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Achromatic Colouring of the Central Graph of Some Specialgraphs

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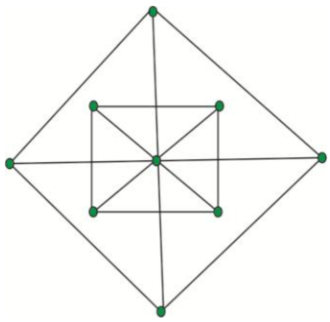
Abstract— In this research investigation, the achromatic number of central graph of double wheel graph, wind mill graph andn- anti prism graph have been studied. In addition the structural properties of these graphs have also been studied.

Key Words: Double wheel graph, Wind mill graph, Anti prism graph, achromatic number, b-chromatic number, Central graph Mathematics subject classification: 05C15

I. INTRODUCTION

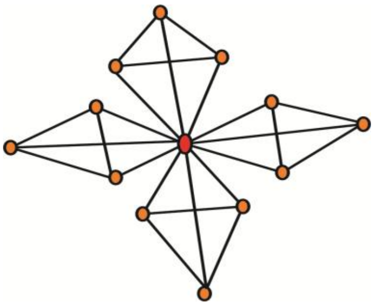
Consider a simple undirected graph G. To form its central graph $C(G)$, we introduce a new node on every edge in G and join those nodes of G that are not adjacent.The achromatic number was introduced by Harary. A proper vertex colouring is said to be achromatic if every pair of colours has at least one edge joining them . The achromatic number $\psi(G)$is the maximum number of colours in an achromatic colouring of G.

A double wheel graph $Dw_n$of size $n$is composed of $2C_n$and K_1. It consists of two cycles of size n where the vertices of the two cycles are connected to a central root vertex.



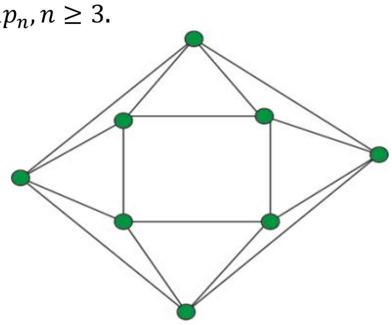
Double wheel Dw_4

The wind mill graph $Wm_n$is obtained by joining$n-$copies of the complete graph $K_n$with a vertex in common.



Wind mill graph Wm_4

An $n-$anti prism $n \geq 3$ is a semi regular polyhedron constructed with two$n-$gons and $2n-$triangles. It is made up of two $n-$gons on the top and bottom separated by a ribbon of $2n-$ triangles, with the two $n-$gons being offset by one ribbon. The graph corresponding to the skeleton of an $n-$anti prism is called the $n-$anti prism graph and is denoted by $Ap_n, n \geq 3$.



Anti prism graph Ap_4

Structural properties of central graph of double wheel graphDw_n

- The number of vertices in $Dw_n = p = 2n + 1$.
- The number of edges in $Dw_n = q = 4n$.
- The maximum degree in $C(Dw_n)$=$\Delta_{C[DW_n]}$= $2n$.
- The number of vertices in $C(Dw_n) = p_{C[DW_n]} = 6n + 1$.
- The number of edges in $C[Wm_n]$=$q_{C[DW_n]} = 2n^2 + 5n$.

Theorem: 1

For a double wheel graphDw_n, the achromatic number of $C(Dw_n)$is $\psi\big(C(Dw_n)\big) = 2n + 1, n \geq 3$.

Proof:

A double wheel graph $Dw_n$of size $n$is composed of $2C_n$and K_1. Consider the two sets of vertices

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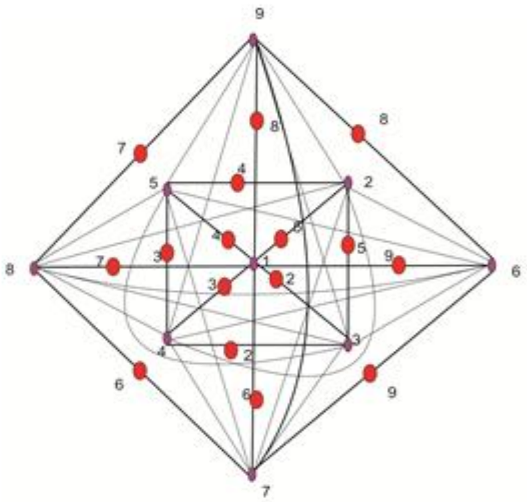
$\{a_0, a_1, \ldots, a_n\}$ and $\{b_1, b_2, \ldots, b_n\}$. Let a_0 be the root vertex. Allocate the names $a_0, a_1, \ldots, a_n$ to the remaining vertices of the inner wheel and $b_1, b_2, \ldots, b_n$ to the vertices of the outer wheel. In $C(Dw_n)$, for $1 \le i \le n-1$, let $a_{i,i+1}$ be the new vertex on the edge a_i, a_{i+1} and let $a_{1,n}$ be the vertex on the edge joining a_1, a_n. Let $b_{i,i+1}$ be the newly introduced vertex on the line joining b_i, b_{i+1} for $1 \le i \le n-1$. Let $a_{0,i}$ be the newly introduced vertex on the edge joining a_0, a_i. For $1 \le i \le n$, let $b_{0,i}$ be the newly introduced vertex on the edge joining a_0, b_i.

Let $C = \{C_0, C_1, C_2, \ldots C_n\}$ and $C' = \{C'_1, C'_2, \ldots C'_n\}$ be two colour sets. Allocate colour C_0 to the root vertex a_0. For $1 \le i \le n$ allocate C_i to a_i and C_i' to b_i. For creating an achromatic colouring, consider the colouring method as follows:

- For $1 \le i \le n-1$ allocate C_i to the vertex $a_{0,i+1}$.
- For $2 \le i \le n-1$ allocate the colour C_i to the vertex $a_{i,i+1}$.
- Allocate the colour C_n to the vertices $a_{0,1}$ and $a_{1,2}$.
- Allocate the colour C_{n-1} to the vertex $a_{1,n}$.
- For $1 \le i \le n-1$ allocate C_i' to the vertex $b_{0,i+1}$.
- For $2 \le i \le n-1$ allocate the colour C_i' to the vertex $b_{i,i+1}$.
- Allocate the colour C_n' to the vertices $b_{0,1}$ and $b_{1,2}$.
- Allocate the colour C_{n-1}' to the vertex $b_{1,n}$.

If a new colour C_{n+1}' is given to any vertex, the other colours will not be adjacent to it. Hence the maximum number of colours for an achromatic colouring is $|C| + |C'| = 2n + 1$.

Example



$\psi(C(Dw_4)) = 9$

Structural properties of the central graph of windmill graph

- The number of vertices in Wm_n=$p = n^2 - n + 1$
- The number of edges in Wm_n =$q = \frac{n^2(n-1)}{2}$.
- The maximum degree in Wm_n =$\Delta_{C[Wm_n]}$= $n(n-1)$.
- The number of vertices in $C[Wm_n]$ =$p_{C[Wm_n]} = \frac{n^3}{2} + \frac{n^2}{2} - n + 1$.
- The number of edges in $C[Wm_n]$ =$q_{C[Wm_n]} = \frac{n^3 - n^2 - 2n + 2}{2}$.

II. OBSERVATION & RESULTS

For a wind mill graph Wm_n, the achromatic number of $C(Wm_n)$ is $\psi(C(Wm_n)) = n^2 - n + 1, n \ge 3$.

Structural properties of the central graph of Antiprism graph Ap_n

- The number of vertices in Ap_n =$p = 2n$.
- The number of edges in Ap_n =$q = 4n$.
- The maximum degree in Ap_n =$\Delta_{C[Ap_n]}$= 4.
- The number of vertices in $C[Ap_n]$ =$p_{C[Ap_n]} = 6n$.
- The number of edges in $C[Ap_n]$ =$q_{C[Ap_n]} = 2n^2 + 3n$.

Theorem: 3

For an anti prism graph Ap_n, the achromatic number of $C(Ap_n)$ is $\psi(C(Ap_n)) = 2n + 1,\ n \ge 3$.

Proof:

Consider the two sets of vertices $U = \{a_1, \ldots, a_n\}$ and $V = \{b_1, b_2, \ldots, b_n\}$. Allocate the names in the set U to the vertices of the inner cycle in the clock wise direction and allocate the names in the set V to the vertices of the outer cycle in the same direction.

- In $C(Ap_n)$, for let $a_{i,i+1}$ be the new vertex on the line connecting a_i and $a_{i,i+1}$, $1 \le i \le n-1$.
- Let $a_{1,n}$ be the vertex on the line connecting a_1 and a_n.
- Let $b_{i,i+1}$ be the newly created vertex on the line connecting b_i and $b_{i,i+1}$,$1 \le i \le n-1$.
- Let $b_{1,n}$ be the vertex on the line connecting b_1 and b_n.
- For $1 \le i \le n$, let ab_i be the newly created vertex on the line connecting a_i and b_i.
- For $1 \le i \le n-1$, let $a_{i+1}b_i$ be the newly created vertex on the line connecting a_{i+1} and b_i.
- Let $a_1 b_n$ be the vertex on the line connecting a_1 and b_n

Let $C = \{C_1, C_2, \ldots C_n\}$ and $C' = \{C_1', C_2', \ldots C'_n, C'_{n+1}\}$ be two sets of colours.

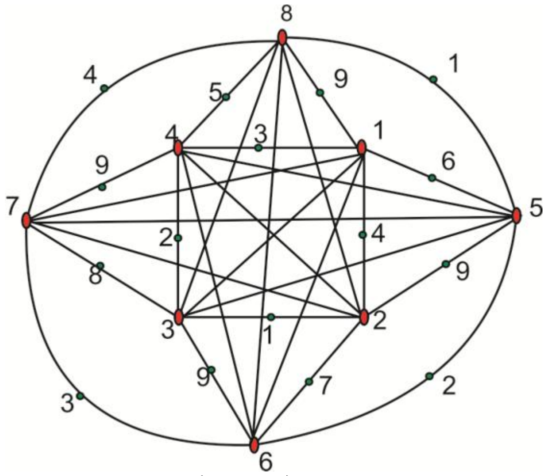
For $1 \le i \le n$ allocate C_i to b_i and C_i' to b_i.

To make this an achromatic colouring, consider the following pattern of colouring:

- For $2 \le i \le n-2$ allocate C_i to the vertex $a_{i+1,i+2}$, C_{n-1} to the vertex $a_{1,n}$ and C_n to $a_{1,2}$.
- For $2 \le i \le n-2$ allocate C_i to the vertex $b_{i+1,i+2}$ and C_{n-1} to the vertex $b_{1,n}$ and allocate C_n to $b_{1,2}$.
- For $1 \le i \le n$ allocate the new colour C_{n+1}' to the vertex $a_{i+1}b_i$.
- For $1 \le i \le n-1$ allocate the new colour C_{i+1}' to the vertex ab_i and allocate the colour C_1' to the vertex ab_n.

By this allotment, the maximum possible number of colours for an achromatic colouring is $|C| + |C'| = 2n + 1$.

Example



$\psi\left(C(Ap_4)\right) = 9$

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