Fuzzy Translation of INK-ideal of INK-algebras

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Abstract: In this paper, the ideas of fuzzy translation (FT) to F-INK-ideals in INK-algebras are presented. The concept of (FE) fuzzy extensions and (FM) fuzzy multiplications of F-INK-ideals with a few related properties are examined. Additionally, the connections between fuzzy translations, fuzzy extensions (FE) and fuzzy multiplications (FM) of F-INK-ideals are investigated.

Keywords: INK-algebra, fuzzy INK-ideal, (FT) fuzzy translation, (FE) fuzzy extension, (FM) fuzzy multiplication.

I. INTRODUCTION

BCI-algebras and BCK-algebras are curtailed to two Boolean algebras. The previous was brought up in 1966 by Imai and Iseki. In 1991, Xi applied the idea of fuzzy sets to BCK-algebras math. In 1993, Jun and Ahmad applied it to BCI-algebras. After that Jun, Meng, Liu and a few specialists explored further properties of fuzzy-BCK-algebras and fuzzy subset(ideal). In 2017, Indhira and Kaviyarasu presented fuzzy-INK-ideal in INK-algebras and presented (IF) intuitionistic fuzzy-INK-ideal in INK-algebras. Lee et al. furthermore, Jun talked about (IT) fuzzy translation, (FE) fuzzy extensions and (FM) fuzzy multiplications of F-subalgebras and ideal in BCK/BCI-algebras. They examined relations among fuzzy translation, F-extension and F-multiplications. In this paper, we investigated all translation properties and extension condition of fuzzy-INK-ideal applied in Fuzzy-INK-algebra.

II. PRELIMINARIES

In this section, some elementary aspects that are necessary for this paper are included. An algebra (U; •, 0) is named a INK-algebra on the off chance that it fulfils the accompanying conditions for every p, q, r ∈ U.

- \((p \circ q) \circ (r \circ q) = 0\)
- \((p \circ r) \circ (q \circ r) = 0\)
- \(p \circ 0 = p\)
- \(p \circ q = 0\) and \(q \circ p = 0\) imply \(p = q\).

A subset A of a INK-algebra Û is named an INK-ideal of Û if,

- \(0 \in A\)
- \((r \circ p) \circ (r \circ q) \in A\) and \(q \in A\) imply \(p \in A\), \(\forall p, q, r \in Û\).

A FsN in a INK-algebra Û is named a fuzzy-INK-ideal of Û if,

- \(N(0) \geq N(p)\)
- \(N(p) \geq \min \{N(r \circ p) \circ (r \circ q), N(q)\}, \forall p, q, r \in Û\).

III. MAIN RESULTS

Throughout this paper, we take \(I = 1, sup\{N(p) / p \in Û\}\) for every fuzzy set \(N\) of Û.

A. Definition Let \(N \subseteq Û\) and \(\hat{a}, \phi \in [0, I]\). Is called F-\(\hat{a}\)-translation (F-\(\hat{a}\)) of \(N\) if it satisfies \(N_{\hat{a}}^{-1}(p) = N(p) + \hat{a}\), for all \(p \in Û\).

B. Theorem Prove that every F\(\hat{a}\)-translation \(N_{\hat{a}}^{-1}\) of \(N\) is a F-INK-ideal of Û, if \(N\) is a fuzzy INK-ideal of Û, for every \(\hat{a} \in [0, I]\).

Proof. We consider \(N\) is a F-INK-ideal of Û and let \(\hat{a} \in [0, I]\).

\(N_{\hat{a}}^{-1}(0) = N(p) + \hat{a} = N(p) + \hat{a}\)

\(\geq \min \{N(r \circ p) \circ (r \circ q), N(q)\} + \hat{a}\)

= \min \{N(r \circ p) \circ (r \circ q), N(q)\} + \hat{a}\)

= \min \{N_{\hat{a}}^{-1}(r \circ p) \circ (r \circ q), N_{\hat{a}}^{-1}(q)\}

for all \(p, q, r \in Û\). Hence complete the proof.

C. Theorem Let \(N\) be a F-subset of Û such that the F-\(\hat{a}\)-translation \(N_{\hat{a}}^{-1}\) of \(N\) is a F-INK-ideal of Û, then \(N\) is a F-INK-ideal of Û.

Proof. \(N_{\hat{a}}^{-1}\) is a F-of Û. Let \(p, q \in Û\).

\(N(0) + \hat{a} = N_{\hat{a}}^{-1}(0)\)

\(\geq N_{\hat{a}}^{-1}(p)\)

\(= N(p) + \hat{a}\)

\(N(0) \geq N(p)\).

Now,

\(N(p) + \hat{a} = N_{\hat{a}}^{-1}(p)\)

\(\geq \min \{N_{\hat{a}}^{-1}\circ (r \circ p) \circ (r \circ q), N_{\hat{a}}^{-1}(q)\}\)

= \min \{N(r \circ p) \circ (r \circ q), N(q)\} + \hat{a}\)

which implies

\(N(p) \geq \min \{N(r \circ p) \circ (r \circ q), N(q)\}\).

Hence \(N\) is a F-INK-ideal of Û.

D. Theorem Let \(N\) be a Fs of Û such that the F-\(\hat{a}\)-translation \(N_{\hat{a}}^{-1}\) of \(N\) is a F of Û, if \((p \circ e) * f = 0\), \(N_{\hat{a}}^{-1}(p) \geq \min \{N_{\hat{a}}^{-1}(p \circ e), N_{\hat{a}}^{-1}(e)\}\),

\(\geq \min \{N_{\hat{a}}^{-1}\circ (p \circ e) \circ (p \circ e), N_{\hat{a}}^{-1}(f)\}\),

\(N_{\hat{a}}^{-1}(e)\).

Proof. Let \(e, b, p \in Û\), then \((p \circ e) * f = 0\).

\(N_{\hat{a}}^{-1}(p) \geq \min \{N_{\hat{a}}^{-1}(p \circ e), N_{\hat{a}}^{-1}(e)\}\),

\(\geq \min \{N_{\hat{a}}^{-1}\circ (p \circ e) \circ (p \circ e), N_{\hat{a}}^{-1}(f)\}\),

\(N_{\hat{a}}^{-1}(e)\).
Therefore, if $U$ is an INK-algebra, then the F-ideal of $\text{INK}_U$ is a F-INK-subalgebra of $U$.

**Proof.** Let $(p_1, q_1) \in U$ such that $p_1, q_1 \in U$ and $p_1 \neq q_1$. Then $\text{INK}_U$ is a fuzzy INK-subalgebra of $U$.

Therefore, $\text{INK}_U$ is a fuzzy INK-subalgebra of $U$.

**G. Theorem.** If $R$ is a fuzzy INK-subalgebra of $U$, then $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$.

**Proof.** Let $(p_1, q_1) \in U$ such that $p_1, q_1 \in U$ and $p_1 \neq q_1$. Then $\text{INK}_R(p_1) \leq \text{INK}_R(q_1)$.

Therefore, $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$.

**H. Theorem.** Let $R$ be a fuzzy INK-algebra such that $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$. Then $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$.

**Proof.** Let $(p_1, q_1) \in U$ such that $p_1, q_1 \in U$ and $p_1 \neq q_1$. Then $\text{INK}_R(p_1) \leq \text{INK}_R(q_1)$.

Therefore, $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$.

**I. Theorem.** If $R$ is a fuzzy INK-algebra such that $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$, then $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$.

**Proof.** Obviously, $\text{INK}_R(p_1, q_1) \leq \text{INK}_R(p_2, q_2)$. Then $\text{INK}_R(p_1, q_1) = \text{INK}_R(p_2, q_2)$.

Therefore, $\text{INK}_R$ is a fuzzy INK-subalgebra of $U$.

**IV. FUZZY $\eta$-MULTIPLICATION OF FUZZY INK-IDEAL.**

**A. Definition.** Let $K$ be a FS of $U$ and $\eta \in [0,1]$, $\text{INK}_K : U \rightarrow [0, 1]$ is said to be a F-\(\eta\)-multiplication of $K$ if it satisfies $\text{INK}_K(p) \geq \eta \text{INK}_K(p)$ for all $p \in U$.

**B. Theorem.** Let $K$ be a FS of $U$ such that $\text{INK}_K$ is a F-INK-ideal of $U$. Then $\text{INK}_K$ is a F-INK-ideal of $U$.

**Proof.** Let $\text{INK}_K$ be a F-INK-ideal of $U$ for some $\eta \in [0,1]$. Then $\text{INK}_K(p) \geq \eta \text{INK}_K(p)$ for all $p \in U$.

**C. Theorem.** If $K$ is a fuzzy INK-ideal of $U$, then $\text{INK}_K$ is a fuzzy INK-ideal of $U$.

**Proof.** Let $\text{INK}_K$ be a fuzzy INK-ideal of $U$. Then $\text{INK}_K(p) \geq \eta \text{INK}_K(p)$ for all $p \in U$.

**D. Theorem.** Let $\eta \in [0,1]$ and let $K$ be a fuzzy INK-ideal of $U$. Then $\text{INK}_K$ is a fuzzy INK-ideal of $U$.

**Proof.** Let $\text{INK}_K$ be a fuzzy INK-ideal of $U$. Then $\text{INK}_K(p) \geq \eta \text{INK}_K(p)$ for all $p \in U$.

**E. Theorem.** The F-\(\eta\)-multiplication $\text{INK}_K$ of $K$ is a fuzzy INK-ideal of $U$.

**Proof.** Let $\text{INK}_K$ be a fuzzy INK-ideal of $U$. Then $\text{INK}_K(p) \geq \eta \text{INK}_K(p)$ for all $p \in U$.
F. **Theorem.** The U and ρF two FT of a F-INK-ideal Κ of $\bar{U}$ is also a F-INK-ideal of $\bar{U}$.

**Proof.** Let $\Lambda_k$, $\Lambda_{k_1}$ be FT of a F-INK-ideal Κ of $\bar{U}$, where $\rho, \vec{\rho} \in [0,1]$. Then $\Lambda_k$, $\Lambda_{k_1}$ are F-INK-ideal of $\bar{U}$.

$$(\Lambda_k \cap \Lambda_{k_1})(p) = \min \{ \Lambda_k(p), \Lambda_{k_1}(p) \} = \min \{ \Lambda_k(p) \},  \Lambda_{k_1}(p) \}$$

And

$$(\Lambda_k \cup \Lambda_{k_1})(p) = \max \{ \Lambda_k(p), \Lambda_{k_1}(p) \} = \max \{ \Lambda_k(p) \}, \Lambda_{k_1}(p) \}$$

Hence $(\Lambda_k \cap \Lambda_{k_1})$ and $(\Lambda_k \cup \Lambda_{k_1})$ are fuzzy INK-ideal of $\bar{U}$.

**V. FUZZY EXTENSIONS OF INK-IDEAL**

**A. Definition.** Let $\Lambda_1$ and $\Lambda_2$ be fuzzy subsets of $\bar{N}$. If $\Lambda_1 \subseteq \Lambda_2$ for all p in $\bar{N}$, then $\Lambda_2$ is a $\mathcal{F}_\lambda$-F-extension of $\Lambda_1$.

**B. Definition.** Let $\Lambda_1$ and $\Lambda_2$ be F-subsets of $\bar{N}$. Then $\Lambda_2$ is named a F-INK-ideal S-extension of $\Lambda_1$ if the succeeding statements are valid:

1. $\Lambda_2$ is a $\mathcal{F}_\lambda$ of $\Lambda_1$.
2. If $\Lambda_1$ is a F-INK-ideal $\mathcal{N}$-extension of $\Lambda_1$, then $\Lambda_2$ is a F-INK-ideal $\mathcal{N}$-extension of $\Lambda_1$.

Consider a INK-algebra $\bar{N} = \{0, 1, a, b\}$.

<table>
<thead>
<tr>
<th>$\Lambda_1$</th>
<th>$\Lambda_2$</th>
<th>$\Lambda_3$</th>
<th>$\Lambda_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>a</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>b</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>a</td>
<td>1</td>
</tr>
</tbody>
</table>

$\Lambda_1(0) = 0.6$, $\Lambda_2(1) = 0.4$, $\Lambda_3(a) = 0.3$, $\Lambda_4(b) = 0.3$.

$\Lambda_5(0) = 0.7$, $\Lambda_6(1) = 0.6$, $\Lambda_7(a) = 0.5$, $\Lambda_8(b) = 0.2$. are fuzzy INK-ideal of $\bar{N}$.

**C. Theorem.** Intersection of any two F-INK-ideal extension of a F-INK-ideal $\mathcal{N}$ of $\bar{N}$ is a F-INK-ideal extension of $\bar{N}$.

**Proof.** Let $\Lambda_1$ and $\Lambda_2$ be a F-INK-ideal extension of a F-INK-ideal $\mathcal{N}$ of $\bar{N}$. Then $\Lambda_1(p) \geq \Gamma(p)$ and $\Lambda_2(p) \geq \Gamma(p)$. Since $\Lambda_1$ is a fuzzy INK-ideal of $U$, $\Lambda_1$ and $\Lambda_2$ are F-INK-ideal of $\bar{N}$. Then $\Lambda_1 \cap \Lambda_2$ is also a F-INK-ideal of $\bar{N}$. Now

$\mathcal{N}_1 \cap \mathcal{N}_2(p) = \min \{ \mathcal{N}_1(p), \mathcal{N}_2(p) \}$

Therefore $\mathcal{N}_1 \cap \mathcal{N}_2$ is a F-INK-ideal extension.

**D. Remark.** Union of F-INK-ideal extension of a F-INK-ideal $\mathcal{N}$ of $\bar{N}$ need not be a F-INK-ideal extension of $\bar{N}$.

Consider the example the fuzzy sets $\mathcal{N}$, $\mathcal{N}_1$, and $\mathcal{N}_2$ of $\bar{N}$ is defined as follows.

<table>
<thead>
<tr>
<th>$\mathcal{N}$</th>
<th>$\mathcal{N}_1$</th>
<th>$\mathcal{N}_2$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>1</td>
<td>1</td>
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<table>
<thead>
<tr>
<th>$\mathcal{N}_1 \cap \mathcal{N}_2(p)$</th>
<th>$\mathcal{N}_1 \cup \mathcal{N}_2(p)$</th>
</tr>
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<tbody>
<tr>
<td>$\min { \mathcal{N}_1(p), \mathcal{N}_2(p) }$</td>
<td>$\max { \mathcal{N}_1(p), \mathcal{N}_2(p) }$</td>
</tr>
</tbody>
</table>

**E. Theorem.** Let $\mathcal{N}$ be a fuzzy INK-ideal of $\bar{N}$. The F-ideal $\mathcal{N}$ is a F-INK-ideal extension of $\mathcal{N}$.

**Proof.** If $\mathcal{N}$ is a F-INK-ideal of $\bar{N}$, then we know that by theorem 1.5, the fuzzy $\mathcal{F}_\lambda$-translation $\mathcal{N}_1 \rangle$ of $\mathcal{N}$ is F-INK-ideal of $\bar{N}$. Now, $\mathcal{N}_1 \rangle(p) = \mathcal{N}(p) \geq \mathcal{N}(p)$, for all p in $\bar{N}$. Hence $\mathcal{N}$ is a fuzzy $\mathcal{F}_\lambda$-translation $\mathcal{N}_1 \rangle$ is a F-INK-ideal extension of $\mathcal{N}$.

**F. Theorem.** Let $\mathcal{N}$ be a F-INK-ideal of $\bar{N}$ and $\tilde{\rho} \geq \eta$, with $\rho, \eta \in [0,1]$, then F-\tilde{\mathcal{F}} translation $\mathcal{N}_1 \rangle \eta$ of $\mathcal{N}$ is a F-INK-ideal extension of the F-\tilde{\mathcal{F}}-translation $\mathcal{N}_1 \rangle \eta$ of $\mathcal{N}$.

**Proof.** Let $\mathcal{N}$ be a fuzzy INK-ideal of $\bar{N}$. Then by theorem 3.1.5 the fuzzy $\mathcal{F}_\eta$-translation $\mathcal{N}_1 \rangle \eta$ of $\mathcal{N}$ and the fuzzy $\mathcal{F}_\eta$-translation $\mathcal{N}_1 \rangle \eta$ of $\mathcal{N}$ are fuzzy INK-ideal of $\bar{N}$, for all $\tilde{\rho}, \eta \in [0,1]$. Since $\tilde{\rho} \geq \eta$, $\mathcal{F}_\eta(p) \geq \mathcal{F}_\eta(p)$.

Therefore, $\mathcal{N}_1 \rangle \eta(p) = \mathcal{N}_1 \rangle \eta(p)$.

Hence $\mathcal{N}_1 \rangle \eta$ is a fuzzy INK-ideal extension of $\mathcal{N}_1 \rangle \eta$.

**G. Theorem.** If $\mathcal{N}$ is a fuzzy INK-ideal of $\bar{N}$, then the fuzzy $\eta$-multiplication of $\mathcal{N}$ is a fuzzy INK-ideal of $\bar{N}$, for all $\eta \in [0,1]$.

**H. Theorem.** Let $\mathcal{N}$ be a FS of $\tilde{\rho} \rho \eta$ in $[0,1]$ and $\eta \eta$ in $[0,1]$. If the F-$\eta$-multiplication $\mathcal{N}_1 \rangle \eta$ of $\mathcal{N}$ is a F-INK-ideal of $\bar{N}$, then the F-$\tilde{\mathcal{F}}$-translation $\mathcal{N}_1 \rangle \eta$ of $\mathcal{N}$ is a F-INK-ideal extension of $\mathcal{N}_1 \rangle \eta$.

**Proof.** Let $\tilde{\rho} \in [0,1]$, $\eta \in [0,1]$ and $\mathcal{N}_1 \rangle \eta$ of $\mathcal{N}$ is a fuzzy INK-ideal of $\bar{N}$. Then $\mathcal{N}$ is a F-INK-ideal of $\bar{N}$. By theorem 3.1.2, $\mathcal{N}_1 \rangle \eta(p) = \mathcal{N}_1 \rangle \eta(p)$ of $\mathcal{N}$. Now, $\mathcal{N}_1 \rangle \eta(p) \geq \mathcal{N}(p) \eta = \mathcal{N}_1 \rangle \eta(p)$.

Therefore $\mathcal{N}_1 \rangle \eta(p)$ is a fuzzy INK-ideal extension of $\mathcal{N}_1 \rangle \eta$.

**VI. CONCLUSION**

In this concept of translation of F-INK-ideal in INK-algebra are familiarized and examined some of their beneficial assets. We have exposed that the F-\tilde{\rho}-translation of a F-INK-ideal is a F-INK-ideal extension but then the converse is not factual. It is correspondingly exposed that intersection of F-INK-ideal extension of a FS- is a F-INK-ideal extension but union of F-INK-ideal extension of a fuzzy subset is not a F-INK-ideal extension. The associations are conversed between FT, FE and F-multiplications of F-INK-I in INK-algebras.
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REFERENCES


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