=M}^{n-1}\frac{(\Delta x_{s+1})^2}{f^2(x_{s+2})}\right)^{1/2} \tag{2.10}$$

$$\sum_{s=M}^{n-1}\left(\frac{(s+1)g(x_{s+2},x_{s+1})\Delta x_{s+1}a_{s+3}\Delta x_{s+3}}{f(x_{s+2})f(x_{s+1})}\right)\le\left(\sum_{s=M}^{n-1}a_{s+3}^2\right)^{1/2} \tag{2.11}$$
$$\sum_{s=M}^{n-1}\left(\frac{(s+1)^2g^2(x_{s+2},x_{s+1})(\Delta x_{s+1})^2(\Delta x_{s+3})^2}{f^2(x_{s+2})f^2(x_{s+1})}\right)^{1/2}$$

$$\sum_{s=M}^{n-1}\left(\frac{(s+1)g(x_{s+2},x_{s+1})\Delta x_{s+1}a_{s+2}\Delta x_{s+2}}{f(x_{s+2})f(x_{s+1})}\right)\le\left(\sum_{s=M}^{n-1}a_{s+2}^2\right)^{1/2}\sum_{s=M}^{n-1}\left(\frac{(s+1)^2g^2(x_{s+2},x_{s+1})(\Delta x_{s+1})^2(\Delta x_{s+2})^2}{f^2(x_{s+2})f^2(x_{s+1})}\right)^{1/2} \tag{2.12}$$

$$\sum_{s=M}^{n-1}\left(\frac{(s+1)g(x_{s+2},x_{s+1})(\Delta x_{s+1})^2a_{s+1}}{f(x_{s+2})f(x_{s+1})}\right)\le\left(\sum_{s=M}^{n-1}a_{s+1}^2\right)^{1/2}\sum_{s=M}^{n-1}\left(\frac{(s+1)^2g^2(x_{s+2},x_{s+1})(\Delta x_{s+1})^4}{f^2(x_{s+2})f^2(x_{s+1})}\right)^{1/2} \tag{2.13}$$

$$\sum_{s=M}^{n-1}\left(\frac{(s+1)p_s\Delta x_s}{f(x_{s+1})}\right)\le\left(\sum_{s=M}^{n-1}(s+1)p_s^2\right)^{1/2}\sum_{s=M}^{n-1}\left(\frac{(s+1)(\Delta x_s)^2}{f^2(x_{s+1})}\right)^{1/2} \tag{2.14}$$

Substituting(2.8), (2.9), (2.10), (2.11), (2.12), (2.13) and (2.14) in (2.7)

$$\left(\frac{(n+1)a_{n+2}\Delta x_{n+2}}{f(x_{n+1})}-2\frac{(n+1)a_{n+1}\Delta x_{n+1}}{f(x_{n+1})}+\frac{(n+1)a_n\Delta x_n}{f(x_{n+1})}\right)-\left(\sum_{s=M}^{n-1}a_{s+3}^2\right)^{1/2}\left(\sum_{s=M}^{n-1}\frac{(\Delta x_{s+3})^2}{f^2(x_{s+2})}\right)^{1/2}+2\left(\sum_{s=M}^{n-1}a_{s+2}^2\right)^{1/2}\left(\sum_{s=M}^{n-1}\frac{(\Delta x_{s+2})^2}{f^2(x_{s+2})}\right)^{1/2}$$
$$-\left(\sum_{s=M}^{n-1}a_{s+1}^2\right)^{1/2}\left(\sum_{s=M}^{n-1}\frac{(\Delta x_{s+1})^2}{f^2(x_{s+2})}\right)^{1/2}+\left(\sum_{s=M}^{n-1}a_{s+3}^2\right)^{1/2}\sum_{s=M}^{n-1}\left(\frac{(s+1)^2g^2(x_{s+2},x_{s+1})(\Delta x_{s+1})^2(\Delta x_{s+3})^2}{f^2(x_{s+2})f^2(x_{s+1})}\right)^{1/2}-2\left(\sum_{s=M}^{n-1}a_{s+2}^2\right)^{1/2}$$
$$\sum_{s=M}^{n-1}\left(\frac{(s+1)^2g^2(x_{s+2},x_{s+1})(\Delta x_{s+1})^2(\Delta x_{s+2})^2}{f^2(x_{s+2})f^2(x_{s+1})}\right)^{1/2}+\left(\sum_{s=M}^{n-1}a_{s+1}^2\right)^{1/2}\sum_{s=M}^{n-1}\left(\frac{(s+1)^2g^2(x_{s+2},x_{s+1})(\Delta x_{s+1})^4}{f^2(x_{s+2})f^2(x_{s+1})}\right)^{1/2}-\left(\sum_{s=M}^{n-1}(s+1)p_s^2\right)^{1/2}$$
$$\sum_{s=M}^{n-1}\left(\frac{(s+1)(\Delta x_s)^2}{f^2(x_{s+1})}\right)^{1/2}=\left(\sum_{s=M}^{n-1}\frac{(M+1)a_{M+2}\Delta x_{M+2}}{f(x_{M+1})}-2\sum_{s=M}^{n-1}\frac{(M+1)a_{M+1}\Delta x_{M+1}}{f(x_{M+1})}+\sum_{s=M}^{n-1}\frac{(M+1)a_M\Delta x_M}{f(x_{M+1})}\right)-\sum_{s=M}^{n-1}(s+1)q_s$$

$$\left(\frac{(n+1)a_{n+2}\Delta x_{n+2}}{f(x_{n+1})}-2\frac{(n+1)a_{n+1}\Delta x_{n+1}}{f(x_{n+1})}+\frac{(n+1)a_n\Delta x_n}{f(x_{n+1})}\right)\to-\infty$$

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$$\frac{(n+1)[a_{n+2}\Delta x_{n+2} - 2(a_{n+1}\Delta x_{n+1}) + a_n \Delta x_n]}{f(x_{n+1})} \to -\infty$$

$$\frac{(n+1)[\Delta(a_{n+2}\Delta x_{n+2}) - a_{n+1}\Delta x_{n+1} + a_n \Delta x_n]}{f(x_{n+1})} \to -\infty$$

Which contradicts our assumption that $\{x_n\}$ is eventually positive.The proof is similar for the case when $\{x_n\}$ is eventually negative.

III. RESULTS & DISCUSSIONS

3.1 Example :

Consider the difference equation

$$\Delta^3(n\Delta x_n) - 8(2n+3)\Delta x_n + 24(2n+3)x_{n+1}^3 = 0 \qquad (3.1)$$

All the condition of theorem are satisfied. Hence every solution of (3.1) is oscillatory.

One such solution is $x_n = (-1)^n$

3.2 Example :

Consider the difference equation

$$\Delta^3\left(\frac{1}{n+2}\Delta x_n\right) + 2\left[\frac{7n^3+72n^2+233n+234}{(n+5)(n+4)(n+3)(n+2)}\right]\Delta x_n \qquad (3.2)$$

$$-2\left[\frac{7n^3+72n^2+233n+234}{(n+5)(n+4)(n+3)(n+2)}\right]x_{n+1}^2 = 0(3.2)$$

Here $a_n = \frac{1}{n+2}$, $p_n = -2\left[\frac{7n^3+72n^2+233n+234}{(n+5)(n+4)(n+3)(n+2)}\right]$ and

$$q_n f(x_{n+1}) = -2\left[\frac{7n^3+72n^2+233n+234}{(n+5)(n+4)(n+3)(n+2)}\right]$$

All the condition of theorem are satisfied. Hence every solution of (3.2) is oscillatory.

One such solution is $x_n = (-1)^{n+1}$

IV. CONCLUSION

In this paper, by using summation averaging techniques some new oscillation criteria for fourth order neutral delay difference equation is obtained. Examples are provided to illustrate the results. Further none of the resultsin the papers [3-9]can be applied to Eq. (1.1) to yield any conclusion.

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