Realization of Memory Effect on Hysteresis Lobe Area of the TiO_2 Based HP Memristor

Nune Pratyusha, Santanu Mandal

Abstract*: In this paper, the memory effect on the hysteresis lobe area of TiO_2 based memristive model is studied. The Green's theorem is used to derive the novel general formula for the area of hysteresis lobe of HP memristor model. Further the memory and boundary values are derived mathematically, where the memory needs to be stable. It is analyzed that in the initial state this nanoscale non-volatile memristor retains its memory. The relation of memory with the lobe area and memristance are also established respectively. The analytical results mentioned above are demonstrated by numerical simulations and graphical representations.

Keywords*: HP memristor, Non-volatile memory, Hysteresis loop, Green's theorem, Neuromorphic computing.

I. INTRODUCTION

In 1971 L.O. Chua [1] introduced a two terminal fundamental element, memristor (memory+resistor). It has a distinctive property of exhibiting self-crossing pinched hysteresis lobe in the $\boldsymbol{v-i}$ plane, which is the most significant fingerprint of the memristor [2].

Due to nonvolatility, high speed switching and high density, memristors have been explored in future storage devices like RRAM (Resistive Random Access Memory) [3,4], in associative memory [5-7]. In multi layered arrays, the horizontal and vertical lines of each layer are controlling the memristor. For encoding the synaptic weights of recurrent artificial neural networks, memristive cross bar arrays are used [8,9]. This architecture reduces the reading/writing costs as well as the energy loss of analog-to-digital conversion.

Biolek et. al. (2012) [10] realized that the area bounded by pinched hysteresis lobe is a measure of the memory of a memristor. They showed that the lobe area is engraved into a triangle where the memristance- R_{max} and R_{min} corresponds to the slopes of the triangle's sides. The authors stated that the area of the hysteresis lobe is equal to two-third area of the triangle. Biolek et.al. (2013) [11] derived the formula of the hysteresis lobe using parameter-vs-state map and polynomial function. In [12], the authors realized the importance of the study on hysteresis lobe area as specific measures of memory-effect of the memristive devices. They computed the lobe area for general mem-systems using OrCAD PSPICE simulation technique. Juhas et al. (2018) [13] introduced another computational technique to determine the lobe area using memristance-vs-state map. In the above research works, there is no mention of the analytical study of memory effect on the lobe area. Here, we have derived the area of hysteresis lobe using Green's theorem. Using this method, we are able to derive the novel lobe area formula in easier way than the other existing approach [10-13]. In 2008 HP Labs designed TiO_2 based physical memristor [14]. Here authors observed the significance of the factor $\frac{1}{D^2}$, where D is the film thickness. They mentioned the significance of the state variable $\boldsymbol{w(t)}$ which is bounded within 0 and D. The state variable remains constant when it reaches either boundaries, till the polarity is not reversed by the voltage. Motivated by this phenomena, in this paper, we considered $\frac{w(t)}{D}$ as 'memory' of the TiO_2 based memristor. The dynamics at the boundaries of $\frac{w(t)}{D}$ needs to be stable, or alternatively, $\frac{w(t)}{D}$ will be bounded in [0, 1]. We derived the formula for the memory and we verified mathematically the boundaries of $\frac{w(t)}{D}$ as 0 and 1. We analyzed further, how memory will have an effect on area of the hysteresis lobe. Moreover, we realized the initial memory inside the memristor in very initial stage, which signifies the non-volatility of HP memristor. All obtained results have been verified by numerical simulations and graphical representations.

We organized the rest of the paper as follows:

In section II, the formula for hysteresis lobe area of the generic memristor using Green's theorem is derived. In section II. A., the novel general formula of the lobe area of TiO_2 memristors established and compared with the existing methodologies [10,11] for such derivations. In section III, the formula for the value of the memory unit, $\frac{w(t)}{D}$ is mathematically established and the boundary of the memory is derived. In section IV, the effect of memory on hysteresis lobe area is analyzed. In each section all theoretical results are verified with numerical simulations. The conclusion is mentioned in section V.

II. THE AREA OF THE HYSTERESIS LOBE USING GREEN'S THEOREM

We consider here, the current-controlled non-volatile memristor model as

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$$v(t) = M(x)i(t)$$
$$\frac{dx}{dt} = f(x,i) \tag{1}$$

Where $M(x)$ is the memristance, which is a function of state variable $x(t)$. We consider A as area of single lobe in first or third quadrants in $v - i$ plane.
Using the Green's theorem, we obtain

$$A = \frac{1}{2}\int_0^{T/2} i(t)dv - v(t)di \tag{2}$$

From (1)

$$A = \frac{1}{2}\int_0^{T/2} M(x)i(t)di - i(t)(M'(x)\frac{dx}{dt}i(t) + M(x)di)dt$$
$$= -\frac{1}{2}\int_0^{T/2} M'(x)\frac{dx}{dt}i^2(t)dt$$
$$A = -\frac{1}{2}\int_0^{T/2} \frac{dM(x)}{dx}i^3(t)dt \tag{3}$$

This is the general formula for lobe area using Green's theorem

A. Computing the lobe area for *TiO_2* based HP memristor

We consider the HP memristor model [14] as follows

$$v(t) = \left[R_{on}\left(\frac{w(t)}{D}\right) + R_{off}\left(1 - \frac{w(t)}{D}\right)\right]i(t) \tag{4}$$

$$\frac{dw(t)}{dt} = \frac{\mu_v R_{on}}{D}i(t) \tag{5}$$

From (5), we get

$$w(t) = \frac{\mu_v R_{on}}{D}q(t) \tag{6}$$

where $i(t)$ is the input signal, μ_v is the dopant mobility, R_{off} and R_{on} are the max and min resistances.
Substituting (6) in (4), we get

$$v(t) = \left[R_{off} - \frac{\mu_v R_{on}}{D^2}q(t)\left(R_{off} - R_{on}\right)\right]i(t) \tag{7}$$

Let $i(t) = U\sin(\omega t)$ where ω is the frequency, t is the time period between 0 to $T/2$, U is the amplitude.
From equation (3), the hysteresis lobe area for the above model

$$A = -\frac{U^3}{2}\int_0^{\pi} -(R_{off} - R_{on})\frac{\mu_v R_{on}}{D^2}\sin^3(\omega)dt$$
$$= \frac{U^3}{2}(R_{off} - R_{on})\frac{\mu_v R_{on}}{D^2(\omega)}\frac{4}{3}$$
$$A = \frac{2}{3}(R_{off} - R_{on})\frac{\mu_v R_{on}}{D^2}\frac{U^3}{\omega} \tag{8}$$

From the above equation (8) we state that the lobe area is inversely proportional to the frequency (i.e. if the frequency increases the lobe area will decrease and converge to straight line) (in Fig.2 and Fig. 3).
Using equation (8) we can convert $\frac{\mu_v R_{on}}{D^2}$ in terms of $\frac{\omega^3}{Q_0}$, where Q_0 is charge necessary for memristance.

Thus we can derive the alternative formula for lobe area as $A = \frac{2}{3}\frac{(R_{off} - R_{on})}{Q_0}Q^3\omega^2$, which was obtained by Biolek et al. (2012)[10] using triangle method.

Again from equation (8) considering $\frac{\mu_v R_{on}}{D^2}$ as material constant k, we obtained lobe area as $A = \frac{2}{3}(R_{off} - R_{on})k\frac{U^3}{\omega}$, which is same as the formula established by Biolek et.al. (2013)[11] using polynomial function method.

Note that, without considering the triangle method or the polynomial function method, directly we can derive the same formula [10,11] using Green's theorem from our derived general formula (8) for area of hysteresis lobe.

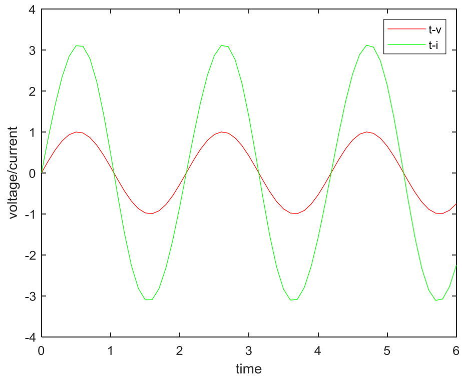

Fig. 1

Fig. 1. The graphical representation depicts the applied voltage (red) $v_0 \sin\omega t$, where $v_0 = 1v$ and resulting current (green) against time *t* for the TiO_2 memristor model.

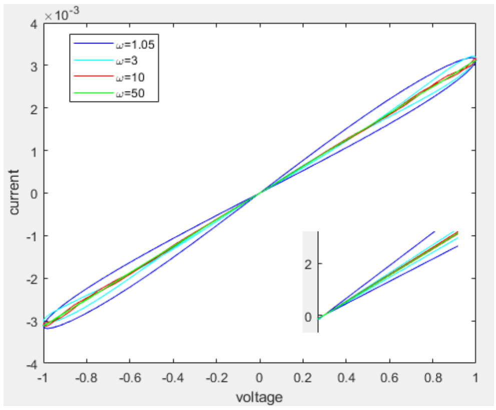

Fig. 2

Fig. 2. This graph depicts pinched hysteresis loop is appeared in first and third quadrant of $v - i$ plane when $D^2/\mu_v = 10ms$, the resistance R_{off} and R_{on} are 320 Ω and 2 Ω espectively. At frequency ω=1.05Hz, non-zero lobe area is appeared whereas the higher frequency ω = 50Hz area shrinks to a straight line.

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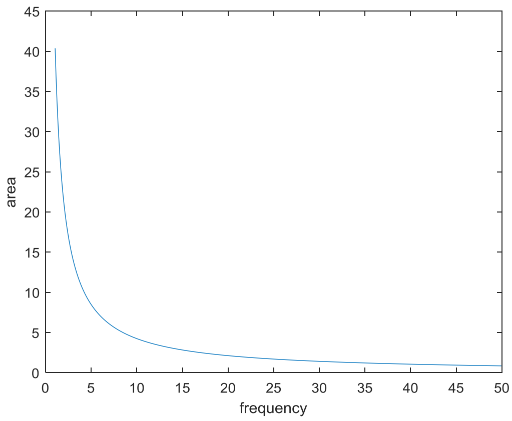

Fig. 3.The lobe area is maximum at $\omega = 1.05$Hz and that area tends to zero at $\omega = 50$Hz

III. MEMORY REALIZATION OF THE HP MEMRISTOR

As we know due to the memory effect memristor exhibits hysteresis loop, we derive mathematically the value of the memory ($\frac{w(t)}{D}$) as follows

From equation (5)

$$\frac{1}{D}\frac{dw(t)}{dt} = \frac{\mu_v R_{on}}{D^2} i(t) \qquad (9)$$

From equations (4) and (9), we get

$$v(t) = \frac{dw(t)}{dt}\left[R_{off} - \frac{w(t)}{D}\left(R_{off} - R_{on}\right)\right]\frac{D}{\mu_v R_{on}} \qquad (10)$$

Considering $\alpha = \frac{R_{off} - R_{on}}{R_{off}}$ and applied voltage $v(t) = v_0 \sin \omega t$, integrating over (10)

$$\frac{w(t)}{D} = \frac{1 - \sqrt{1 - 2\alpha\left(\frac{w(0)}{D} - \alpha\frac{1}{2}\left(\frac{w(0)}{D}\right)^2 + \frac{\mu_v R_{on} v_0}{D^2 R_{off}}\left(\frac{\cos \omega t - 1}{\omega}\right)\right)}}{\alpha} \qquad (11)$$

We consider '- 'sign instead of '+/- 'so that the solution of equation (11) lies in the interval [0,1]

Now, for the equation (11) if we consider

$$p = 2\alpha\left(\frac{w(0)}{D} - \alpha\frac{1}{2}\left(\frac{w(0)}{D}\right)^2 + \frac{\mu_v R_{on} v_0}{D^2 R_{off}}\left(\frac{\cos \omega t - 1}{\omega}\right)\right)$$

Assuming $\alpha = 0.9$, we observe that the p lies between 0 and 1, (in Fig. 4).

Therefore, it is observed that, the memory: $\frac{w(t)}{D}$ is bounded between 0 and 1(in Fig. 5).

Further from equation (9) we have

$$\frac{w(0)}{D} = 0.2c \qquad (12)$$

The above expression represents the initial memory state of the memristor where its response depends on the past history c (the integral constant).

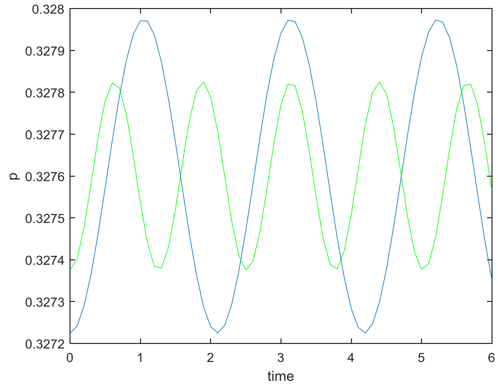

Fig. 4. The diagram confirms p value lies between 0 and 1

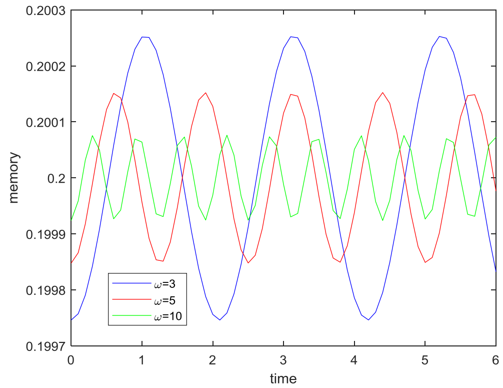

Fig. 5. The memory: $\frac{w(t)}{D}$ is bounded between 0 and 1

IV. MEMORY EFFECT ON HYSTERESIS LOBE AREA

Using equations (8) and (9), the lobe area can be expressed as

$$A = \frac{2}{3}\left(R_{off} - R_{on}\right)\frac{w(t)}{D}\frac{1}{q(t)}\frac{U^3}{\omega} \qquad (13)$$

From the above equation we observe that the lobe area is directly proportional to memory (in Fig.6 and Fig.7) and memory effect on lobe area is realized.

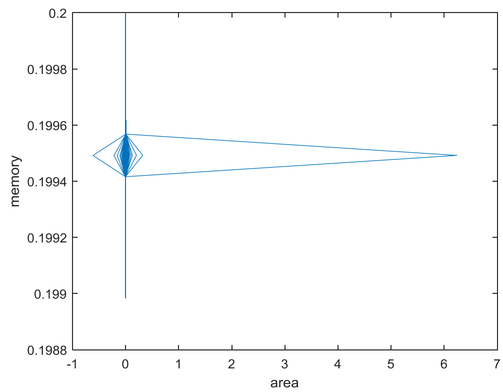

Fig. 6. Relation between area and memory at $\omega = 1.05$Hz

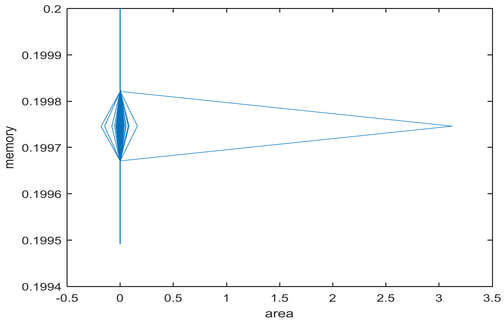

Fig. 7. Relation between area and memory at ω = 3Hz

The above figures (Fig.6 and Fig.7.) show that though the area tends to zero the memristor still stores some memory and the memory does not vanish. We can notice that the memory varies within 0 and 1 for little variation of nonzero lobe area. On the other way, from equation (4), the memristance of the HP memristor

$$M(q) = \left[R_{on}\left(\frac{w(t)}{D}\right) + R_{off}\left(1 - \frac{w(t)}{D}\right)\right]$$

$$\Rightarrow \frac{w(t)}{D} = \frac{R_{off} - M(q)}{R_{off} - R_{on}} \qquad (14)$$

From the above relation (14) it can be observed that if memristance reaches the highest resistance R_{off}, then memristor loses its memory state and other side, at lowest resistance R_{on}, the memory is closer to '1'. This study tells us about the relation between memristance and memory which are inversely proportional to each other and, $\frac{w(t)}{D}$ will lies in its boundary limits 0 and 1(in Fig. 8).

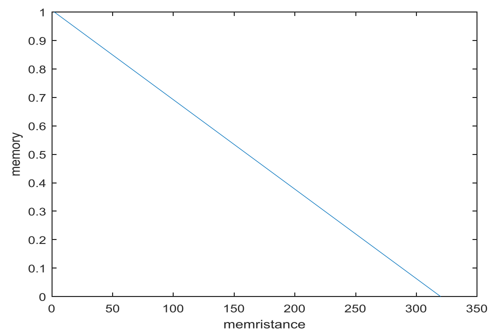

Fig. 8. Relation between memory and memristance

V. CONCLUSION

This paper discussed memory and its effect on area of hysteresis lobe of TiO_2 memristor model. A simpler general method is adopted and compared with the existing analysis [10,11] to derive the lobe area. The benefit of the Green's theorem to derive the novel general formula for the lobe area is realized. This method can be applicable on other nonvolatile memristive model also. The memory of the HP memristor is formulated mathematically. It is realized mathematically that the memory does not exactly equals to zero or is not more than one for any input signal. The boundaries of the memory are derived analytically. It is observed that the memory and lobe area are directly proportional to each other whereas, the memory and memristance are inversely proportional. Further it is observed that when the lobe area is almost zero in high frequency, the memristor still retains some memory which signifies the non-volatility of the memristor. Hence, it is concluded that the response of the memristor depends on its past history and there is a significant impact of memory on hysteresis lobe area.

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