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A Stability of Force Convection Boundary Layer Laminar flow in Diverging Chennal

Iyyappan G, A. K . Singh

Abstract: ***The objective of this paper works is to study, a stability analysis of two dimensional boundary layer forced convection over a fixed vertical plate vary in viscosity and prandtl number with time dependent . The governing equations are transformed to non linear coupled partial differential equations with boundary conditions have been solved numerically by finite difference scheme in combination with quasi-linearization technique with suitable step size along the stream- wise direction. Results are showed different values of mass transfer parameter (A) . It was found that the solutions is stable when increases the suction/injection parameter A.***

Keywords: Force convection, coupled PDE Non-similar solution, fixed vertical plate, Wall suction or injection , quasi-linearization technique, finite difference scheme, Stability Analysis.

I. INTRODUCTION

Now a days handling of heating and cooling of a heavy fluid most encountered problem in engineering problems. The laminar boundary layer flow in diverging channel, when a fluid with uniform velocity enters a straight with increasing the diameter if channel a velocity of the fluid is decreases. C.F. Dewey, J.F. Gross [1] are discussed flow configuration and methods of the solution of non-similar coupled partial differential equations. Later B.J. Venkatachala, G. Nath[2] are analyzed incompressible laminar boundary fluid flow fixed in a flat plate . After several decades discussed the flow of nature and stability of velocity and temperature profiles. S. Roy et al [3-7] are discussed the flow of three dimension with rotating sphere with the help of fine difference scheme using quasi-linearization technique. The same work to extended for laminar flow in diverging channel with variable viscosity and Prandtl number and the effects of viscous dissipation number by Iyyappan. G et al [8-9]. In this article, fully focused on force convection flow of fluid flow over a vertical fixed plate with variable viscosity and prandtl number. The mass transfer parameter (A) is significant on the velocity and temperature profiles and numerical results are showed numerical and graphically.

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***** Correspondence Author
Iyyappan G.*, Assistant Professor, Department of Mathematics, Hindustan Institute of Technology and Science, Chennai, (Tamil Nadu), India.
A. K. Singh, Assistant Professor, Department of Mathematics Division, Vellore Institute of Technology Chennai, (Tamil Nadu), India.

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II. MATHEMATICAL FORMULATION

We consider the two-dimensional steady laminar incompressible boundary layer in forced convection over a diverging channel, assumed the range of temperature considered (i.e., 0◦C − 40◦C), specific heat (Cp) and density (ρ). Moreover, the thermal conductivity (k) , variation of viscosity (μ) and prandtl number (Pr) with temperature are quite significant. The equations of conservation of mass, momentum and energy equations governed by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\left(\frac{du_e}{dx}\right) + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (2)$$

The boundary condition are given by

$$y = 0; u = 0 \quad ; V = v_w(x)$$

$$y \to \infty ; u \to u_e(x); T \to T_\infty \quad (3)$$

$$\xi = \frac{x}{L}; \ \eta = \left(\frac{u_e}{\nu x}\right)^{\frac{1}{2}}; \ y = \psi(x,y) = (\nu u_e x)^{\frac{1}{2}} f(x,y)$$

$$u = \frac{\partial\psi}{\partial y}; \quad v = -\frac{\partial\psi}{\partial x}; \quad u = u_e f_\eta; \quad u = u_w + u_\infty;$$

$$v = -\frac{(\nu u_e x)^{\frac{1}{2}}}{2x}\left\{\left(\frac{x}{u_e}\frac{du_e}{dx}+1\right)f + 2\xi f_\xi - \eta\left(\frac{x}{u_e}\frac{du_e}{dx}-1\right)f_\eta\right\}. \quad (4)$$

Eq (1) identically satisfied , Eq (2) reduce to

$$(NF_\eta)_\eta + \left(\frac{m+1}{2}\right)f F_\eta + (1-F^2)m = \xi\left(FF_\xi - f_\xi F_\eta\right) \quad (5)$$

Where

$$N = \frac{\mu}{\mu_\infty} = \frac{b_1 + b_2 T_\infty}{b_1 + b_2 T} = \frac{1}{a_1 + a_2\theta}$$

$$a_2 = c_1 + c_2 T_\infty \ ; \ a_3 = c_2 \Delta T_w \ ; \ T_w = T_w - T_\infty$$

$$E_c = \frac{u_\infty^{\ 2}}{C_p \Delta T_w}$$

Transformed boundary conditions are

$$F = 1-\varepsilon; \quad \theta = 1; \quad at \quad \eta = 0$$

$$F = \varepsilon; \quad \theta = 0; \quad at \quad \eta = \infty \qquad (6)$$

The physical quantities of wall shear stress and heat transfer rate defined as (Nusselt number $R_e^{1/2} N_u$ and skin friction coefficient $R_e^{1/2} C_f$) these coefficients are defined by

$$\mathrm{Re}^{\frac{1}{2}} C_f = 2\xi^{\frac{-1}{2}} \left(1-\varepsilon e^{\xi}\right)^{\frac{1}{2}} f_{\eta\eta}(\xi,0)$$

$$\mathrm{Re}^{-\frac{1}{2}} N_u = \xi^{\frac{1}{2}} \left(1-\varepsilon e^{\xi}\right)^{\frac{1}{2}} G_{\eta}(\xi,0) \qquad (7)$$

III. NUMERICAL SOLUTION

The boundary value problem represented by Eq (9) with boundary (6) solved by numerically using implicit finite difference scheme with Quasilinearization technique. After applying Quasilinearization technique in equations (5) with boundary condition (6) We get

$$X_1^k F_{\eta\eta}^{k+1} + X_2^k F_{\eta}^{k+1} + X_3^k F^{k+1} + X_4^k F_{\xi}^{k+1} + X_5^k \theta_{\eta}^{k+1} + X_6^k \theta^{k+1} = X_7^k \qquad (8)$$

The coefficient in equation (7) is given by

$$X_1^k = N \;; X_2^k = -a_1 \theta_{\eta} N^2 + \left(\frac{m+1}{2}\right) f + \xi f_{\xi}$$

$$X_3^k = -\xi F_{\xi} - 2mF \;;\; X_4^k = -\xi F \;;\; X_5^k = -a_1 F_{\eta} N^2$$

$$X_6^k = -a_1 N^2 F_{\eta\eta} + 2a_1^2 N^3 F_{\eta} \theta_{\eta}$$

$$X_7^k = 2a_1^2 N^3 F_{\eta} \theta_{\eta} \theta - a_1 F_{\eta} \theta_{\eta} N^2 - a_1 N^2 F_{\eta\eta} \theta - \xi F_{\xi} F - m\left(1+F^2\right)$$

The resulting sequence of equation is reduced to system linear algebraic equation with tri-diagonal matrices which are solved using varga's algorithm[12]. The solution is assumed converge the difference reaches strictly less than 10^{-5}.

$$i.e., Max\left\{\left|\left(F_{\eta}\right)_w^{i+1} - \left(F_{\eta}\right)_w^{i}\right|\right\} \le 10^{-5}$$

IV. STABILITY ANALYSIS

The existence of solutions drives the interest discuss the stability analysis. Consider unsteady case for equations (2)

$$N_{\eta}\left(\frac{\partial^2 f}{\partial \eta^2}\right) + \left(\frac{\partial^3 f}{\partial \eta^3}\right) + \left(\frac{m+1}{2}\right) f\phi \left(\frac{\partial^2 f}{\partial \eta^2}\right) + \left(1-\left(\frac{\partial f}{\partial \eta}\right)^2\right) m\phi + \frac{\xi \mathrm{Re}\, \phi_{\tau} \phi^{-1}}{1-\varepsilon e^{\xi}}\left(1-\left(\frac{\partial f}{\partial \eta}\right)\right) - \frac{\xi \mathrm{Re}}{1-\varepsilon e^{\xi}}\left(\frac{\partial^2 f}{\partial \eta \partial \tau}\right)$$
$$= \phi\xi\left[\left(\frac{\partial f}{\partial \eta}\right)\left(\frac{\partial f}{\partial \eta}\right)_{\xi} - \left(\frac{\partial^2 f}{\partial \eta^2}\right)\left(\frac{\partial f}{\partial \xi}\right)\right] \qquad (9)$$

With boundary conditions are

$$F = 1-\varepsilon; \quad \theta = 1; \quad at \quad \eta = 0$$

$$F = \varepsilon; \quad \theta = 0; \quad at \quad \eta = \infty \qquad (10)$$

$f = f_0$, $G = G_0$ are the perturbation of the basic flow it can be detected stability of the flow, with the following disturbances $f = f_0 + 1 - e^{-\gamma\tau} F$

$$N_{\eta} F''' + N_{\eta} F'' + \left(\frac{m+1}{2}\right)\left(f_0 F'' + F f_0''\right) + \left(1-F'^2\right) m\phi + \frac{\xi \mathrm{Re}\, \phi_{\tau} \phi^{-1}}{1-\varepsilon e^{\xi}}\left(1-F'\right) - \frac{\xi \mathrm{Re}\, \gamma F'}{1-\varepsilon e^{\xi}}$$
$$= \phi\xi\left[f_{\eta} F_{\xi} - F'' f_{\xi}\right] \qquad (11)$$

With transformed boundary conditions

$$F = 0; \; F' = 0; \; \theta = 0; \quad at\ \eta = 0$$

$$F = 0; \; \theta = 0; \quad at\ \eta \to \infty \qquad (12)$$

For infinite values of $\gamma_1 < \gamma_2 < \gamma_3$........ can be found by solving the eigenvalue problems (10). The stability of the corresponding steady flow solutions determined by the smallest eigenvalue γ_1 for $f = f_0$ *and* $G = G_0$. If the value of γ_1 is positive then flow is considered as stable, the flow is unstable and initial growth of disturbances exists then the value of γ_1 negative

V. RESULT AND DISCUSSION

The Ordinary differential equation (8) , dimensionless non-linear partial differential equation (11) with boundary conditions (12) have been solved numerically by Quasi-linearization technique [8-10]. The computations have been carried out , the edge of the boundary layer η_{∞} has been taken in between 0.0 and 6.0.

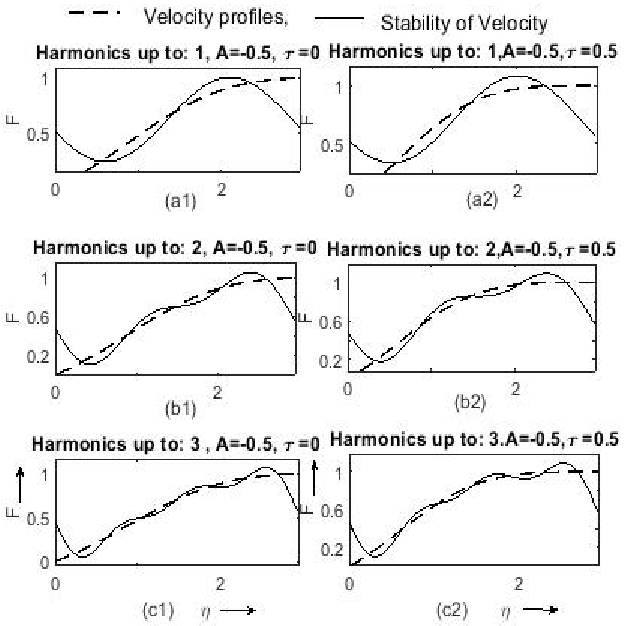

Fig 1. Distribution and Stability of Velocity profiles (Injection A < 0)

Fig.1 shows the effects of injection(A<0) and τ on velocity profiles F(ξ,η,τ) for ε =0.01 ξ =0.0, α=2.0, Ec=0.5, Re=100, ΔTw=10°C and T_∞=18.7°C. It can be noticed in Fig.1 that the injection parameter controls the velocity of the fluid. In Fig.1, the impact of τ on velocity is displayed from left to right and shows stability of velocity profiles from top to bottom as discussed via harmonic analysis. From the sub figures a_1-a_2, the solid lines represent velocity distribution of the fluid at η =0 to η =6.0 in different time intervals and dotted line indicates stability of velocity of the fluid from figures a_1 ,b_1 and c_1.

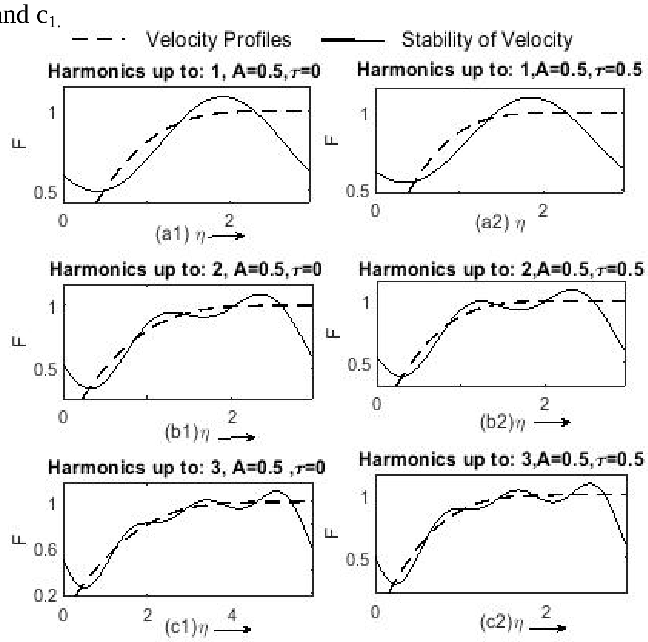

Fig 2. Distribution and Stability of Velocity profiles (*Suction A* > 0)

Fig.2 display the effects of suction *A>0* and time *τ* on F for *α =2.0, Ec=0.5, Re=100*. When the fluid reaches near the wall, the velocity of the fluid is independent of time. It can be noticed that the velocity profiles decrease due to the increase of *A*. The physically effect is that the fluids moved from towards the wall when suction occurs. Therefore, the mass transfer parameter *A* is more significant to controls the fluid flow in the diverging channel. shows that the fluid flow at different time intervals, the distribution of velocity profiles present in different time intervals from left to right and stability of velocity profiles in different nodes from up to down when suction occurs . The fluid flow can be analysed using the velocity profiles through harmonic analysis.

Fig. 3 the effects of temperature profile at different time intervals are shown from left to right, and the behavior of temperature distribution showed graphically from top to bottom using harmonics. It can be noticed that initially *τ =0* (First harmonic) the temperature distribution near the wall is stable for different time intervals.

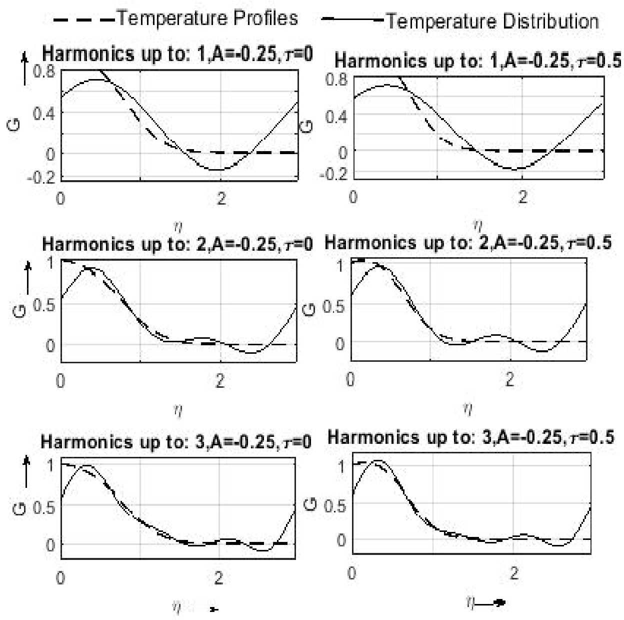

Fig 3. Distribution of Temperature profiles (*A* <0)

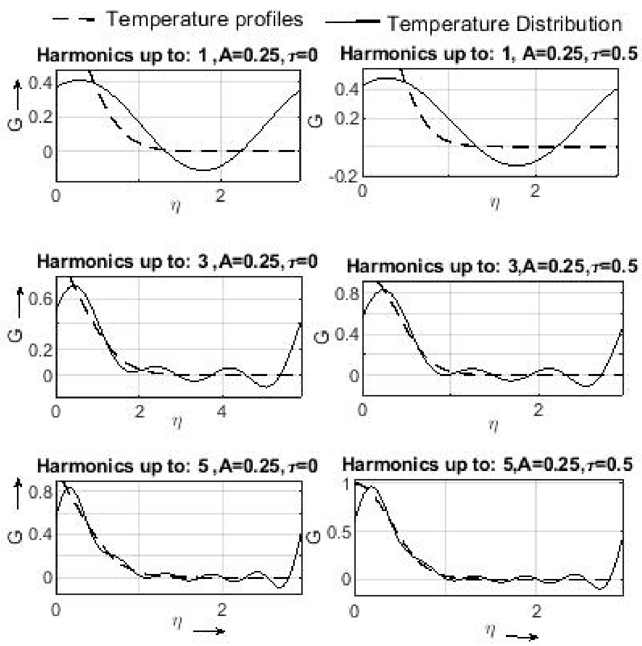

Fig 4.Distribution of Temperature profiles (*A* > 0)

Fig. 4 shows the distribution of temperature of fluid in the different time intervals when suction occurs. When fluid reaches the plate near the wall, the average velocity of the fluid is zero and the parabolic velocity profiles is not found at $\tau = 0$ The temperature profiles suddenly increased between the time intervals $\tau = 0$ .5 to $\tau = 1.0$ with the increase of suction parameter *(A>0)*, which are evident from the figures 8 and 9. In case of suction, the fluid at neighborhood of the wall reduces the temperature of fluid and thermal boundary layer thickness.

VI. CONCLUSIONS

The forced convection flow is considered over a vertical plate in presence of suction/injection, the numerical solutions are obtained using Quasi-linearization technique in association with finite difference method. The stability of velocity and temperature profiles analyzed using harmonic analysis. We conclude that the solution is stable when for all positive values of γ also the mass transfer parameter is significant effect observed on velocity, temperature profiles.

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AUTHORS PROFILE

Iyyappan G. is a Assistant Professor of Department of Mathematics at Hindustan Institute of Technology and Science, Chennai, India. Currently he doing PhD Degree in the field of Boundary Layer Theory at Vellore Institute of Technology , Chennai , India . He Obtained his M.Phil degree at Madurai Kamaraj University, Madurai and Master degree at Anna University Chennai.

A. K. Singh is a Assistant Professor of Mathematics Division at Vellore Institute of Technology Chennai. He is doing Research in Fluids, Heat Transfer process and Boundary Layer Theory. He Obtained his PhD Degree at Indian Institute of Technology Madras, Chennai and Master degree at University of Allahabad at U.P , India.