

Reliable, Effective and Fault-Tolerant Design of LeafyCube Interconnection Network Topology

Nibedita Adhiakri, Archana Singh and Nirmal Keshari Swain



Abstract: This paper attempts to derive the performance properties of the Leafycube (LC) interconnection network. The Leafycube is already observed to have quite superior topological properties in comparison to the other contemporary networks. The various performance parameters of the LC network are studied and compared with the existing HC and its variants. The routing and broadcasting algorithms are proposed and the time complexities are also compared. The paper attempts to evaluate the cost effectiveness, reliability and fault tolerance aspects of LC interconnection network in order to justify the novelty in the design of the proposed structure. The leafy structure helps to retain the original hypercube while improving the node packing density in the interconnection network.

Keywords: HC, star crossed cube, cef, tecf, reliability

I. INTRODUCTION

In today's scenario there is a very high demand for faster computing in every aspect. The 24x7 internet connectivity has made a habit of staying connected. The speed of change seen now-a-days is due to inventions and innovation in technology. The velocity of such transformation has made unpredictability as the new normal. So the multiprocessor systems with a focus on shared memory are providing quite better solution. The parallel and distributed systems are thus receiving much attention. The parallel interconnection networks being the back bone of parallel systems are thus have become a basis for research in the last few decades (Bhuyan et al. 1989, Dally et al. 2004 and Duato et al. 2003). A surprising amount of revision is being visible with an expansion of reliability and availability as well as fault tolerance. Since this field is vast and quickly moving the emphasis is on the design of a new interconnection network keeping in view the cost effectiveness. Numerous articles are available describing the topological and performance parameters of various interconnection networks (Feng 1981). The number of different topologies that are discussed in different publications is really difficult to count but only a few have been implemented as actual parallel processors. For faster computing faster communication is also needed.

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To achieve the desired improvement several techniques are adopted like folding, extending, balancing, hybrid, embedding etc. and each technique has resulted in a new topology. To establish their superiority, the performance parameters are studied.

Among the well known and surveyed networks much work has been done on cube based and permutation graphs. The hypercube (HC) has been the most popular among the researchers due to its existing efficient parameters such as regular and symmetric structure, small diameter, low node degree, and link complexity (Saad et al. 1988). However, with the enhancement in the network dimension in a HC the associated link density also raises, as a result the performance of the network degrades to some extent. After a huge research is done on cube based networks, its several variations have been proposed, such as Folded hypercube (FHC) (Ahmed et al. 1991 & Duh et al. 1995), Crossed cube (CQ) (Efe 1992), varietal hypercube (Cheng et al.1994), Extended Hypercube (Kumar et al. 1992), Floded crossed hypercube (Adhikari et al. 2010), Extended-crossed cube (ECQ) (Adhiakri 2017). Dual cube and Meta cube are two level structures also derived from the cube topology (Li et al. 2001 & 2004). The Folded hypercube (FHC) is designed with addition of extra edges in hypercube where as the crossed cube (CC) and varietal cube are designed by changing the interconnection structure. In FHC the link complexity is increased. In CC due to crossed links, the routing and embedding are a bit complicated. The Extended crossed cube is another variation with network controllers (NC) at each level of the network. Due to addition of NCs the routing process is improved but the NC's never take part in computation. Though the number of Processing Elements (PEs) or nodes is increased but there is no improvement in computation. The Dual cube and Meta cube are clustered networks useful for large scale computing. The two level network structures make a complex routing and message traffic density is high. The original cube structure is not retained.

The n-Star being permutation graph is also studied and several variations of it are suggested (Akers et al. 1987 & Day et al. 1994). The fault tolerance and packing density along with other parameters of star graphs are extensively studied. Incomplete star (Latifi et al. 1994), Star cube (SC) (Tripathy et al. 2004), Star-crossed cube (SCQ) (Adhikari et al. 2014) are suggested with an intention to increase number of nodes. The n-star with $n!$ number of nodes with degree $(n-1)$ is a permutation graph and grows to its higher level very fast. Several variation of star graph like Starcube and Star crossed cube try to bridge the gap in between two consecutive levels of n-star. But being two level structures they are not exact replica of the single original structure.



The link complexity, bisection width is also increased.

A major disadvantage of hypercube networks is the link complexity. The tremendous improvement in performance of interconnection networks is offset by the remarkable increase in cost with enhancement in size (Gram et al. 1993 & 2003). This motivates engineers/ designers to invent new topologies which can scale up at low cost, low latency and increased fault tolerance. The evaluation of success of parallel systems mostly depends upon the nature of communication in the workload of parallel programs that are to be solved on the processing units. Few structures are also derived from the clustered network meta cube (Adhikari et al. 2009, 2011, 2012). In the two level structure or hybrid network the link complexity increases though they contain more number of nodes. The complexity of routing algorithms also increases. While keeping all these in view, the authors have tried to improve certain features of hypercube and proposed a new variant of it called the Leafy-cube (LC), which is actually a standard Hyper-cube embedded with tree (Adhikari et al. 2019). There are some extra links connecting few leaf nodes. The purpose of the network is to improve packing density while keeping the degree and diameter same or as low as possible with respect to the original structure. Also the bisection width is exactly same as that of HC. This paper concentrates in deriving the performance parameters of the Leafycube topology. Also attempts have been made to propose routing and broadcasting algorithms. Further the fault tolerance and reliability issues are also addressed to establish the superiority.

II. PRELIMINARIES

The parallel interconnection networks are designed with an intention to provide faster computing with low cost to make availability all the time a success. This intention is the sole spirit of the current research. As the hypercube is the widely accepted and implemented structure, it is again tired for modification as follows.

A. Leafycube System Structure

The n -dimensional hypercube $HC(n)$ is a regular graph containing 2^n number of PEs. Every node in the $HC(n)$ has n number of neighbors at hamming distance 1 and hence the node degree is n (Saad et al. 1998). To the regular hypercube few nodes are added to propose a new parallel interconnection network which are treated as child nodes or leaf nodes. Thus the new topology is named as Leafy cube (LC) network (Adhikari et al. 2019). For construction of leafy cube n -dimensional hypercube is considered. In the basic structure additional ' n ' number of nodes are joined to each vertex of the basic structure using n additional edges. Here the original cube nodes become root nodes and are connected to n leaf nodes. The edge connecting root node to leaf node is called leaf edge and the edge connecting root nodes (original cube nodes) are called cube edges. The resulting topology is a regular and hierarchical network. The new hybrid structure of dimension two and three are shown in Figure 1 and 2 respectively.

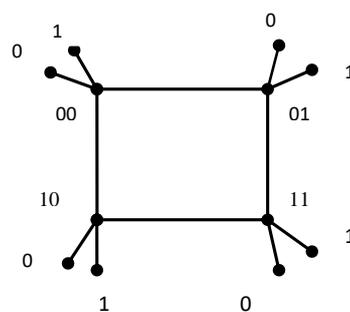


Fig. 1: Leafycube of Degree 2, LC (2)

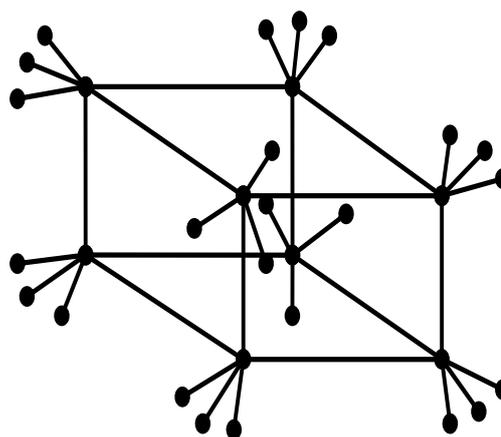


Fig. 2: Leafy Cube of Degree 3, LC(3)

B. Topological Properties

The various topological properties are already derived (Adhikari et al. 2019). They are again discussed here without supporting proof for further analysis.

2.2.1 Degree

This defines the number of immediately adjacent nodes to a particular node. These nodes should be immediate neighbors.

Theorem 1: The degree of Leafy cube of degree n that is $LC(n)$ is $(n+n)$.

2.2.2 Total number of nodes

Theorem 2: The total number of nodes in $LC(n)$ is $V = 2n(n+1)$.

2.2.3 No of Edges

Theorem 2: The no of edges of $LC(n)$ is, $E = n2^{n-1} + 2^n n$

2.2.4 Diameter

It is defined as the maximum distance between the farthest nodes of the interconnection network.

Theorem 4: The diameter of $LC(n)$ is $(n+2)$.

2.2.5 Cost

For a symmetric network the cost factor is defined as the product of the diameter & the degree of the nodes.

Theorem 5: The cost of $LC(n)$ is $(n+n)(n+2)$.

2.2.6 Bisection Width

It is defined as the number of edges whose deletion results in two distinct sub networks. Then each equal halves will contain exactly half number of nodes.

Theorem 5: The bisection width of the $LC(n)$ is $\frac{2^n}{2}$.

2.2.7 Average Node Distance

Theorem 6: The average node distance in $LC(n)$ is $\bar{d}_{avg} = \frac{n}{2} + 2$.

2.2.8 Message Traffic Density

Theorem 7: The message traffic density of LC(n) network is

$$\rho = \frac{n+1}{3}$$

The derived topological parameters are compared against the contemporary networks in Table 1.

Table1. Comparison of the Topological Parameters

Parameters	HC	FHC	Star cube (m,n)	CQ	SCQ(m,n)	LC
Degree	n	n+1	m+n-1	n	m+n-1	2n
Nodes	2 ⁿ	2 ⁿ	m!2 ⁿ	2 ⁿ	m!2 ⁿ	2 ⁿ + n2 ⁿ
Diameter	N	$\lceil \frac{n}{2} \rceil$	$m + \lceil \frac{3}{2}(n-1) \rceil$	$\lceil \frac{n+1}{2} \rceil$	$\lceil \frac{m+1}{2} \rceil + \lceil \frac{3}{2}(n-1) \rceil$	n+2
Cost	n ²	(n+1)* $\lceil \frac{n}{2} \rceil$	(m+n-1)(m+ $\lceil \frac{3}{2}(n-1) \rceil$)	n $\lceil \frac{n+1}{2} \rceil$	(m+n-1) $\lceil \frac{m+1}{2} \rceil$ + $\lceil \frac{3}{2}(n-1) \rceil$	(2n)(n+2)
No of edges	n* 2 ⁿ⁻¹	(n+1)* 2 ⁿ⁻¹	m! 2 ⁿ⁻¹ (m+n-1)	2 ⁿ⁻¹	m! 2 ⁿ⁻¹ (m+n-1)	n2 ⁿ⁻¹ + n2 ⁿ
Bisection width	$\frac{2^n}{2}$	$\frac{2^n}{4}$	2 ⁿ⁺¹	$\frac{2^n}{2}$	2 ⁿ⁺¹	$\frac{2^n}{2}$
Average Node distance	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2} + m - 4 + \frac{2}{n} + \sum_{i=1}^m \frac{1}{i}$	$\frac{n}{2}$	$\frac{11x+4y}{8} + n - 4 + \frac{2}{m} + \sum_{i=1}^m \frac{1}{i}$	$\frac{n}{2}$
Message Traffic Density	1	$\frac{n}{n+1}$	$\frac{2\bar{d}_{sc}}{(m+n-1)}$	1	$\frac{2\bar{d}_{scq}}{(m+n-1)}$	$\frac{n+1}{3}$

III. PERFORMANCE PARAMETERS OF LC NETWORK

This section is devoted to study the performance parameters of LC network. A PIN is specified by its topology, routing algorithms, and flow control mechanisms. Performance analysis determines the failure or success of a project forecast using various parameters. From Performance analysis we can determine the total cost of a system as it reproduce important aspect of a multiprocessor system. To make the network more attractive, here more emphasis is given to Cost effectiveness (CEF) & Time cost effectiveness (TCEF). In some cases, extra links are added in real machines to the simple topologies in order to improve the performance and reliability. In the subsequent paragraphs the fault tolerance and reliability issues are also discussed.

A. Cost Effectiveness Factor

It is nothing but ratio of cost effectiveness and efficiency. Here processor cost and communication link cost are taken into consideration.

Theorem 7: The cost effectiveness factor of the LC (n) is $\frac{1}{1+p(\frac{n}{2})}$, where p is the ratio of the link to processor cost.

Proof: In general no of links = f(no. of nodes). In the proposed network, the no of nodes is $p = 2^n(n+1)$.

The total no of links is given by $E = n2^{n-1} + n2^n = 2^n(n+1) \lceil \frac{3n}{2(n+1)} \rceil = p \left(\frac{3n}{2(n+1)} \right) = f(p)$

$$g(p) = \frac{f(p)}{p} = \frac{p \left(\frac{3n}{2(n+1)} \right)}{p} = \frac{3n}{2(n+1)}$$

$$CEF(p) = \frac{1}{1+pg(p)} = \frac{1}{1+p \left(\frac{3n}{2(n+1)} \right)}$$

B. Time Cost Effectiveness Factor

Theorem 8: The TCEF of LC (n) network is given by network is given by $\frac{1+\sigma}{1+p \left(\frac{n}{2} \right) + \frac{1}{2^n(n+1)}}$.

Proof: The TCEF is given by $TCEF(p, T_p) = \frac{1+\sigma T_1^{\alpha-1}}{1+pg(p)+\frac{\sigma}{p}}$

, where T₁ is the time required to solve the problem by a single processor using the fastest sequential algorithm, T_p is the time required to solve the problem by a parallel algorithm using a multiprocessor system having p processors and σ is the ratio of the cost of penalty to cost of processors. For linear time penalty in T_p, α is chosen as 1[7].

As per Theorem above $g(p) = \frac{3n}{2(n+1)}$ & σ and α are assumed to be 1.

$$\text{So TCEF} = \frac{1+\sigma}{1+\rho g(p)+\left(\frac{\sigma}{p}\right)} = \frac{1+\sigma}{1+\rho\left(\frac{3n}{2(n+1)}\right)+\frac{\sigma}{2^n(n+1)}} \blacksquare$$

C. Fault Tolerance

The probability of failure grows with the size of the computing system and draws attention towards fault tolerance aspects. Fault tolerance plays a major role in parallel computing systems as failure of nodes or links will drastically reduce the performance. While evaluating it, the node connectivity is a major issue. For network topology it is defined as the maximum number of vertices whose removal will result in a connected network, it means fault tolerance of a network should be one less than its total connectivity. So for $LC(n)$, fault tolerance is one less than total no of connectivity, i.e. $2n-1$.

D. Fault Diameter

When a node from the network is removed in case fault occurs, then it impacts the diameter directly. The fault diameter value need to be close to the original diameter, so the original diameter of $LC(n)$ increases by unity i.e. $n+3$.

E. Reliability

When the performance parameters are considered reliability is treated with utmost attention. It is the probability that the system will successfully perform its desired operations for a given time under some predefined operating conditions. There are many measures of reliability but the terminal reliability is discussed here in this research work.

Terminal Reliability (TR)

Terminal Reliability is the probability that at least one path exists from a specific input node to another output node. TR is always associated with link between source and destination nodes which is a direct one-to-one connection established between an input port (source) and an output port (destination). If it fails to establish a connection from a given source to its desired destination then the interconnection network is termed as a faulty system.

Reliability Analysis of $LC(n)$

In the current work only Terminal Reliability (TR) is taken into consideration. Generally TR is used to measure the robustness of a communication network. It is the probability that there exist at least one flawless path between a designated source and destination node pair (Adhikari et al. 2010).

Suppose 'S' and 'D' are the source and destination nodes. There should be 'n' number of paths with distinct nodes lying between S and D. Let r_i be the number of edges involved in the path i , where $1 \leq i \leq n$. Thus there exist r_i-1 number of nodes in i th path.

Let $P(E_i)$ be the probability of affluent routing using the i^{th} path. Then the link reliability denoted as R_l with link failure rate is 0.0001. Next R_n be the node reliability having failure rate 0.001,

$$R_l = e^{-\lambda t}, \text{ where } \lambda = 0.0001 \text{ and } t=1000 \text{ and, } R_n = e^{-\lambda t} \text{ where } \lambda = 0.001 \text{ and } t=1000.$$

Theorem 9: For $LC(n)$ network, the two terminal reliability is given by

$$TR = 1 - \prod_{i=1}^n (1 - R_l^{r_i} R_n^{r_i-1}).$$

Proof: Here all nodes and links are identical and have statistically independent and exponentially distributed failure rates. Now the probability of existence of an affluent link between the sending and receiving PEs can be given by

$$P(E_i) = R_l^{r_i} R_n^{r_i-1}$$

$$\text{So TR} = P(E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n)$$

$$= 1 - \prod_{i=1}^n (1 - R_l^{r_i} R_n^{r_i-1}) \blacksquare$$

IV. ROUTING IN $LC(n)$ NETWORK

This section is devoted to discuss routing algorithms for $LC(n)$. In the cube based networks, the routing process depends upon the shortest path decided using Hamming distance. There are four cases and are proposed here one by one.

Before going to algorithm certain basics are discussed. Let 's' is source and 'd' is destination node for all four cases:

Case I: s and d both are leaf nodes,

Case II: s is Leaf node and d is Cube node,

Case III: s is cube node and d is leaf node,

Case IV: both the nodes s and d are cube nodes.

The Routing algorithm for all the above cases is given below.

Algorithm Case I

Para do

Step-1: If s-leaf node then

```
{
Move to next cube node (root)
Perform Cube routing
```

Else if s-cube node and d-leaf node then travel one step to reach destination node

```
}
```

In the above case source and destination nodes both are leaf node. If source node is leaf node then route the message from leaf node to cube nodes, then cube nodes to cube nodes and at last from cube node to destination node, i.e. neighbor node in one step and hopping stops. For cube nodes normal cube routing will be used and these steps are limited by diameter value.

Algorithm Case - II

Step-1: if s-leaf node then

```
{
move to nearest cube node (root)
```

Step-2: Perform Cube routing, travel n steps to reach destination node

```
}
```

Here if the source node is a leaf node and target node is a cube node then route the message from the leaf node to the root node and then route the message using simple cube routing.

Algorithm Case - III

s is cube node and d is leaf node

Step-1: if s-cube node then

```
{
Perform Cube routing
```

```
{
```

Step-2: Cube node to leaf node one step to reach destination node

```
}}
```

Here source node is cube node so it performs cube routing to route the message from source node to the root node of destination node and then one step to reach destination.

Algorithm Case- IV

s and d both are Cube nodes

Perform cube routing (self-routing in Hypercube)

In the last case both the nodes are cube nodes so directly cube routing will be done or on the other word we can say self-routing will be done.

V. BROADCASTING

This journal uses double-blind review process, which means that both the reviewer (s) and author (s) identities concealed from the reviewers, and vice versa, throughout the review process. All submitted manuscripts are reviewed by three reviewer one from India and rest two from overseas. There should be proper comments of the reviewers for the purpose of acceptance/ rejection. There should be minimum 01 to 02 week time window for it.

Broadcasting is the process of information dissemination in an interconnection network by which a message initiated at a node is transferred to all other processing elements. The broadcast process finds extensive application in the control of distributed systems and in parallel computing. For instance, in parallel computing, there are many tasks, distributed among several PEs and their result need to be updated at other processors in order to continue the processing.

In this section attempts have been made to discuss two distinct broadcasting algorithms one-to-all and all-to-all in the proposed LC network.

A. One-to-all Broadcast Algorithm for LC

Before going to algorithm certain basics are discussed, s is source node and d is destination node same as previously discussed in routing algorithm.

The proposed broadcasting process satisfies the below stated conditions.

1. A PE can only send (receive) the message to (from) one of its neighbors (i.e. one port communication).
2. A node can receive the message exactly once in the whole broadcasting process to avoid duplication of the received message.

One-to-All

```
{
Step1: Suppose the source node is leaf node, then message travels one step to the reach the root node
Step2: Next, the cube node (root) broadcast simultaneously to their respective cube neighbors using binomial trees.
Step3: Now the loaded PEs in the network will perform a broadcast inside the network utilizing above two steps.
Step 4: Next a spanning broadcast tree (SBT) is designed for LC, where each PE is linked either by cube edge or by leaf edge.
}
```

In SBT cube edge connects each node, if the hamming distance is less than or equal to n or a leaf edge is used if the destination is a leaf node with hamming distance 1. Thus the spanning broadcast tree is having height at most n+2. Hence, the broadcasting is done in O(n) time that is exactly similar to hypercube. in spite of packing comparatively large number of nodes. The routing process is illustrated below through a simple example.

B. Illustration of Routing in LC Network

The illustration is given for three dimensional LC network. The farthest nodes in this network be denoted as S and D. the respective node addresses are (000,0) and (111,1). S is the leaf node of the cube node R₀(000). As per the spanning broadcast tree shown below in Figure 3, the bold lines show the route from source to destination. In the figure the R_i's are cube nodes. The leaf edges are shown through dashed lines and cube edges are shown through solid lines. The path from S to D is as follows

VI. PREPARE YOUR PAPER BEFORE STYLING

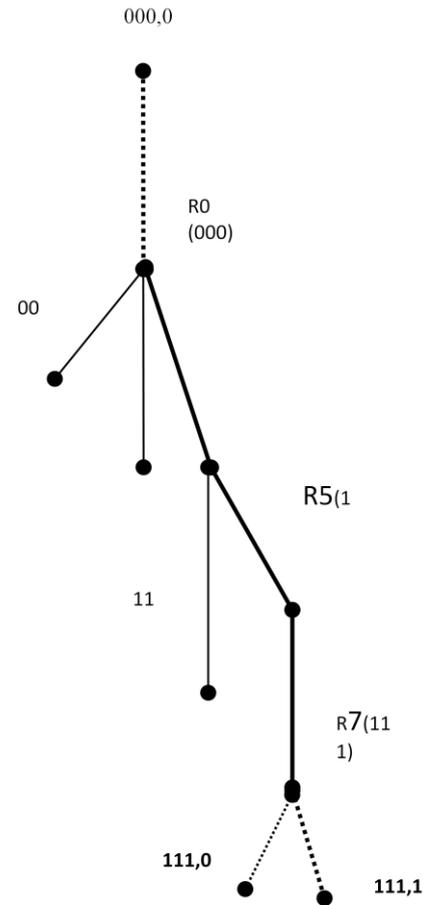


Fig. 3: Spanning Broadcast Tree for LC(3) Network
S(000,0)->R₀ (000) ->R₄(100)-> R₅(101)-> R₇(111)->D(111,1) as shown in Figure3. Further the distance is 5.

C. All-to-All Broadcast Algorithm for LC Network

Hence the one to all algorithms can be executed for each node for complete broadcasting of the message in the network.

a. Time Complexity

The term simply describes that how much amount of time is taken to run, it is simply calculated by calculating the no of elementary operations performed by the algorithm. The routing and broadcasting algorithms are developed with lesser time complexity for Leafycube.

b. Routing Time Complexity of LC

In this subsection we are going to calculate the Routing time complexity in LC network. As previously discussed in routing algorithm there are four cases possible in LC network, those are discussed one by one.

Case-I: Here 's' and 'd' both are leaf nodes. Each cube node has 'n' no of leaf nodes at one step distance. The time complexity of hyper- cube is O(n). If leaf nodes are taken into consideration then two extra nodes are travelled in single hop. Hence the time complexity is $T(n) = O(n) + 2$.

Case II: In case of 's' is Leaf node and 'd' is Cube node, the time complexity will be one step addition to the time complexity of hyper-cube. i.e. $T(n) = 1 + O(n)$.



Case III: In case of 's' is cube node and 'd' is a leaf node, $T(n) = O(n) + 1$ is the time complexity

Case IV: In the last case where 's' and 'd' both are Cube nodes, the time complexity of LC is same as time complexity of Hyper-cube i.e. $T(n) = O(n)$.

When n is too large the complexity of routing in Leafy cube is $O(n)$ that is equal to Hypercube. In spite of packing a large number of nodes the LC network efficiently does communication in equal time as that of Hypercube. The structure helps in improving the efficiency of the proposed network. The leaf nodes are one hop distance and this helps in faster communication for all nodes simultaneously.

VII. RESULTS AND DISCUSSIONS

This section is devoted to make comparison of results obtained. As there are several emerging interconnection architectures, so there is a need to compare their performance parameters. The concept of cost effectiveness is applied in planning and management of the INs. It is widely used to compare among the competing designs for various factors. Effectiveness is the concept of being able to achieve a desired result and it measures the extent to which the resources are fully utilized for the intended task. Here the cost effectiveness is defined as the product of two factors: one is the architectural features of the processing units and the other factor is efficiency of the algorithm. For any value of $p > 1$, the $CEF(p) < 1$ where p is the total number of PEs.

The comparison of CEF and TCEF are shown in Figure 4 and 5. The values are plotted against network dimension. It is monotonically decreasing function like the hypercube.

The terminal reliability comparison is shown in Figure 6 and 7. The comparison is done with Hypercube, and its two alternatives namely Crossed cube and Folded crossed cube as they are direct derivative of hypercube possesses nearly similar structure. Star and its derivatives are not considered here as they are two level structures. The Leafycube exhibits better reliability than hypercube and crossed cube. Though the folded crossed cube is having

Table 2: Cost Effectiveness Factor of LC(n)

Dimension	$\rho=0.1$	$\rho=0.2$	$\rho=0.3$	$\rho=0.4$
3	0.357143	0.217391	0.15625	0.121951
4	0.25	0.142857	0.1	0.076923
5	0.181818	0.1	0.068966	0.052632
6	0.136986	0.073529	0.050251	0.038168
7	0.106383	0.05618	0.038168	0.028902
8	0.084746	0.044248	0.02994	0.022624
9	0.068966	0.035714	0.024096	0.018182
10	0.057143	0.029412	0.019802	0.014925
11	0.048077	0.024631	0.016556	0.012469
12	0.040984	0.020921	0.014045	0.010571
13	0.035336	0.017986	0.012063	0.009074
14	0.030769	0.015625	0.010471	0.007874
15	0.027027	0.013699	0.009174	0.006897

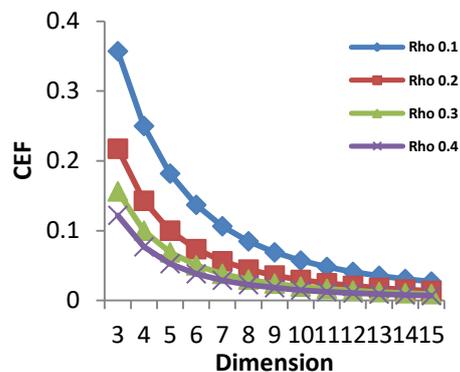


Fig. 4: Cost Effectiveness Factor of LC (n)

Table 3: Time Cost Effectiveness Factor of LC (n), $\rho=0.1$, $\alpha = 1$

Dimension	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$	$\sigma = 4$
2	1.008403	1.451613	1.860465	2.238806
3	0.706402	0.861617	1.000814	1.128518
4	0.498442	0.464154	0.498644	0.53026
5	0.363292	0.266093	0.272352	0.278089
6	0.273889	0.173423	0.17428	0.175066
7	0.212744	0.12483	0.124921	0.125005
8	0.169485	0.095324	0.095332	0.095339
9	0.137929	0.075539	0.07554	0.075541
10	0.114285	0.061459	0.061459	0.061459
11	0.096154	0.051032	0.051032	0.051032
12	0.081967	0.043075	0.043075	0.043075
13	0.070671	0.036858	0.036858	0.036858
14	0.061538	0.031903	0.031903	0.031903
15	0.054054	0.027889	0.027889	0.027889

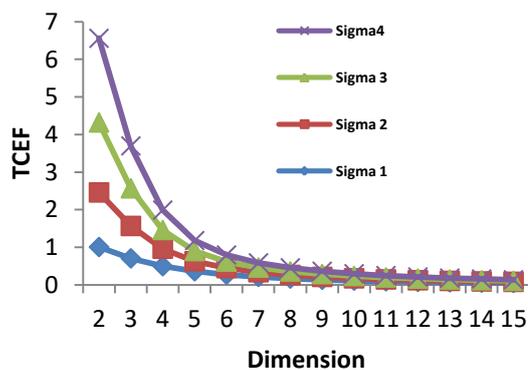


Fig. 5: Time Cost Effectiveness Factor of LC

higher values they all have equal number of nodes but FCC has more edges due to application of folding technique. However, the FCC network contains

Table 4: Comparison of Reliability of LC
($\lambda_l = 0.0001, \lambda_p = 0.001, t=1000\text{Hrs}$)

Dimension	HC	CC	FCC	LC
3	0.029853	0.108906	0.824142	0.67583
4	0.039605	0.117863	0.825910	0.708331
5	0.049259	0.12673	0.827660	0.737574
6	0.058816	0.135508	0.829392	0.763884
7	0.068276	0.144198	0.831107	0.787557
8	0.077642	0.152800	0.832805	0.808856
9	0.086913	0.161316	0.834485	0.82802
10	0.096091	0.169747	0.836149	0.845263
11	0.105177	0.178092	0.837796	0.860776
12	0.114172	0.186354	0.839427	0.874735

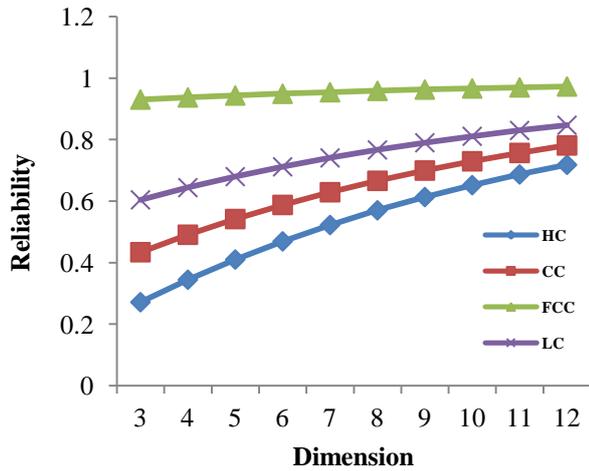


Fig. 6: Comparison of Reliability for LC Network
Table 5: Reliability Comparison with Time for LC($\lambda_l = 0.0001, \lambda_p = 0.001, n = 10$)

Time (Hrs)	HC	CC	FCC	LC
1000	0.652323	0.729968	0.966914	0.999998
2000	0.096091	0.169747	0.836149	0.998658
3000	0.010032	0.03611	0.743418	0.978196
4000	0.00101	0.009131	0.670653	0.903824
5000	0.000101	0.00257	0.606571	0.773833
6000	1.02E-05	0.000756	0.548816	0.621376
7000	1.02E-06	0.000226	0.496586	0.478532
8000	1.02E-07	6.78E-05	0.449329	0.360675
9000	1.02E-08	2.04E-05	0.40657	0.270018
10000	1.03E-09	6.15E-06	0.367879	0.202715

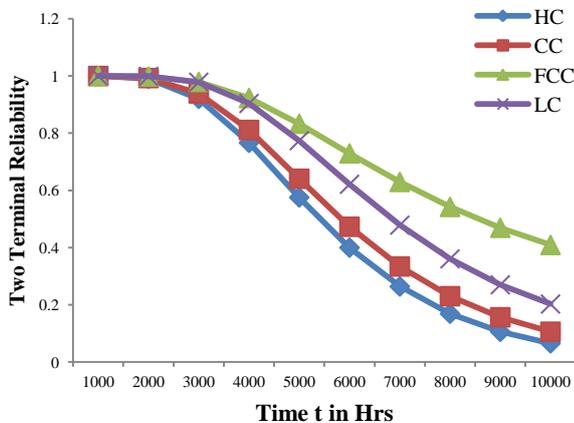


Fig. 7: Comparison of Reliability against Mission Time for n=10

exactly same number of nodes as hypercube but LC contains more number of nodes. The Figure 7 shows the superiority of

LC over others in terms of reliability while keeping the dimension n fixed at 10 with mission time varying from 1000 to 10,000. The computed values for the LC network and others are listed in Table 4 and 5 respectively.

VIII. CONCLUSION AND SCOPE FOR FUTURE WORK

In the current research, attempts have been made to improve few features of the popular n-cube. The aim was to suggest a new derivative of Hypercube which can pack more number of nodes with low node degree. The Leafycube satisfied the same and emerged as a fine derivative. The leafy structure is proved to be an improved topology due to addition of extra nodes at one hop distance. The average distance, bisection width is exactly equal and message density of LC network is almost equal to that of hypercube. Thus in spite of packing more nodes routing and broadcasting algorithms of LC network are more efficient. The Leafycube is more cost effective and reliable than the parent network and other derivatives like the Crossed cube, FCC and FCC.

As the interconnection networks are the backbone of high performance computing systems, the width with which the installed system can grow need to be studied. The growth in volume of computation, size of input data and reduction in response time; are the few parameters that lead to an ever increasing demand for scalable system. Also the load needs to be balanced to avoid faults and wastage of resources in the network. Thus further research can be done to study the scalability and load balancing features of Leafycube to further increase the overall performance of the system (Cybenko et al. 1989, Jan et al. 2003, & Adhikari 2018).

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