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A Queueing Inventory System with Server Working Breakdown

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Abstract*: This paper considers a single server inventory queueing system with two types of server breakdowns, say Type 1(T1) breakdown and Type 2 (T2) breakdowns, and working breakdown. T1 breakdown occurs in regular service period whereas T2 breakdown occurs in the duration of working breakdown. The inter arrival time between any two customers and the times of occurrences of both types of breakdowns are all independent exponential distributions. The commencement of repairing process of T1 breakdown follows Bernoulli's whereas the commencement of repairing process of T2 breakdown is instantaneous. By using matrix method, we obtain the steady state probability vector of the finite capacity queueing inventory system. Finally, the numerical examination of model sensitiveness is performed.

Keywords: Server Working Breakdown, Markovian inventory, (s,Q) policy.

I. INTRODUCTION

Usually customers are provided with additional services during the purchase for proper utilization of the products. For instance, electronics and electrical products appliances are provided with procedure to use the product with some safety measures and along with the directions to use it in a best possible manner. Through these kinds of services, the manufactures can gain attention and good reputation from the customers by acknowledging the quality of that product. The customers are queued for benefiting from these kinds of quality services. The inventory queuing theory is documented based on the above mentioned scenarios.

At times, the flow of service is disturbed due to certain server failures and once when the failures are repaired, the service is resumed.

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Hence many studies were done on the basis of combining server interruption and inventory queuing system. White and Christiare [4] exhibits for the first time as to how a server interruption affects the various performances in M/M/1 queuing system. Due to certain failures, the normal functioning of the server may be disturbed. In spite of these disturbances, the server may provide limited services with limited operations. This may avoid the disappointment of the customers towards the service provided. If needed in due course, the server repairs completely which reduces the expected waiting time of a customer and expected number of customers lost. Few papers in queuing theory has studied the possibilities of the services done with defective servers. Such kind of service policy is known as working breakdown. Kalidass and Kasturi [2] discussed a slower rate of service with a queue, when the server is in breakdown condition. Further, with reference [1,3] interruption on various inventory problems are studied.

The paper is organized as follows. The immediate section deliberates mathematical formulation of our model. In section 3, the analysis of the system is obtained with steady state. Further, we exhibit some important performance measures and the interpretations of some numerical evidences are given in section 4 and 5 respectively.

II. MODEL DESCRIPTION AND NOTATIONS

In this paper, we have considered a single server inventory queueing system with working breakdowns and along with repairing policy of the server. The features of the model are given below:

The arrival process of customers is according to Poisson with rate λ and the items are replenished under (s,Q) ordering policy, in which the quantity Q is ordered whenever the level of inventory drops to s. The life time of an item and the lead time of an order follows independent exponential distributions with rate γ and β respectively. The capacity of the waiting hall is finite, say, N. The arriving customer may be lost if the waiting hall is full. According to FCFS basis, the server commences the service with positive inventory. Otherwise, the server is in idle state. The service time follows exponential with rate μ_1. During the service time, the server may attain two consecutive breakdowns. The first break down and the second breakdown are respectively called as type 1 breakdown and type 2 breakdown. Duration of all activities taken before the first breakdown is known as regular service period. In regular service period, the time of occurrence of the first breakdown follows exponential distribution with rate η_1. After the first breakdown, the server may continue the service with the slower rate μ_2 ($\mu_2 < \mu_1$).

This is known as working breakdown. The decision for providing the service in working breakdown with rate (1-p) η_1 or going to repair state with rate $p\eta_1$. In the duration of working breakdown, the server may attain the second breakdown with exponential of rate η_2. After the second breakdown, he simply goes to repair state. The repair times of any type of breakdowns follows exponential distribution with rate α.

- After completion of repair, the server commences the regular services whenever the inventory level and customer level are both positive. Otherwise, he is in idle state in regular service period.

III. Analysis of the Model

Let a_t and b_t be the inventory level and present state of the server respectively.

$$b_t = \begin{cases} 1, & \text{if the server is idle at time } t \\ 2, & \text{if the server is busy during regular service period (RSP) at time } t \\ 3, & \text{if the server is busy during working breakdown period (WBP) at time } t \\ 4, & \text{if the server is in repair state at time } t. \end{cases}$$

let c_t be the customers level (waiting and being served) in the waiting hall. It is easy to perceive that state space (SS) of the multi-dimensional continuous time Markov chain (CTMC) $z_t = \{a_t, b_t, c_t ; t \geq 0\}$, is $U = U_1 \cup U_2 \cup U_3$, where
$U_1 = \{(0,v,w) \mid v=1,4,\ w=0,1,2,\dots,N\}$
$U_2 = \{(u,v,0) \mid u=1,2,\dots,S,\ v=1,4\}$
$U_3 = \{(u,v,w) \mid u=1,2,\dots,S,\ v=2,3,4,\ w=1,2,\dots,N\}$.
The rate matrix ψ of the CTMC $z_t = \{a_t, b_t, c_t ; t \geq 0\}$ has a form:

$$[\Psi]_{l_1 m_1} = \begin{cases} A_0, & a' = Q, & a = 0 \\ A_1, & a' = Q + a, & a = 1,2,..,s \\ D_a, & a' = a - 1, & a = 1,2,..,S \\ C_a, & a' = a & a = 0,1,2,..,S \\ 0, & otherwise. \end{cases}$$

To obtain the sub matrices of ψ see [1].
From the well known structure of Ψ, that the CTMC, $\{z_t ; t \geq 0\}$ with the finite state space U is regular. Therefore the Steady state distribution

$$X^{(u,v,w)} = \lim_{t \to \infty} Pr[a_t = u, b_t = v, c_t = w \mid Z_1(0), Z_2(0), Z_3(0)]$$

exists. Let $X = (X^{(0)}, X^{(1)}, \ldots, X^{(S-1)}, X^{(S)})_{1 \times (s+1)}$, be the limiting distribution Then,

$$X\psi = 0 \qquad (1)$$
$$Xe = 1, \qquad (2)$$

The equation gives the following:

$$X^{(u)} C_u + X^{(u+1)} D_{u+1} = 0, \quad u = 0,1,2,\dots,Q-1,$$
$$X^{(u-Q)} A_0 + X^{(u)} C_u + X^{(u+1)} D_{u+1} = 0, \quad u = Q,$$
$$X^{(u-Q)} A_1 + X^{(u)} C_u + X^{(u+1)} D_{u+1} = 0, \quad u = Q+1, Q+2, \dots, S-1,$$
$$X^{(u)} C_u + X^{(u-Q)} A_1 = 0, \quad u = S,$$

After lengthy simplification, we have $X^{(u)}$ for all u, see [1].

IV. Important measures

In this section, we establish average of some measures like, inventory level (E_h), reorder rate (E_s), perishable rate (E_p), customers in the waiting hall (WH) (E_{wh}), customers enter into the waiting hall(WH) (E_{we}), waiting time (E_w), interruption rate (E_i) and repair rate (E_r). Using this performance measures, we can obtain the total cost (TC).

$$E_h = \sum_{u=1}^{S} u\,(X^{(u,1,0)} + X^{(u,4,0)}) + \sum_{u=1}^{S}\sum_{v=2}^{4}\sum_{w=1}^{N}(u\, X^{(u,v,w)})$$

$$E_s = (s+1)\gamma\,(X^{(s+1,1,0)} + X^{(s+1,4,0)}) + \sum_{v=2}^{4}\sum_{w=1}^{N}((s+1)\gamma\, X^{(s+1,v,w)})$$
$$+ \sum_{w=1}^{N}(\mu_1 X^{(s+1,2,w)} + \mu_2 X^{(s+1,3,w)})$$

$$E_p = \sum_{u=1}^{S} u\gamma(X^{(u,1,0)} + X^{(u,4,0)}) + \sum_{u=1}^{S}\sum_{v=2}^{4}\sum_{w=1}^{N} u\gamma(X^{(u,v,w)})$$

$$E_{wh} = \sum_{w=1}^{N} w(X^{(0,1,w)} + X^{(0,4,w)}) + \sum_{u=1}^{S}\sum_{v=2}^{4}\sum_{w=1}^{N}(w\, X^{(u,v,w)})$$

$$E_{we} = \sum_{w=0}^{N-1} \lambda\,(X^{(0,1,w)} + X^{(0,4,w)}) + \sum_{u=1}^{S} \lambda(X^{(u,1,0)} + X^{(u,4,0)})$$
$$+ \sum_{u=1}^{S}\sum_{v=2}^{4}\sum_{w=1}^{N-1} \lambda\, X^{(u,v,w)}$$

$$E_w = \frac{E_{wh}}{E_{we}}$$

$$E_l = \lambda\,(X^{(0,1,N)} + X^{(0,4,N)}) + \sum_{u=1}^{S}\sum_{v=2}^{4} \lambda(X^{(u,v,N)})$$

$$E_i = \sum_{u=1}^{S}\sum_{w=1}^{N}(\eta_1 X^{(u,2,w)} + \eta_2 X^{(u,3,w)})$$

$$E_h = \sum_{w=0}^{N}(\alpha\, X^{(0,4,w)}) + \sum_{u=1}^{S}\sum_{w=0}^{N}(\alpha\, X^{(0,4,w)})$$

V. Cost Analysis and Numerical Results

Let c_h be the inventory holding host(/unit/unit time), c_s be be the setup cost(/order), c_p be the waiting cost(/unit time/customer), c_l be the lost cost (/unit time/customer), c_n be the interruption cost (/unit time) and c_r be the repair cost (/unit time). Define a cost function TC of the system and is given by

$$TC(S,s,N) = c_h \times E_h + c_s \times E_s + c_p \times E_p + c_w \times E_w + c_l \times E_l + c_n \times E_n + c_r \times E_r.$$

Example:

Suppose the parameters of the given system are λ=10, β=0.3, γ=0.2, μ_1=19, μ_2=12, α=10, η_1=1, η_2=0.3, p=0.5, N=25 and the associated costs are c_h=0.001, c_s=40, c_p=0.3, c_w=2, c_l=5, c_n=4, c_a=7. Using Tables 1 - 3, under the given range of S and s (as mentioned in the tables) we observe the optimum expected total cost(OETC) and the corresponding optimum maximum inventory level(OMIL) (S^*) and reorder level (OMRL) (s^*) for each given value of p (i.e, p=0, p=0.5, p=1). With corresponding OMIL S^* and OMRL s^* * of each value of p, we further obtain the OECT for varying N (as given in Table 4). From Tables 1 - 4, the following discussions are made.

1. The value of p lies between 0 and 1. Further if p=0 interprets the server goes to working breakdown whenever the breakdown occurs,

P = 1 interprets the server goes to repair state whenever the breakdown occurs, and p = 0.5 indicates the server is having an equal chance for going to take either a repair state or a working breakdown.

2 Due to the certain extent of server breakdown, the server can be severed the customers with a slower rate. This is known as working breakdown. Instead of that the sever goes to repair state whenever the breakdown occurs, it leads to increase the expected total cost (OETC) in the long run under the certain circumstances. This can be noticed from tables 1-3. The reason is that the repair cost per breakdown and the rate of occurrence of first breakdown are more sensitive when the sever goes to repair state often. Also the duration of occurrence of the second breakdown takes longer whenever the server severs the customers with a slower rate. Further, we examine the following optimum results under the given ranges of S and s and the values of other parameters and costs are fixed as mentioned above.

3 Under the given range of S and s, the minimum expected total cost (METC) occurs at p=0 is 24.575355and the optimum maximum inventory level and re order level are all maximum in the given range (i.e) S* = 200 and s* = 11 (Table 1).

4 Under the same given range of S and s, the minimum expected total cost (METC) occurs at p=0.5 is 25.582990 and the optimum maximum inventory level (OMIL) and re order level (OMRL) are all middle values in the given ranges of S and s respectively (i.e) S* = 190 and s* = 9(Table 2).

5 Under the given range of S and s, the minimum expected total cost (METC) occurs at p=1 is 27.253806 and the optimum maximum inventory level (OMIL) and re order level (OMRL) are all minimum in the given range (i.e) S* = 180 and s* = 7 (Table 3).

6 Under the optimum value of S and s (i.e.S* and s*), for different combinations of N and p, we obtain the (OETC) expected total cost as shown in Table 4. For each p, we also understand from the table that the expected total cost function can be convex over the interval of N, $30 \leq N \leq 60$ and it will yield a minimum cost at p=0.

7 In the whole discussion of this case, we observe that the expected total cost is minimum when the working breakdown is allowed.

Table- 1: TC(S,s,25) as a function of S and s at p=0

S \ s	7	8	9	10	11
180	24.690873	24.674937	24.661624	24.650859	24.642574
185	24.670545	24.653182	24.638375	24.626047	24.616126
190	24.656336	24.637717	24.621593	24.607884	24.596518
195	24.647690	24.627968	24.610685	24.595759	24.583116
200	24.644112	24.623424	24.605123	24.589125	**24.575355**

Table- 2: TC(S,s,25) as a function of S and s at p=0.5

S \ s	7	8	9	10	11
180	25.593202	25.592850	25.594688	25.598657	25.604709
185	25.587394	25.585651	25.586027	25.588463	25.592905
190	25.586862	25.583899	**25.582990**	25.584075	25.587098
195	25.591113	25.587078	25.585041	25.584937	25.586709
200	25.599705	25.594733	25.591706	25.590556	25.591226

Table- 3: TC(S,s,25) as a function of S and s at p=1

s / S	7	8	9	10	11
180	**27.253806**	27.282989	27.313704	27.345930	27.379654
185	27.275846	27.303746	27.333088	27.363848	27.396007
190	27.301393	27.328180	27.356328	27.385811	27.416606
195	27.330069	27.355890	27.383002	27.411373	27.440979
200	27.361547	27.386531	27.412741	27.440145	27.468714

Table 4: N versus TC(S^*, s^*)

N	p = 0	p = 0.5	p = 1
30	23.906809	24.815665	26.370465
35	23.585743	24.358942	25.731582
40	**23.533433**	24.152851	25.319328
45	23.684852	**24.141539**	25.094943
50	23.991726	24.279327	**25.010457**
55	24.420174	24.532938	25.025817
60	24.946613	24.880013	25.116334

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