Voltage Stability Indices: Formulation and Classification Perspectives

Priti Prabhakar

Abstract: Voltage instability had been observed as the foremost cause of blackout incidents worldwide in last three decades. In order to deploy an appropriate countermeasure and enhance voltage stability margin, voltage stability predictor is of utmost importance. Therefore, much research had been focussed to propose voltage stability indices (VSIs) that can identify weak areas and overall condition of power systems. In this paper systematic review covering imperative aspects of formulation theory, expressions, critical values and applicability of VSIs has been presented in chronological order. A broad categorization of VSIs is also addressed. An inclusive review provides a strong foundation for further research in the perspective of voltage stability evaluation for real-time control applications.

Keywords: Voltage instability, Voltage collapse, Voltage stability indices, Indices classification

I. INTRODUCTION

The first major power failure occurred in the U.S. and Canada in 1965 that affected around 30 million people. Great Northeast Blackout in both, making it the single in U.S. history at the time. A significant number of black-outs have been occurred worldwide since then. Some eminent incidents of blackout amongst them are [1- 3]: August 2003-Northeast US and Canada, August 2005-Indonesia, January & February 2008- Central Chinese city of Changzhou, November 2009- Itapúa Paraguay-Brazil, July 2012-Northern grid of India, December-2014 Detroit US, , and. The collapse of Northern grid in India on 30th and 31st July 2012 had been the worst blackout all over the world and affected about 620 million people [4]. The rate of recurrence of these collapses along with their social and economical impacts led to significant research in identifying the cause. The investigations revealed that voltage instability (VI), different from angular stability, led to blackouts in significant part of power systems. The key factors contributing were gradual or sudden increase in load, mal operation of protective devices, limits of reactive power resources and consequence of tap changing transformers action. Therefore, to control the menace of VI vital measures are reactive power compensation, rescheduling of generators, LTC blocking, load reduction, fast capacitor switching, fast load shedding, fault clearing FACTS and HVDC with fast controllers.

Proceeding to abrupt decline in voltages, unusual reactive powers flows in transmission lines were observed. An irrepressible decrease in voltage profiles along with the action of and reactions of power system components leads to partial or overall voltage collapse (VC) [5] - [7]. These symptoms that identify the system state need to be quantified to initiate protective and corrective control in time. The measure of closeness of the current operating state and system instability state is termed as voltage stability index (VSI).

This quantification should have scalar magnitude that varies smoothly as the system parameters changes. The desirable attributes of these VSIs are predictable shape, computationally inexpensive and sufficiently accurate. Therefore, developing VSIs suitable for different application are of great interest to the power utility engineers, operators and researchers.

The motive of present work is to provide a gist of various indexes developed and widely accepted so far for voltage stability assessment. The various aspects covered are formulation of criterion, derivation of equations and limiting conditions for stability assessment. A straightforward approach has been applied to elaborate the theory of indices development. The crisp review and classification based on the concept, methods and type in chronological order can be of significant importance for in-depth research in the area of voltage stability analysis. Apposite identification of weaker areas with an efficient index may avoid the irrecoverable losses due VC.

Outline of the research work is as follows: Section II of the paper elaborates the various voltage collapse detection criterion. Power flow Jacobian based indices are also presented in tabular form in this section. In Section III the classification based on the method of computation has been shown in pictorial form. Section IV reviews the VSIs that are classified as bus, line and overall indices depending on the part of power system prone to voltage collapse. Section V concludes the discussions.

II. VOLTAGE COLLAPSE DETECTION CRITERION

Various criterions for developing voltage stability indices are:

- Load flow Jacobian
- PV-VQ curves
- Maximum power transfer theorem
- Power transfer limit of a line
- Reactive power consumption of a line
- Lyapunov stability theory

A. Load Flow Jacobian [8-17]

The first criterion was proposed by Venikov et al. [8]. It was shown that the steady state limit is directly related to load flow Jacobian (J). Singularity of J signifies the instability state and hence the Eigen values of the linearized system matrix can be an index of proximity to VC. Since then numerous indices had been derived from the condition of non-solvability of load flow equations and hence singularity J.

The power balance equations at the buses can be represented by non-linear equations as:

\[
\begin{align*}
\Delta P_i &= P_{isp} - \sum_{j=1}^{n} V_j (G_{ij}\cos \theta_{ij} + B_{ij}\sin \theta_{ij}) = 0 \\
\Delta Q_i &= Q_{isp} - \sum_{j=1}^{n} V_j (G_{ij}\sin \theta_{ij} - B_{ij}\cos \theta_{ij}) = 0
\end{align*}
\]

(1)
Linearizing above power balance equations by Taylor’s series expansion and neglecting higher order derivatives yields:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \begin{bmatrix}
\frac{\partial P}{\partial V} & \frac{\partial P}{\partial \theta} \\
\frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial \theta}
\end{bmatrix}
\begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix}
\]

Or

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \begin{bmatrix}
J_{PV} & J_{PQ} \\
J_{QV} & J_{QQ}
\end{bmatrix}
\begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix}
\Rightarrow \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = J \begin{bmatrix}
\Delta V \\
\Delta \theta
\end{bmatrix}
\]

(2)

Where:
\(\Delta P, \Delta Q\): Active and reactive power errors respectively
\(\Delta V, \Delta \theta\): Voltage and angle corrections respectively
\(J\): Jacobian matrix with partial derivative terms

Assuming the loose coupling between active power and voltage magnitudes and setting \(\Delta P = 0\):

\[
\Delta Q = \left[J_{QV} - J_{PQ} + J_{PV} \right] \Delta V
\]

(3)

Where: \(J_{kB}\): Reduced Jacobian matrix

The voltage stability limits were derived as the closeness of the determinant of \(J\) or \(J_{kB}\) near to zero.

Table 1 shows the Jacobian matrix based indices.

### Table 1: Jacobian Matrix Based Indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Expression</th>
<th>Stability Condition</th>
<th>Concept Used</th>
<th>Author</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Det ( J )</td>
<td>( U =</td>
<td>J_{PV} - J_{QV} + J_{QV} + J_{PP}</td>
<td>)</td>
<td>(</td>
<td>U</td>
</tr>
<tr>
<td>2. Singular / Eigen values Modal analysis</td>
<td>Minimum Eigen value</td>
<td>All Eigen values should be positive</td>
<td>Solvability of load flow equations</td>
<td>Lof et al., 1975</td>
<td>[9]</td>
</tr>
<tr>
<td>3. Test functions</td>
<td>Test functions</td>
<td>Test functions</td>
<td>Singularity of ( J )</td>
<td>Chiang et al., 1995</td>
<td>[10]</td>
</tr>
<tr>
<td>4. Sensitivity factors</td>
<td>( VSF_i = \max_j \left[ \frac{\Delta V_j}{\Delta Q_i} \right] )</td>
<td>( VSF ), should be small positive</td>
<td>Singularity of ( J )</td>
<td>Olsadina et al., 1990</td>
<td>[11]</td>
</tr>
<tr>
<td>5. Inverse sensitivity factors</td>
<td>( IVSF = 1 / VSF )</td>
<td>( IVSF = 0 )</td>
<td>Inverse of reduced Jacobian ( J_i )</td>
<td>Elshafei et al., 1990</td>
<td>[12]</td>
</tr>
<tr>
<td>6. Second order index</td>
<td>( 1 = \frac{\phi_{max}}{\theta_{max}} )</td>
<td>( i &gt; 0 )</td>
<td>Singularity of ( J )</td>
<td>A. Barjazi, 1998</td>
<td>[13]</td>
</tr>
<tr>
<td>7. P and Q angle</td>
<td>( \alpha_i = \cos ^{-1} \left[ \left( \frac{\Delta V_{ij}}{\Delta Q_{ij}} \right) \right] )</td>
<td>( \alpha_i &lt; 90^\circ )</td>
<td>Singularity of ( J )</td>
<td>Wang L et al., 1996</td>
<td>[14]</td>
</tr>
<tr>
<td>8. Tangent vector index</td>
<td>( TVI_i = \frac{\Delta V_i}{\Delta Q_i} )</td>
<td>( TVI_i )</td>
<td>Inverse Sensitivity factor</td>
<td>De Souza et al., 1996</td>
<td>[15]</td>
</tr>
</tbody>
</table>

The useful singular and Eigen value orthogonal decomposition techniques to evaluate the smallest Eigen singular value [9, 10] had been applied in many researches as a measure of VI.

Chiang et al. in [11] had utilized the family of scalar functions discussed in [12] to voltage collapse studies. These test functions have quadratic shape and better than the non-linear indices. Other well known indices are sensitivity factors that are used by utilities all over the world [13]. The sensitivity factors indicate the variation of the generated reactive power with shape. Therefore, inverse sensitivity factors with more predictable loading parameters. Critical generators and critical loads have large sensitivity factors. These indices are of non-linear loads shape and computationally inexpensive are also utilized [14]. Linearity based indices may be inadequate as these display large discontinuities when generators and transformer taps hit their control limits.

To overcome inadequacy of first order indices, a second order index that also includes the effect of sensitivity of index is suggested in

Also the associated right singular / Eigen vectors indicate the most critical buses and the right Eigen singular value gives the most sensitive direction for power injections changes [15]. The index P-Q angle had been derived from the geometrical construal of power flow equations in [16]. At VC the angle between the tangent to the curve and the gradient to the surface becomes 90°. Another index similar to sensitivity factors had been explained in [17]. The index TVI becomes zero at VC and can be easily calculated from the Newton-Raphson iteration.

B. PV-QV Curves

**Loading margin** [18-20] from P-V and V-Q plots of PQ (load) buses is the most fundamental and conventional index predicting proximity to VC. It has been extensively used as benchmark in the voltage stability studies to compare the performance of other indices. Loading margin at a given operating state is the amount of added load that would result
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Most of the line indices have been obtained from the limitation on power transfer through a line. The maximum power limit reaches when no real solution exists for the receiving end voltage.

\[ V_r^4 + 2(R_p + X_Q)t + V_i^2 - X_r^2V_i^2 + (R_r^2 + X_r^2)(P_r^2 + Q_r^2) = 0 \]  
(6)

Therefore, equating the expression for the discriminant of equation (6) to zero becomes the criterion for deriving the critical limits of various line VSI's.

Where:
- \( V_r \): Sending end bus voltage
- \( S_r(P_r + jQ_r) \): Sending end power
- \( V_r \): Receiving end bus voltage
- \( S_r(P_r + jQ_r) \): Receiving end power
- \( R, X, Y \): line Resistance, reactance and shunt admittance respectively.

Some indices in terms of ABCD parameters have also been derived from the pie model of transmission line as shown in the Fig. 5[49, 52].

E. Reactive power consumptions of lines

An overloaded line consumes high reactive power and consequently restricts transfer of reactive power to receiving end connected loads. Near the point of VC, enhancing the apparent power from the sending end of the line no longer yields an addition to the apparent power delivered to loads. Therefore, increase in reactive power losses is the indicator of voltage instability [25, 28].

F. Lyapunov stability theory

Lyapunov stability theory had been initially established for direct stability investigation of power systems. Hence a technique based on it had been effectively applied and implemented as a voltage stability predictor in [62]. This scalar function had been shown to directly related with the area encircled by the Q- In V curve or P- \( \alpha \) curve assuming PQ loads and thus a measure of VI/VC [63].
A difficulty with this index is that it cannot be readily modified to include more complex system models. Therefore, in literature using any of the following conditions and simplifying Equations (5)-(6), the indices to predict the voltage stability condition, are derived:

1. Load flow Jacobian matrix
2. Load flow solutions (System variables and parameters)
3. Maximum power transfer theorem
4. Power transfer limit of a Line
5. Feasible voltage solutions of the load buses

6. Reactive power consumption of the line
7. Lyapunov stability theory

III. CLASSIFICATION OF VOLTAGE STABILITY INDICES

A. Based on Criterion/Method used

Classification of indices based on formulation and calculation methods is shown in the Fig.6.

![Classification of Indices](image)

**Fig. 6. Classification of Indices based on the criterion/method of calculation**

B. Based on Power System Parameters and Variables

Power system variables can be obtained from either load flow solutions or the measurements of voltages, currents and powers injected of various buses. VSIs calculated from this system variable and parameters distinguish the weak parts of power system for initiating the corrective action against VI/VC.

Therefore, from the identification of the part prone to VI/VC, these are classified as:

1. Bus Indices
2. Line Indices
3. Overall Indices

Bus Indices can identify the buses nearest to voltage stability limits and thus the suitable candidate buses for installation of shunt FACTs controllers. Similarly line indices have been utilized for providing series compensation in weak lines. The overall indices anticipate the stability state of the whole power system instead of indicating weak buses or lines.

An inclusive survey has been done and the imperative indices developed so far are reported in the chronological order along with the criterion used for their formulation and presented in section IV.

C. Based on Measurements

With the inception of phasor technology with Phasor Measurement Unit (PMU) and advanced communication technologies, time tagged phasors can be obtained. Analog voltages and currents are converted into phasors and these time tagged phasors are delivered to phasor data concentrators via advanced communication modes with minimum delay. These phasors are converted into useful information predicting the state of power system at the monitoring and control centres. Therefore, many indices had been derived with phasor information for early anticipation of power system instability.

Voltage stability surveillance and controls based on measurement can be classified as follows:

1. Local Measurements [22, 24-29]
2. Wide Area Monitoring [33, 55, 58-61]

Network equivalent at a critical/monitoring bus may be determined from voltage and current phasors measurements with PMUs installed at these buses. Theses buses are analysed independently as no synchronization and information of other buses (Voltages/currents) is required.

The dynamic and wide area tracking of the power system state is quite obvious with the phasor measurement units placed at optimal locations for complete observability.

IV. BUS, LINE AND OVERALL VOLTAGE STABILITY INDICES

A. Bus Indices: Bus indices that predict the vulnerable buses are tabulated in Table II.
<table>
<thead>
<tr>
<th>Index</th>
<th>Expression</th>
<th>Stability Condition</th>
<th>Concept Used</th>
<th>Author</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. L-Index</td>
<td>$L_j = 1 - \sum_{i \in \alpha_j} \frac{E_i V_j}{V_j}$</td>
<td>$L_j &lt; 1$</td>
<td></td>
<td>Kessel P et al, 1986</td>
<td>[21]</td>
</tr>
<tr>
<td>2. Voltage Index Predictor</td>
<td>$\Delta S = \frac{(V_j - Z_{th} I_{ij})^2}{4Z_{th}}$</td>
<td>$\Delta S &gt; 0$</td>
<td>Maximum power transfer theorem</td>
<td>Julian D. E. et al, 2000</td>
<td>[22]</td>
</tr>
<tr>
<td>3. P-Q Boundary</td>
<td>$P_{cr} = -\frac{E^2 R_{th}}{2X_{th}} + \frac{</td>
<td>Z_{th}</td>
<td>E}{2X_{th}}$</td>
<td>$P_L &lt; P_{cr}$</td>
<td>Feasible solution of voltage equations</td>
</tr>
<tr>
<td>4. Voltage Collapse Prediction Index</td>
<td>$VCPI_k = 1 - \sum_{m \in \alpha_j} \frac{V_m}{V_k}$</td>
<td>$VCPI_k &lt; 1$</td>
<td>Feasible solution of power flow and voltage equations</td>
<td>Balamourougan V. et al, 2004</td>
<td>[24]</td>
</tr>
<tr>
<td>5. S Difference Criterion</td>
<td>$SDC = 1 + \frac{\Delta V_{Ir}^2}{V_{Ir} \Delta I_r^2}$</td>
<td>$SDC &gt; 0$</td>
<td>Increase in reactive power losses of the line</td>
<td>Verbic G. et al., 2004</td>
<td>[25]</td>
</tr>
<tr>
<td>6. Impedance Stability Index</td>
<td>$ISI = \frac{Z_L - Z_{th}}{Z_L}$</td>
<td>$ISI &gt; 0$</td>
<td>Maximum power transfer theorem</td>
<td>Smon I. et al., 2006</td>
<td>[26]</td>
</tr>
<tr>
<td>7. $Z_L/Z_S$ ratio</td>
<td>$\frac{Z_L}{Z_S} = \frac{M + 1}{M \cos \beta + [(M \cos \beta)^2 M^2 + 1]}$</td>
<td>$\frac{Z_L}{Z_S} &gt; 1$</td>
<td>Maximum power transfer theorem</td>
<td>Wiszniewski A., 2007</td>
<td>[27]</td>
</tr>
<tr>
<td>8. Voltage Stability Index</td>
<td>$VSI_i = \left[1 + \left(\frac{1}{V_i}\right)^{\frac{1}{\Delta V_{Ir}}\Delta I_r^2}\right]^{u}$</td>
<td>$VSI_i &gt; 0$</td>
<td>Increase in reactive power losses of the line</td>
<td>Haque M. H., 2007</td>
<td>[28]</td>
</tr>
<tr>
<td>9. Equivalent Node Voltage Collapse Index</td>
<td>$ENVCI = 2E_k V_n \cos \theta_{kn} - E_k^2$</td>
<td>$ENVCI &gt; 0$</td>
<td>Voltage quadratic equation solution</td>
<td>Wang Y., et al., 2009</td>
<td>[29]</td>
</tr>
<tr>
<td>10. Linearized motor voltage stability index</td>
<td>$LMVSI_i = \frac{MVSI_i}{\det(A_i)}$</td>
<td>$LMVSI_i &gt; 0$</td>
<td>Singularity of the equivalent load state matrix with dynamic model of induction motor</td>
<td>Gu W. et al., 2010</td>
<td>[30]</td>
</tr>
</tbody>
</table>
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11. DSY (Derivative of the load apparent power with respect to its admittance)

\[ DSY = \frac{\Delta S_i}{V_i^2 \Delta Y_i} \]

DSY > 0

Maximum power transfer theorem

Parniani M. et al., 2010 [31]

12. Voltage stability Risk indices

\[ VSR_i = \frac{1}{N} \sum_{j=i}^{N} (d_j + d_{j-1}) \Delta t \]

Most negative index, shows the highest risk of voltage instability

Transient variation of the system voltages

Seethalekshmi K. et al., 2010 [32]

13. Simplified Voltage Stability Index

\[ SVSI_i = \frac{\Delta V_i}{\beta V_i} \]

SVSI_i < 1

Maximum power transfer theorem

Pérez-Londoño S. et al., 2014 [33]

14. Linear M-index

\[ M_i = 1 - \frac{|V_i| |P_{eff_i} - |V_i||^2}{|P_{eff_i}||V_i|} \]

M_i > 0

Voltage quadratic equation solution

Matavalam A.R.R. et al., 2015 [34]

15. \( \frac{dV}{dQ} \) index

\[ \Pi_i \pm \sum_{j \in E} \frac{\partial V_i}{\partial Q_j} \cdot i \in L \]

\( \Pi_i \rightarrow +\infty \) at VC

Singularity of J

Simpson-Porco J. W. et al., 2016 [35]

16. \( \frac{dV_i}{dV_G} \) index

\[ \Pi_i \pm \sum_{k \in G} \frac{\partial V_i}{\partial V_k} \cdot \, i \in L \]

\( \Pi_i \rightarrow +\infty \) at VC

Singularity of J

Simpson-Porco J. W. et al., 2016 [35]

17. \( \frac{dQ^a}{dQ^l} \) index

\[ \Pi_i \pm \sum_{k \in G} \frac{\partial Q^a}{\partial Q^l} \cdot \, i \in L \]

\( \Pi_i \rightarrow -\infty \) at VC

Singularity of J

Simpson-Porco J. W. et al., 2016 [35]

18. P-index

\[ P-index_i = \frac{-\frac{P_i}{V_i} \frac{dV}{dP}}{1 - \frac{P_i}{V_i} \frac{dV}{dP}} \]

P - index < 1

Load flow Jacobian matrix

Kamel M. M. et al., 2017 [36]

B. Line Indices: Table III shows the imperative line indices.

Table-III: Line Indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Expression</th>
<th>Stability Condition</th>
<th>Concept Used</th>
<th>Author</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Line Stability Factor</td>
<td>( LQP = 4 \left( \frac{X}{V_s^2} \right) \left( Q_r + \frac{P_s^2 X}{V_s^2} \right) )</td>
<td>( LQP &lt; 1 )</td>
<td>Power transfer limit of a line</td>
<td>Mohamed A. et al, 1989</td>
<td>[37]</td>
</tr>
<tr>
<td>2. Voltage Stability Load Index</td>
<td>( VLSI = 4 \left[ V_s V_r \cos \delta - V_s^2 \cos \delta \right] )</td>
<td>( VLSI &lt; 1 )</td>
<td>Power transfer limit of a line</td>
<td>Rahman TKA et al., 1995</td>
<td>[38]</td>
</tr>
<tr>
<td>3. Transmission Path Stability Index</td>
<td>( TPSI = 0.5V_g - \Delta V_d )</td>
<td>( TPSI &gt; 0 )</td>
<td>Maximum power transfer theorem</td>
<td>Gubina F. et al, 1995</td>
<td>[39]</td>
</tr>
<tr>
<td>4. Line Stability Index</td>
<td>( L_{mn} = \frac{4X Q_j}{V_j \sin(\theta - \delta) \gamma} )</td>
<td>( L_{mn} &lt; 1 )</td>
<td>Power transfer limit of a line</td>
<td>Moghadam M. M. et al, 1998</td>
<td>[40]</td>
</tr>
</tbody>
</table>
5. **Line Stability Index**  
\[ L_P = \frac{4RP}{V_c\cos(\theta - \delta)} \]  
\[ L_P < 1 \]  
Power transfer limit of a line  
Moghavemmi M. et al, 2001 [41]

6. **Fast Voltage Stability Index**  
\[ FVSI = \frac{4Z^2Q}{V_i^2X} \]  
\[ FVSI < 1 \]  
Power transfer limit of a line  
Musirin I. et al, 2002 [42]

7. **L_{sr} Index**  
\[ L_{sr} = \frac{S_r}{S_{r(max)}} \]  
\[ L_{sr} < 1 \]  
Power transfer limit of a line  
Albuquerque MA et al, 2003 [43]

8. **Volatges stability Load Bus Index**  
\[ VLSBI = \frac{V_r}{\Delta V} \]  
\[ VLSBI > 1 \]  
Power transfer limit of a line  
Milosevic B. et al., 2003 [44]

9. **Voltage Stability Margin Index**  
\[ VSMI = \frac{\delta_{max} - \delta}{\delta_{max}} \]  
\[ VSMI > 0 \]  
Maximum power transferable through a line  
He T. et al., 2004 [45]

10. **Power Transfer Stability Index**  
\[ PTSI = \frac{2S_rZ(1+\cos(\theta - \delta))}{V_s^2} \]  
\[ PTSI < 1 \]  
Maximum power transferable through a line  
Nizam M. et al, 2006 [46]

11. **New Line Stability Index**  
\[ NLSI = \frac{P_iR + Q_iX}{V_s^2/4} \]  
\[ NLSI > 1 \]  
Maximum power transferable through a line  
Yazdanpanah-Goharrizi A. et al, 2007 [47]

12. **Line Stability Index**  
\[ L_{ij} = \frac{4Z^2Q_iX}{V_s^2(R\sin\delta - X\cos\delta)^2} \]  
\[ L_{ij} < 1 \]  
Maximum power transferable through a line  
Subramani C. et al, 2009 [48]

13. **Line collapse Proximity Index**  
\[ LCPI = \frac{4A\cos(a_i)(P_iB\cos\beta + Q_iB\sin\beta)}{(V_c\cos\delta)^2} \]  
\[ LCPI < 1 \]  
Voltage quadratic equation solution  
Tiwari R. et al., 2012 [49]

14. **New Voltage stability Index**  
\[ NVSI = \frac{2Z_i\beta^2 + Q_i^2}{2Q_iX - V_s^2} \]  
\[ NVSI < 1 \]  
Maximum power transferable through a line  
Kanimozhi R. et al, 2013 [50]

15. **Critical Boundary Index**  
\[ CBI_{ik} = \sqrt{\Delta P_{ik}^2 + \Delta Q_{ik}^2} \]  
\[ CBI_{ik} \]  
Voltage stability equation  
VLSI  
Milosevic M. et al., 2018 [51]

\[ LVI < 1 \]

**C. Overall Indices:**

Overall indices are generally Jacobian based and well analyzed in [53]. Overall indices give the measure of the proximity to VC taking into account the whole power system model. Some of these are enumerated in the table IV.

**Table IV: Overall Indices**

<table>
<thead>
<tr>
<th>Index</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1. Voltage Instability Proximity Index</td>
<td>[ VIPI = \cos^{-1} \left( \frac{y_i^2 y(a)}{|y_i||y(a)|} \right) ]</td>
<td>[ VIPI &gt; 0 ]</td>
<td>Feasible solution of Power flow equations</td>
<td>Tamura Y. et al, 1989</td>
<td>[54]</td>
</tr>
<tr>
<td>2. Sensitivity Matrix</td>
<td>[ S_{\phi\phi} = -\varphi_s^t \varphi_s^{-1} \varphi_s ]</td>
<td>[ S_{\phi\phi} &gt; 0 ]</td>
<td>At VC ( S_{\phi\phi} ) changes sign through infinity</td>
<td>M. Glavic et al, 2009</td>
<td>[55]</td>
</tr>
</tbody>
</table>
3. Area of Voltage Stability Region

\[ AVSR = \int_{P_t}^{P_l} [Q(P) - Q_i] dP \]

\[ AVSR > 0 \]

Feasible solution of Power flow equations

Lee C. Y. et al, 2010

[56]

4. Network sensitivity approach

\[ SG_p = \frac{P_{st}}{P_{st}} \]

\[ SG_q = \frac{Q_{st}}{Q_{st}} \]

\[ SG_p \rightarrow \infty \text{ at VC} \]

\[ SG_q \rightarrow \infty \text{ at VC} \]

P-V curve

Althowibi et al, 2012

[57]

5. Voltage Instability Monitoring Index

\[ V\text{IMI}_k = W_1(k) \frac{\text{DFR}_k}{\text{DFR}_{\text{max}}} + W_2(k) \frac{\text{CVD}_k}{\text{CVD}_{\text{max}}} \]

\[ V\text{IMI}_k < 1 \]

Voltage Deviation and consecutive voltage deviation

Sodhi R. et al, 2012

[58]

V. CONCLUSIONS

The earlier indices are mainly derived from the conditions of singularity of Jacobian obtained in Newton-Raphson method of load flow analysis. These indices are suitable in offline applications like planning and design of power grids. The disadvantages of Jacobian based indices are high computational effort and accurate prediction is possible only when extremely close to instability point. Sensitivity factors are well recognized indices and implemented in several utilities all over the world. Sensitivity factors that show the variation of generated reactive powers with the changes in loading parameters involves simple calculations in off-line studies. Loading margin is the benchmark index of voltage collapse. But it requires more computation effort at far away point from the current state.

System variables and parameters based indices have been derived from the concept of maximum power transfer theorem and maximum power flow through a line. These indices can be evaluated using load flow solutions or measurements. Measurement based indices can be determined either using local measurements or global measurements. Thevenin’s equivalent model is obtained and utilized for calculation of local measurement based indices. Although the approach is simple and computationally less expensive, time window frame size poses inaccuracy in Thevenin’s equivalent estimation and hence difficulties in actual implementations.

The comprehensive survey concludes that some indices are more suitable for real time control as these can be calculated easily in spite of less accuracy. On the other hand, some indices involve higher computational costs with more accuracy and hence more suitable for offline studies. All the aforesaid indices have their advantages and limitations. Therefore, to acclaim a specific index is not practical as it depends on the application and viability.

REFERENCES


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