

# Fuzzy Translation of INK-ideal of INK-algebras



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**Abstract:** In this paper, the ideas of fuzzy translation (FT) to F-INK-ideals in INK-algebras are presented. The concept of (FE) fuzzy extensions and (FM) fuzzy multiplications of F-INK-ideals with a few related properties are examined. Additionally, the connections between fuzzy translations, fuzzy extensions (FE) and fuzzy multiplications (FM) of F-INK-ideals are investigated.

**Keywords:** INK-algebra, fuzzy INK-ideal, (FT) fuzzy translation, (FE) fuzzy extension, (FM) fuzzy multiplication.

## I. INTRODUCTION

BCK-algebras and BCI algebras are curtailed to two Boolean algebras. The previous was brought up in 1966 by Imai and Iseki, and the last was natives around the same time due to Iseki. In 1991, Xi applied the idea of fuzzy sets to BCK-algebras math. In 1993, Jun and Ahmad applied it to BCI-algebras. After that Jun, Meng, Liu and a few specialists explored further properties of fuzzy-BCK-algebras and fuzzy subset(ideal). In 2017, Indhira and Kaviyarasu presented fuzzy-INK-ideal in INK-algebras and presented (IF) intuitionistic fuzzy-INK-ideal in INK-algebras. Lee et al. furthermore, Jun talked about (IT) fuzzy translation, (FE) fuzzy extensions and (FM) fuzzy multiplications of F-subalgebras and ideal in BCK/BCI-algebras. They examined relations among fuzzy translation, F-extension and F-multiplications. In this paper, we investigated all translation properties and extension condition of fuzzy-INK-ideal applied in Fuzzy-INK-algebra.

## II. PRELIMINARIES

In this section, some elementary aspects that are necessary for this paper are included.

An algebra  $(\dot{U}; \circ, 0)$  is named a INK-algebra on the off chance that it fulfils the accompanying conditions for every  $p, q, r \in \dot{U}$ .

- $((p \circ q) \circ (p \circ r)) \circ (r \circ q) = 0$
- $((p \circ r) \circ (q \circ r)) \circ (p \circ q) = 0$
- $p \circ 0 = p$
- $p \circ q = 0$  and  $q \circ p = 0$  imply  $p = q$ .

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A  $F_S \aleph$  in a INK-algebra  $\dot{U}$  is named a F-INK-subalgebra of  $\dot{U}, \aleph(p \circ q) \geq \min\{\aleph(p), \aleph(q)\}, \forall p, q \in \dot{U}$ .

Let  $(\dot{U}, \circ, 0)$  be a INK-algebra. A subset  $Y$  of  $\dot{U}$  is named a INK-subalgebra of  $\dot{U}$  if  $p \circ q \in Y$ .

A subset  $A$  of a INK- algebra  $\dot{U}$  is named an INK-ideal of  $\dot{U}$  if,

- $0 \in A,$
- $(r \circ p) \circ (r \circ q) \in A$  and  $q \in A$  imply  $p \in A. \forall p, q, r \in \dot{U}.$

A  $F_S \aleph$  in a INK-algebra  $\dot{U}$  is named a fuzzy-INK- ideal of  $\dot{U}$ , if,

- $\aleph(0) \geq \aleph(p)$
- $\aleph(p) \geq \min\{\aleph(r \circ p) \circ (r \circ q), \aleph(q)\}, \forall p, q, r \in \dot{U}.$

## III. MAIN RESULTS

Throughout this paper, we take  $\top = 1 - \sup\{\aleph(p) / p \in \dot{U}\}$  for every fuzzy set  $\aleph$  of  $\dot{U}$ .

**A. Definition** Let  $\aleph \subseteq \dot{U}$  and  $\tilde{\alpha}, \in [0, \top]. \aleph_{\tilde{\alpha}, \top}: \dot{U} \rightarrow [0, 1]$  is called a F- $\tilde{\alpha}$ -translation (F- $\tilde{\alpha}$ ) of  $\aleph$  if it satisfies  $\aleph_{\tilde{\alpha}, \top}(p) = \aleph(p) + \tilde{\alpha}$ , for all  $p \in \dot{U}$ .

**B. Theorem** Prove that every F $\tilde{\alpha}$ -translation  $\aleph_{\tilde{\alpha}, \top}$  of  $\aleph$  is a F-INK-ideal of  $\dot{U}$ , if  $\aleph$  is a fuzzy INK-ideal of  $\dot{U}$ , for very  $\tilde{\alpha} \in [0, \top]$ .

**Proof.** we consider  $\aleph$  is a F-INK-ideal of  $\dot{U}$  and let  $\tilde{\alpha} \in [0, \top]$ .

$$\begin{aligned} \aleph_{\tilde{\alpha}, \top}(0) &= \aleph(0) + \tilde{\alpha} = \aleph_{\tilde{\alpha}, \top}(p) \\ \aleph_{\tilde{\alpha}, \top}(p) &= \aleph(p) + \tilde{\alpha} \\ &\geq \min\{\aleph(r^*p) * (r^*q), \aleph(q)\} + \tilde{\alpha} \\ &= \min\{\aleph(r^*p) * (r^*q) + \tilde{\alpha}, \aleph(q) + \tilde{\alpha}\} \\ &= \min\{\aleph_{\tilde{\alpha}, \top}((r^*p) * (r^*q)), \aleph_{\tilde{\alpha}, \top}(q)\} \end{aligned}$$

for all  $p, q, r \in \dot{U}$ . Hence complete the proof.

**C. Theorem.** Let  $\aleph$  be a F-subset of  $\dot{U}$  such that the F- $\tilde{\alpha}$ -translation  $\aleph_{\tilde{\alpha}, \top}(p)$  of  $\aleph$  is a F-INK- ideal of  $\dot{U}$  then  $\aleph$  is a F-INK-ideal of  $\dot{U}$ .

**Proof.**  $\aleph_{\tilde{\alpha}, \top}$  is a IF of  $\dot{U}$ . Let  $p, q \in \dot{U}$ .

$$\begin{aligned} \aleph(0) + \tilde{\alpha}, &= \aleph_{\tilde{\alpha}, \top}(0) \\ &\geq \aleph_{\tilde{\alpha}, \top}(p) \\ &= \aleph(p) + \tilde{\alpha}, \\ \aleph(0) &\geq \aleph(p). \end{aligned}$$

Now,

$$\begin{aligned} \aleph(p) + \tilde{\alpha} &= \aleph_{\tilde{\alpha}, \top}(p) \\ &\geq \min\{\aleph_{\tilde{\alpha}, \top}((r^*p) * (r^*q)), \aleph_{\tilde{\alpha}, \top}(q)\} \\ &= \min\{\aleph((r^*p) * (r^*q)) + \tilde{\alpha}, \aleph(q) + \tilde{\alpha}\} \\ &= \min\{\aleph((r^*p) * (r^*q)), \aleph(q)\} + \tilde{\alpha} \end{aligned}$$

which implies

$$\aleph(p) \geq \min\{\aleph((r^*p) * (r^*q)), \aleph(q)\}.$$

Hence  $\aleph$  is a F-INK-ideal of  $\dot{U}$ .

**D. Theorem** Let  $\aleph$  be a Fs of  $\dot{U}$  such that F- $\tilde{\alpha}$ - translation  $\aleph_{\tilde{\alpha}, \top}$  of  $\aleph$  is a FI of  $\dot{U}$ , if  $(p^*e) * f = 0$  then,  $\aleph_{\tilde{\alpha}, \top}(p) \geq \min\{\aleph_{\tilde{\alpha}, \top}(e), \aleph_{\tilde{\alpha}, \top}(b)\}$ .

**Proof.** Let  $e, b, p \in \dot{U}$ , then  $(p * e) * f = 0$ .

$$\begin{aligned} \mathfrak{N}_{\tilde{\alpha}}^{\top}(p) &\geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(p * e), \mathfrak{N}_{\tilde{\alpha}}^{\top}(e) \} \\ &\geq \min \{ \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(p * e) * b \}, \mathfrak{N}_{\tilde{\alpha}}^{\top}(f) \}, \mathfrak{N}_{\tilde{\alpha}}^{\top}(e) \} \\ &= \min \{ \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(0), \mathfrak{N}_{\tilde{\alpha}}^{\top}(f) \}, \mathfrak{N}_{\tilde{\alpha}}^{\top}(e) \} \\ &= \min \{ \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(f) \}, \mathfrak{N}_{\tilde{\alpha}}^{\top}(e) \} \\ \mathfrak{N}_{\tilde{\alpha}}^{\top}(p) &= \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(e) \}, \mathfrak{N}_{\tilde{\alpha}}^{\top}(f) \} \end{aligned}$$

**E. Theorem.** Let  $\tilde{\alpha}$  in  $[0, \top]$  and let  $\mathfrak{N}$  be a F-INK-ideal of  $\dot{U}$ . if  $\dot{U}$  is a INK-algebra, then the F- $\tilde{\alpha}$ -translation  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  of  $\mathfrak{N}$  is a F-INK-subalgebra of  $\dot{U}$ .

**Proof.**  $\mathfrak{N}_{\tilde{\alpha}}^{\top}(p * q) = \mathfrak{N}(p * q) + \tilde{\alpha}$

$$\begin{aligned} &\geq \min \{ \mathfrak{N}(r * (p * q)) * (r * q), \mathfrak{N}(q) \} + \tilde{\alpha} \\ &\geq \min \{ \mathfrak{N}((p * q) * q), \mathfrak{N}(q) \} + \tilde{\alpha} \\ &\geq \min \{ \mathfrak{N}(0), \mathfrak{N}(q) \} + \tilde{\alpha} \\ &\geq \min \{ \mathfrak{N}(p), \mathfrak{N}(q) \} + \tilde{\alpha} \\ &= \min \{ \mathfrak{N}(p) + \tilde{\alpha}, \mathfrak{N}(q) + \tilde{\alpha} \} \\ &= \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(p), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}. \end{aligned}$$

Hence  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  is a fuzzy INK-subalgebra of  $\dot{U}$ .

**F. Theorem** If the F- $\tilde{\alpha}$ -translation  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ , then  $\mathfrak{N}$  is a F-INK-subalgebra of  $\dot{U}$ .

**Proof.** Let  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ .

$$\begin{aligned} \mathfrak{N}(p * q) + \tilde{\alpha} &= \mathfrak{N}_{\tilde{\alpha}}^{\top}(p * q) \\ &\geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(r * (p * q)) * (r * q) \}, \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \} \\ &\geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(0), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \} \\ &= \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(p), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \} \\ &= \min \{ \mathfrak{N}_{\tilde{\alpha}}(p) + \tilde{\alpha}, \mathfrak{N}_{\tilde{\alpha}}(q) + \tilde{\alpha} \} \end{aligned}$$

$$\mathfrak{N}(p * q) + \tilde{\alpha} = \min \{ \mathfrak{N}_{\tilde{\alpha}}(p), \mathfrak{N}_{\tilde{\alpha}}(q) \} + \tilde{\alpha}$$

$$\mathfrak{N}(p * q) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}(p), \mathfrak{N}_{\tilde{\alpha}}(q) \}.$$

Therefore,  $\mathfrak{N}$  is a fuzzy INK-subalgebra of  $\dot{U}$ .

**G. Theorem.** If the F- $\tilde{\alpha}$ -translation  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ . Then if necessity be a FI of  $\dot{U}$ .

**Proof.** Let f- $\tilde{\alpha}$ -translation  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  of  $\dot{U}$ . Then we have

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}((r * p) * (r * q)), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

put  $r = 0$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}((0 * p) * (0 * q)), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(p * q), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

Therefore,  $\mathfrak{N}_{\tilde{\alpha}}^{\top}(p)$  is fuzzy INK-ideal of  $\dot{U}$ .

**H. Theorem.** Let the F- $\tilde{\alpha}$ -translation  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ ,  $\tilde{\alpha} \in [0, \top]$ . If  $p \leq q$ . Then  $\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \mathfrak{N}_{\tilde{\alpha}}^{\top}(q)$ .

**Proof.** Let  $p, q \in \dot{U}$  such that  $p \leq q$ . then  $p * q = 0$  and hence,

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}((r * p) * (r * q)), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

put  $r = 0$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}((0 * p) * (0 * q)), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(p * q), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}(0), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \mathfrak{N}_{\tilde{\alpha}}^{\top}(q).$$

**I. Theorem.** If  $\mathfrak{N}$  be a FS of  $\mathfrak{N}$  such that the F- $\tilde{\alpha}$ -translation  $\mathfrak{N}_{\tilde{\alpha}}^{\top}$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ , the set  $I_{\mathfrak{N}} = \{ p \in \dot{U} \mid \mathfrak{N}_{\tilde{\alpha}}^{\top}(p) = \mathfrak{N}_{\tilde{\alpha}}^{\top}(0) \}$  is fuzzy INK-ideal of  $\dot{U}$ .

**Proof.** Obviously,  $0 \in I_{\mathfrak{N}}$ . Let  $p, q, r \in \dot{U}$  be such that

$(r * p) * (r * q) \in I_{\mathfrak{N}}$  and  $q \in I_{\mathfrak{N}}$ . Then,

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}((r * p) * (r * q)), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) = \mathfrak{N}_{\tilde{\alpha}}^{\top}(0) = \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \text{ and}$$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \min \{ \mathfrak{N}_{\tilde{\alpha}}^{\top}((r * p) * (r * q)), \mathfrak{N}_{\tilde{\alpha}}^{\top}(q) \}$$

$$\mathfrak{N}_{\tilde{\alpha}}^{\top}(p) \geq \mathfrak{N}_{\tilde{\alpha}}^{\top}(0), \text{ this implies that}$$

$$\mathfrak{N}(p) + \tilde{\alpha} = \mathfrak{N}(0) + \tilde{\alpha} \text{ or}$$

$$\mathfrak{N}(p) = \mathfrak{N}(0) \text{ so that } q \in I_{\mathfrak{N}}.$$

Therefore,  $I_{\mathfrak{N}}$  is a INK-ideal of  $\dot{U}$ .

#### IV. FUZZY $\tilde{\alpha}$ -MULTIPLICATION OF FUZZY INK-IDEAL

**A. Definition** Let  $\mathfrak{N}$  be a FS of  $\dot{U}$  and  $\eta \in [0, 1]$ .  $\mathfrak{N}_{\eta}^M : \dot{U} \rightarrow [0, 1]$  is said to be a F- $\eta$ -multiplication of  $\mathfrak{N}$  if it mollifies  $\mathfrak{N}_{\eta}^M(p) = \eta \cdot \mathfrak{N}(p), \forall p$  in  $\dot{U}$ .

**B. Theorem** let  $\mathfrak{N}$  be a FS of  $\dot{U}$  such that F- $\eta$ -multiplication  $\mathfrak{N}_{\eta}^M$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}, \forall \eta$  in  $[0, 1]$ , then  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ .

**Proof.** Let  $\mathfrak{N}_{\eta}^M$  is a F-INK-ideal of  $\dot{U}$  for some  $\eta$  in  $(0, 1]$ .

$$\text{Then } \eta \mathfrak{N}(0) = \mathfrak{N}_{\eta}^M(0)$$

$$\geq \mathfrak{N}_{\eta}^M(p)$$

$$= \eta \mathfrak{N}(0)$$

$$\text{And so } \mathfrak{N}(0) \geq \mathfrak{N}(p)$$

$$\eta \mathfrak{N}(p) = \mathfrak{N}_{\eta}^M(p)$$

$$\geq \min \{ \mathfrak{N}_{\eta}^M(r * p) * (r * q), \mathfrak{N}_{\eta}^M(q) \}$$

$$\geq \min \{ \eta \mathfrak{N}(r * p) * (r * q), \eta \mathfrak{N}(q) \}$$

$$\eta \mathfrak{N}(p) = \eta \min \{ \mathfrak{N}(r * p) * (r * q), \mathfrak{N}(q) \}$$

And so

$$\mathfrak{N}(p) \geq \min \{ \mathfrak{N}(r * p) * (r * q), \mathfrak{N}(q) \}.$$

Hence  $\mathfrak{N}$  is a fuzzy INK-ideal of  $\dot{U}$ .

**C. Theorem.** If  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ , then the F- $\eta$ -multiplication  $\mathfrak{N}_{\eta}^M$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ .

**Proof.** Let  $\mathfrak{N}$  be a F-INK-ideal of  $\dot{U}$ .

$$\mathfrak{N}_{\eta}^M(0) = \eta \mathfrak{N}(0)$$

$$\geq \eta \mathfrak{N}(p)$$

$$= \mathfrak{N}_{\eta}^M(p)$$

And

$$\mathfrak{N}_{\eta}^M(p) = \eta \mathfrak{N}(p)$$

$$\geq \eta \min \{ \mathfrak{N}((r * p) * (r * q)), \mathfrak{N}(q) \}$$

$$= \min \{ \eta \mathfrak{N}((r * p) * (r * q)), \eta \mathfrak{N}(q) \}$$

$$= \min \{ \mathfrak{N}_{\eta}^M((r * p) * (r * q)), \mathfrak{N}_{\eta}^M(q) \}$$

Hence  $\mathfrak{N}_{\eta}^M$  of  $\mathfrak{N}$  is a fuzzy INK-ideal of  $\dot{U}$ .

**D. Theorem.** Let  $\eta$  in  $[0, 1]$  and let  $\mathfrak{N}$  be a fuzzy INK-ideal of a INK-algebra  $\dot{U}$ . Then the F- $\eta$ -multiplication  $\mathfrak{N}_{\eta}^M$  of  $\mathfrak{N}$  is a F-INK-subalgebra of  $\dot{U}$ .

**Proof.**  $p, q \in \dot{U}$ . Now we have,

$$\mathfrak{N}_{\eta}^M(p * q) = \eta \mathfrak{N}(p * q)$$

$$\geq \eta \min \{ \mathfrak{N}((r * p) * (r * q)), \mathfrak{N}(q) \}$$

$$\geq \eta \min \{ \mathfrak{N}((p * q) * q), \mathfrak{N}(q) \}$$

$$\geq \eta \min \{ \mathfrak{N}(0), \mathfrak{N}(q) \}$$

$$= \eta \min \{ \mathfrak{N}(p), \mathfrak{N}(q) \}$$

$$= \min \{ \eta \mathfrak{N}(p), \eta \mathfrak{N}(q) \}$$

$$\mathfrak{N}_{\eta}^M(p * q) \geq \min \{ \mathfrak{N}_{\eta}^M \mathfrak{N}(p), \mathfrak{N}_{\eta}^M \mathfrak{N}(q) \}.$$

**E. Theorem** If the F- $\eta$ -multiplication  $\mathfrak{N}_{\eta}^M$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}, \eta \in [0, 1]$ , then  $\mathfrak{N}$  is a F-INK-subalgebra of  $\dot{U}$ .

**Proof.** let us take  $\mathfrak{N}_{\eta}^M$  of  $\mathfrak{N}$  is a F-INK-ideal of  $\dot{U}$ , Then

$$\eta \mathfrak{N}(p * q) = \mathfrak{N}_{\eta}^M(p * q)$$

$$\geq \min \{ \mathfrak{N}_{\eta}^M((r * p) * (r * q)), \mathfrak{N}_{\eta}^M(q) \}$$

$$\geq \min \{ \mathfrak{N}_{\eta}^M((p * q) * q), \mathfrak{N}_{\eta}^M(q) \}$$

$$\geq \min \{ \mathfrak{N}_{\eta}^M(0), \mathfrak{N}_{\eta}^M(q) \}$$

$$= \min \{ \mathfrak{N}_{\eta}^M(p), \mathfrak{N}_{\eta}^M(q) \}$$

$$= \min \{ \eta \mathfrak{N}(p), \eta \mathfrak{N}(q) \}$$

$$\eta \mathfrak{N}(p * q) = \min \eta \{ \mathfrak{N}(p), \mathfrak{N}(q) \}$$

$$\mathfrak{N}(p * q) \geq \min \{ \mathfrak{N}(p), \mathfrak{N}(q) \}.$$

Hence  $\aleph$  is a fuzzy INK-subalgebra of  $\hat{U}$ .

**F. Theorem.** The  $\cup$  and  $\cap$  of two FT of a F-INK-ideal  $\aleph$  of  $\hat{U}$  is also a F-INK-ideal of  $\hat{U}$ .

**Proof.** let  $\aleph_{\tilde{q}}$  and  $\aleph_v$  be FT of a F-INK-ideal  $\aleph$  of  $\hat{U}$ . Where  $\tilde{q}, v \in [0, 1]$ ,  $\tilde{q} \leq v$ . Then  $\aleph_{\tilde{q}}$  and  $\aleph_v$  are F-INK-ideal of  $\hat{U}$ .  
 $(\aleph_{\tilde{q}} \cap \aleph_v)(p) = \min \{ \aleph_{\tilde{q}}(p), \aleph_v(p) \}$   
 $= \min \{ \aleph(p) + \tilde{q}, \aleph(p) + v \}$   
 $= \aleph(p) + \tilde{q}$   
 $= \aleph_{\tilde{q}}(p)$ .

And

$$(\aleph_{\tilde{q}} \cup \aleph_v)(p) = \max \{ \aleph_{\tilde{q}}(p), \aleph_v(p) \}$$

$$= \max \{ \aleph(p) + \tilde{q}, \aleph(p) + v \}$$

$$= \aleph(p) + v$$

$$= \aleph_v(p)$$

Hence  $(\aleph_{\tilde{q}} \cap \aleph_v)$  and  $(\aleph_{\tilde{q}} \cup \aleph_v)$  are fuzzy INK-ideal of  $\hat{U}$ .

### V. FUZZY EXTENSIONS OF INK-IDEAL

**A. Definition** Let  $\aleph_1$  and  $\aleph_2$  be fuzzy subsets of  $\tilde{N}$ . If  $\aleph_1 \leq \aleph_2$  for all  $p$  in  $\tilde{N}$ , then  $\aleph_2$  is a  $(F_E)$ -extension of  $\aleph_1$ .

**B. Definition** Let  $\aleph_1$  and  $\aleph_2$  be F-subsets of  $\tilde{N}$ . Then  $\aleph_2$  is named a F-INK-ideal  $\hat{S}$ -extension of  $\aleph_1$  if the succeeding statements are valid:

- $\aleph_2$  is a  $F_E$  of  $\aleph_1$ .
  - If  $\aleph_1$  is a F-INK-ideal of  $\tilde{N}$ , then  $\aleph_2$  is a F-INK-ideal of  $\tilde{N}$ .
- Consider a INK-algebra  $\tilde{N} = \{0, 1, a, b\}$ .

$\circ$	0	1	a	b
0	0	1	a	b
1	1	0	b	a
a	a	b	0	1
b	b	a	1	0

$\aleph_1(0) = 0.6, \aleph_1(1) = 0.4, \aleph_1(a) = 0.3, \aleph_1(b) = 0.3$ .  
 $\aleph_2(0) = 0.7, \aleph_2(1) = 0.6, \aleph_2(a) = 0.5, \aleph_2(b) = 0.2$ . are fuzzy INK-ideal of  $\tilde{N}$ .

**C. Theorem.** Intersection of any two F-INK-ideal extension of a F-INK-ideal  $\aleph$  of  $\tilde{N}$  is a F-INK-ideal extension of  $\tilde{N}$ .

**Proof.** Let  $\aleph_1$  and  $\aleph_2$  be a F-INK-ideal extension of a F-INK-ideal  $\aleph$  of  $\tilde{N}$ . Then  $\aleph_1(p) \geq \aleph(p)$  and  $\aleph_2(p) \geq \aleph(p)$ . Since  $\aleph$  is a fuzzy INK-ideal of  $\hat{U}$ .  $\aleph_1$  and  $\aleph_2$  are F-INK-ideal of  $\tilde{N}$ . Then  $\aleph_1 \cap \aleph_2$  is also a F-INK-ideal of  $\tilde{N}$ . Now

$$\aleph_1 \cap \aleph_2(p) = \min \{ \aleph_1(p), \aleph_2(p) \}$$

$$\geq \min \{ \aleph(p), \aleph(p) \} = \aleph(p)$$

Hence  $\aleph_1 \cap \aleph_2$  is a F-INK-ideal extension.

**D. Remark.** Union of F-INK-ideal extension of a F-INK-ideal  $\aleph$  of  $\tilde{N}$  need not be a F-INK-ideal extension of  $\aleph$ . Consider the example the fuzzy sets  $\aleph, \aleph_1$  and  $\aleph_2$  of  $\tilde{N}$  is defined as follows

$\circ$	0	1	2	3
0	0	1	2	3

1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$\aleph_1(0) = 0.6, \aleph_1(1) = 0.4, \aleph_1(2) = 0.3, \aleph_1(3) = 0.3$ .  
 $\aleph_2(0) = 0.7, \aleph_2(1) = 0.6, \aleph_2(2) = 0.5, \aleph_2(3) = 0.2$ .  
 we take  $p = 3, q = 1, r = 0$ .

$$\aleph_1 \cup \aleph_2(3) \geq \min \{ \aleph_1 \cup \aleph_2((0*3) * (0*1)), \aleph_1 \cup \aleph_2(1) \}$$

$$\aleph_1 \cup \aleph_2(3) \geq \min \{ \aleph_1 \cup \aleph_2(3*1), \aleph_1 \cup \aleph_2(1) \}$$

$$\max \{ \aleph_1(3), \aleph_2(3) \} \geq \min \{ \aleph_1 \cup \aleph_2(2), \aleph_1 \cup \aleph_2(1) \}$$

$$\max \{ 0.3, 0.2 \} \geq \min \{ \max \{ \aleph_1(2), \aleph_2(2) \}, \max \{ \aleph_1(1), \aleph_2(1) \} \}$$

$$\max \{ 0.3, 0.2 \} \geq \min \{ \max \{ 0.3, 0.5 \}, \max \{ 0.4, 0.6 \} \}$$

$$0.3 \geq \min \{ 0.5, 0.6 \}$$

$$0.3 \not\geq 0.5$$

**E. Theorem** Let  $\aleph$  be a fuzzy INK-ideal of  $\tilde{N}$ . The  $F-\tilde{q}$ -translation  $\aleph_{\tilde{q}}$  is a F-INK-ideal extension of  $\aleph$ .

**Proof.** If  $\aleph$  is a F-INK-ideal of  $\tilde{N}$ , then we know that by theorem A, the fuzzy  $\tilde{q}$ -translation  $\aleph_{\tilde{q}}$  of  $\aleph$  is F-INK-ideal of  $\tilde{N}$ . Now,  $\aleph_{\tilde{q}}(p) = \aleph(p) + \tilde{q} \geq \aleph(p)$ , for all  $p \in \tilde{N}$ . Hence fuzzy  $\tilde{q}$ -translation  $\aleph_{\tilde{q}}$  is a F-INK-ideal extension of  $\aleph$ .

**F. Theorem.** Let  $\aleph$  be a F-INK-ideal of  $\tilde{N}$  and  $\tilde{q} \geq \eta$ , with  $\tilde{q}, \eta \in [0, T]$ , then  $F-\tilde{q}$ -translation  $\aleph_{\tilde{q}}$  of  $\aleph$  is a F-INK-ideal extension of the  $F-\eta$ -translation  $\aleph_{\eta}$  of  $\aleph$ .

**Proof.** Let  $\aleph$  be a fuzzy INK-ideal of  $\tilde{N}$ . Then by theorem 3.1.5 the fuzzy  $\tilde{q}$ -translation  $\aleph_{\tilde{q}}$  of  $\aleph$  and the fuzzy  $\eta$ -translation  $\aleph_{\eta}$  of  $\aleph$  are fuzzy INK-ideal of  $\tilde{N}$ , for all  $\tilde{q}, \eta \in [0, T]$ . Since  $\tilde{q} \geq \eta, \aleph(p) + \tilde{q} \geq \aleph(p) + \eta$ .

Therefore,  $\aleph_{\tilde{q}}(p) \geq \aleph_{\eta}(p)$ .

Hence  $\aleph_{\tilde{q}}$  is a fuzzy INK-ideal extension of  $\aleph_{\eta}$ .

**G. Theorem** If  $\aleph$  is a fuzzy INK-ideal of  $\tilde{N}$  then the fuzzy  $\eta$ -multiplication of  $\aleph$  is a fuzzy INK-ideal of  $\tilde{N}$ , for all  $\eta \in [0, 1]$ .

**H. Theorem.** Let  $\aleph$  be a FS of  $\tilde{N}$ .  $\tilde{q}$  in  $[0, T]$  and  $\eta$  in  $(0, 1]$ . If the  $F-\eta$ -multiplication  $\aleph_{\eta}^M(p)$  of  $\aleph$  is a F-INK-ideal of  $\tilde{N}$  is a F-INK-ideal of  $\tilde{N}$ , then the  $F-\tilde{q}$ -translation  $\aleph_{\tilde{q}}$  of  $\aleph$  is a F-INK-ideal extension of  $\aleph_{\eta}^M$ .

**Proof.** Let  $\tilde{q}$  in  $[0, T], \eta \in (0, 1]$  and  $\aleph_{\eta}^M(p)$  of  $\aleph$  is a fuzzy INK-ideal of  $\tilde{N}$ . Then  $\aleph$  is a F-INK-ideal of  $\tilde{N}$ . By theorem 3.1.2,  $\aleph_{\tilde{q}}$  of  $\aleph$  is a F-INK-ideal of  $\tilde{N}$ . Now,  $\aleph_{\tilde{q}}(p) = \aleph(p) + \tilde{q} \geq \aleph(p) \geq \aleph(p) \eta = \aleph_{\eta}^M(p)$ .

Therefore,  $\aleph_{\tilde{q}}$  of  $\aleph$  is a fuzzy INK-ideal extension of  $\aleph_{\eta}^M$ .

### VI. CONCLUSION

In this concept of translation of F-INK-ideal in INK-algebra are familiarized and examined some of their beneficial assets. We have exposed that the  $F-\tilde{q}$ -translation of a F-INK-ideal is a F-INK-ideal extension but then the converse is not factual. It is correspondingly exposed that intersection of F-INK-ideal extension of a FS- is a F-INK-ideal extension but union of F-INK-ideal extension of a fuzzy subset is not a F-INK-ideal extension. The associations are conversed between FT, FE and F-multiplications of F-INK-I in INK-algebras.

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