International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-8 Issue-12, October 2019

Two Stage Flow Shop Scheduling Model Including Transportation Time with Equipotential Machines at Every Stage

Deepak Gupta, Sonia Goel

Abstract: ***This study presents a solution algorithm for the problem of minimizing the makespan on equipotential parallel machines at every stage in two stage flow shop scheduling model. The processing time of all the jobs on all the two machines is given and the time for which parallel equipotential machines are available is also given. Transportation time for moving the jobs from first machine to second machine is also taken into consideration. A mathematical illustration is also given in support of the algorithm proposed.***

Keywords: Scheduling, Elapsed time, equipotential machines, operating cost, transportation time

I. INTRODUCTION

Scheduling models concern with the determination of an optimal sequence in which to service customers, or to perform a set of jobs, in order to minimize total elapsed time or another suitable measure of performance. Scheduling of jobs has been a fruitful area of research for many decades, in which order of processing of jobs is decided.

Scheduling problems are used in production concerns where the production of some items is made into distinct but successive stages .At each stage there is one or more than one machine to perform the required set of jobs.

In our daily life there are many practical situations in which we are required to sequence the jobs as well as allotment of machines for processing in given set of jobs. In these types of problems there is more than one machine of one type like printing of books in a publication house. Let A denote a printing machine, B denote the binding machine. Let there are m equipotential printing machines and p equipotential binding machines. Let the jobs be processed in order A, B. It is supposed that each job for processing on machine A,B may be done in different parts, each part being processed on one of the equipotential machine of type A or type B . Similarly each job for processing on machine B may be done in different parts, each part being processed on one of the equipotential machine of type B. When all the different parts of the same job are processed completely on machine A, the job as a whole is processed on machines B in the similar manner. The essential investigation in flow shop scheduling has been made by Johnson (1954) [1]. The effort was explored by Ignall and Scharge (1965) [3] and other researchers by considering various parameters.

Revised Manuscript Received on October 30, 2019.
***** Correspondence Author

Deepak Gupta*, Professor, Department of Mathematics, M. M. (Deemed to be University), Mullana (Ambala) Haryana, India.

Sonia Goel, Research Scholar, Department of Mathematics, M. M. (Deemed to be University), Mullana (Ambala) Haryana, India.

Lomnieki(1965)[2] gives Branch and Bound technique to minimize the elapsed time in two or three stage flow shop scheduling problem. Ignall, E. and Scharge L. (1965) [4] gives Application of Branch and Bound method to some scheduling problems. Narain Laxmi(2003) [6], Narain Laxmi and Bagga P.C. (2005)[7] worked on scheduling problems in various hiring situations .Gupta D., Singla P., Singh H. (2012)[10], Gupta D., Singla P., Bala S. (2013)[11], studied two stage flow shop scheduling problems using Branch and Bound technique including a variety of parameters.

Lee, H.T. (1991) [5] studied some class of parallel machines problems. The conventional parallel machine scheduling includes scheduling of n jobs over m identical parallel machines .Every job is measured with a deterministic dispensation time with the objective of optimizing different criterion .various researchers work on parallel machine problems under deterministic environment and fuzzy environment with the objective of optimizing different criterion like elapsed time, number of tardy jobs, weighted flow time, maximum completion time etc. Deepak Gupta and Sameer Sharma (2015)[12] worked on parallel machines involving weighted flow time and maximum tardiness .Sharma S., Sharma S., Gupta D., Gulati N (2016)[13] explored Bi-objective parallel machines problems under fuzzy environment. But in actual life situations in most problems of production there are two or three machines of different type and at each stage there are more than one machine of each type. Various researchers like Sunita and Singh (2008)[8] studied the problems that include m identical parallel machine at first stage and single machine at second and third stage that includes transportation time in moving the jobs from one machine to other. Deepak Gupta and Sonia Goel (2018) [14] deal with m parallel machines for the first machine by associating probabilities with processing time of jobs. In this research, we extend the work done by, Sunita Bansal (2008) [8], Deepak Gupta (2018) [14] by taking the parallel machines at each stage including transportation time.

Notations

a_i = Dispensation time of i^{th} job on machine A
b_i = Dispensation time of i^{th} job on machine B
A_j = m equipotential machines for machine A
B_j = p equipotential machines for machine B
t_i = Transportation time of moving the jobs from machine A to machine B
C_{ij} (i = 1, 2….n; j = 1, 2, 3…..m) unit cost of i^{th} job on A_j^{th} parallel machine
D_{ij} (i = 1, 2….n; j = 1, 2, 3…..p) unit cost of i^{th} job on B_j^{th} parallel machine

II. PROBLEM FORMULATION

Suppose there are m equipotential machines $A_1, A_2, \ldots, A_m$ of type A, p equipotential machines $B_1, B_2, \ldots, B_p$ of type B. Suppose n jobs are to be processed on machines A and B in the order AB, with no passing allowed. Also the jobs are assumed to be completed in the parts on like/equipotential machines of type A .After completing on machine A the jobs are assumed to be processed in the parts on like/equipotential machines of type B in the similar manner .Let t_i be the time of transporting the semi finished jobs from machine A to machine B for further processing .The processing time for which equipotential machines $A_1, A_2, \ldots, A_m$ and $B_1, B_2, \ldots, B_p$ are available is given by $t_{11}, t_{12}, \ldots, t_{1m}$ and $t_{21}, t_{22}, \ldots, t_{2p}$ respectively. Let a_i and b_i (i = 1, 2, n) be the processing time of jobs on machine A and B respectively. Let C_{ij} denote the operating cost per unit time of job i on machine A_j for all i= 1,2,…,n and j=1,2,...,m. D_{ij} denote the operating cost per unit time of job i on machine B_j for all i= 1,2,…,n and j=1,2,…,p. The goal of this research is to find the best possible schedule of jobs which will diminish the total elapsed time and allocates in an optimal manner processing time of jobs of machine A on like/equipotential machines A1, $A_2, \ldots, A_m$, processing time of jobs of machine B on like/equipotential machines B1 B_2 $B_3, \ldots$, Bp, so that total cost of doing the jobs is minimized.

III. MATHEMATICAL MODEL

Table 1. Mathematical model of the problem

	A			B	
Jobs / Machines	A_1 A_2 …A_m	a_i	t_i	B_1 B_2 …B_p	b_i
(1)	C_{11} $C_{12}\ldots C_{1m}$	a_1	t_1	D_{11} $D_{12}\ldots D_{1p}$	b_1
(2)	C_{21} $C_{22}\ldots C_{2m}$	a_2	t_2	D_{21} $D_{22}\ldots D_{2p}$	b_2
(3)	C_{31} $C_{32}\ldots C_{3m}$	a_3	t_3	D_{31} $D_{32}\ldots D_{3p}$	b_3
.	.			.	
(n)	C_{n1} $C_{n2}\ldots C_{nm}$	a_n	t_n	D_{n1} $D_{n2}\ldots D_{np}$	b_n
Available Time	t_{11} $t_{12}\ldots t_{1m}$			t_{21} $t_{22}\ldots t_{2p}$	

Assumptions

1. The jobs can be done in parts i.e. either a job can be completed wholly on one machine of type A or B or on two or more than two machines of type A or B respectively.
2. Each machine follows the same sequence of operations.
3. A_{ij} denotes the optimal allocation of dispensation time a_i of job i to machine A_j (j=1, 2, m), $A_{ij} \geq 0$, $A_{ij} = 0$, means the job i will not visit the machine A_j.
4. Each job after being completed on first machine A is then processed on machine B.
5. All the parallel machines of type A and type B can start at the same time.
6. It is not necessary for a job to be processed on each of the m similar machines of type A, p parallel machines of type B.
7. Each machine of type A and type B has different operating cost.

1. ALGORITHM

Step I Create two fabricated machines G and H with their dispensation time a'_i and b'_i as follows:

$a'_i = a_i + t_i$ and $b'_i = b_i + t_i$

Step II Check the condition

$$\sum_{j=1}^{m} t_{1j} = \sum_{i=1}^{n} Gi \;,\; \sum_{j=1}^{p} t_{2j} = \sum_{i=1}^{n} H_i$$

Find out the optimal allocation of processing time of all the jobs on machine A and machine B to the like/equipotential machines A_j and B_j respectively given by A_{ij} for j = 1,2…m and B_{ij} for j = 1,2,…,p by applying Modified distribution method

Step III Calculate

$$g_1 = \max\left\{\sum_{i=1}^{n} A_{ij}\right\}_{j=1,2,\ldots m} + \min_{i\in j_r'}\left\{\max B_{ij}\right\}_{j=1,2..p}$$

$$g_2 = \left\{\max_{i\in j_r} A_{ij}\right\}_{j=1,2,..m} + \max\left\{\sum_{i=1}^{n} B_{ij}\right\}_{j=1,2,\ldots,p}$$

Step IV Calculate

$$g = \max\{g_1, g_2\}$$

Step V We calculate g for the n-module of permutations, starting from 1, 2, 3,...,n correspondingly, having categorized the suitable vertices of the scheduling hierarchy by such values.

Step VI Investigate the apex for the least possible label. Calculate g for the (n-1) sub-modules initiating with this vertex. Once more deliberate on the least possible label vertex. Progressing like this, until we accomplish the end of the tree represented by two sole permutations for which we estimate the entire work length. In this way, we get the most favourable schedule of the jobs.

Step VII Organize in/out table for the most favourable order obtained in step IV and obtain the least amount total elapsed time.

Step VIII Show the diagrammatic representation for the Branch and Bound technique (Numerical problem).

IV. NUMERICAL PROBLEM

Find the most favourable sequence of jobs to diminish the elapsed time where four jobs are processed on two machines A, B in the order AB given in Table 2:

Table 2.Numerical problem

Jobs / Machines	A			t_i	B		
I	A_1	A_2	a_i		B_1	B_2	b_i
1	9	11	9	1	12	8	8
2	13	8	8	1	9	11	10
3	15	12	9	2	7	10	13
4	8	7	11	1	15	12	12
Available Time	18	24			30	18	

Where there are two machines A_1, A_2 of type A, two machines B_1, B_2 of type B available for different time, having different operating costs which are given as follows:

SOLUTION

As per **step I** create two fabricated machines G and H given by Table 3.

Table 3.Fabricated machines G and H

Jobs / Machines	G			H		
I	A_1	A_2	a'_i	B_1	B_2	b'_i
1	9	11	10	12	8	9
2	13	8	9	9	11	11
3	15	12	10	7	10	15
4	8	7	12	15	12	13
Available Time	18	24		30	18	

Step 2: Find the optimal allocation of processing time a'_i (i = 1,2,3,4) , b'_i (i = 1,2,3,4) of job i to like/equipotential machine A_j (j = 1,2) ,B_j (j = 1,2) by applying modified distribution method as shown in table 4 and table 5:

Table 4.Optimal allocation of processing time a'_i on machine A_1 and A_2

Jobs / Machines	A_1	A_2
1	10	0
2	0	9
3	0	11
4	8	4

Table 5. Optimal allocation of processing time b'_i on machine B_1 and B_2

Jobs / Machines	B_1	B_2
1	0	9
2	11	0
3	15	0
4	4	9

Now the reduced problem can be written as shown in Table 6.

Table 6.Reduced problem

Jobs / Machines	A_1	A_2	B_1	B_2
1	10	0	0	9
2	0	9	11	0
3	0	11	15	0
4	8	4	4	9

As per **step 3** and **step 4** calculate lower bound for one scheduled job and find the maximum value.

LB (1) = Max {24+9, 10+30} = Max {33, 40} = 40

Similarly,

LB (2) = Max {24+9, 9+30} = Max {33, 39} =39

LB (3) = Max {33, 41} = 41

LB (4) = 38.

Minimum value of lower bound is 38 that correspond to job 4. Hence we fix job 4 at the first place in the optimal sequence and proceed to fix the second job of the optimal sequence.

As per **step-5** and **Step-6** calculate lower bound for two scheduled jobs.

LB (41) = Max {35, 48} = 48

LB (42) = Max {33, 43} = 43.

LB (43) = Max {33, 45} = 45

Minimum value of lower bound is 43 and it is for the subsequence 42. Hence we fix job 2 at second place in the optimal sequence. Fixing the job 4 and job 2 at first and second position respectively, we proceed to find the third job to be done in the optimal sequence.

Now we calculate the lower bound of three scheduled jobs as

LB (421) = Max {39, 48} = 48.

LB (423) = 54.

Here minimum value of lower bound is 48 that correspond to subsequence 421. Hence, 4, 2, 1 jobs will have first, second and third places in the optimal sequence respectively and the remaining job 3 will have the fourth position

Therefore, the optimum sequence is (4213).

As per step 7 make In-out table for the optimal sequence shown in Table 7 and find the total elapsed time.

Table 7. In-out table for optimal sequence

jobs	A_1	A_2	B_1	B_2
4	0-8	0-4	8-12	8-17
2	-	4-13	13-24	-
1	8-18	-	-	18-27
3	-	13-24	24-39	-

Total elapsed time = 39 hours

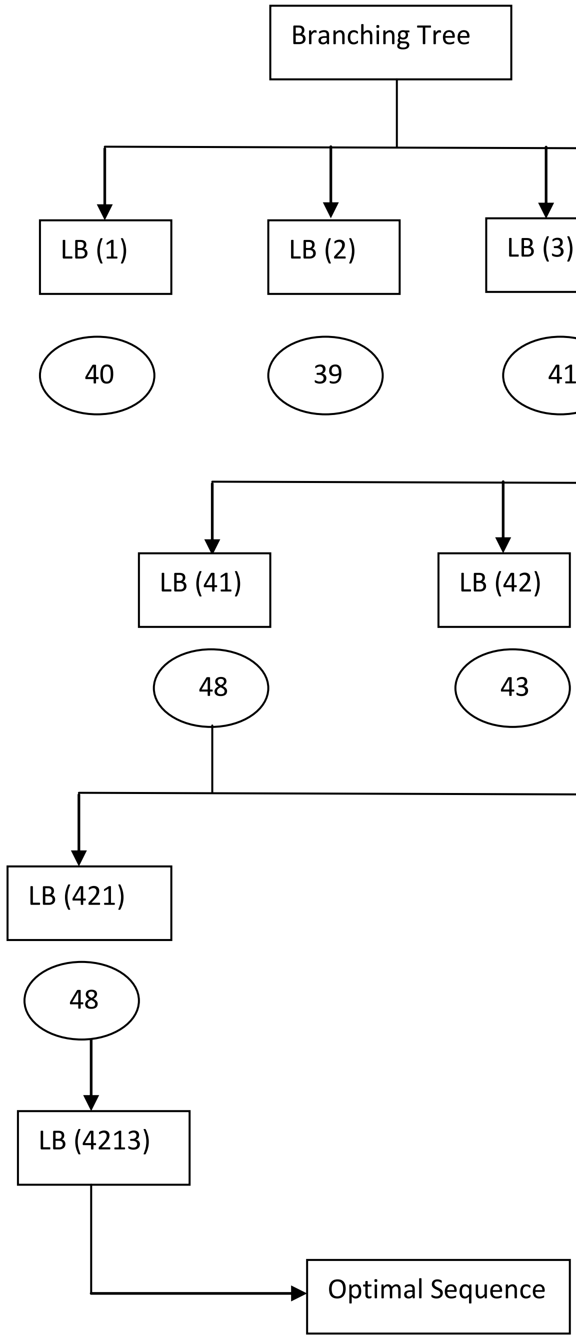

Figure 1. Diagrammatic representation for Branch and Bound technique (example)

As per step VIII show the diagrammatic representation of Branch and Bound technique (Numerical Problem) in Figure 1.

V. REMARKS

The algorithm proposed in this paper gives minimum elapsed time. In the stated problem if we take single machine of type B, work is similar to that of Sunita Bansal et.al. (2008). Work can further be extended by taking three machines A, B, C with equipotential machines at every stage or by taking the processing time of all the jobs in fuzzy environment.

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AUTHORS PROFILE

Dr. Deepak Gupta is renowned personality working in the field of Operation Research. He is M.Sc., M. Phil., Ph.D. and working as a Professor & Head in the Department of Mathematics, Maharishi Markandeshwar Engineering College, Maharishi Markandeshwar (deemed to be University), Mullana, Ambala, Haryana, India. He is prolific author of over eighteen books of higher mathematics. He has had many statutory positions at Maharishi Markandeshwar University. He has to his credit more than two hundred research papers published in National/International Journal of repute. He has supervised eighteen candidates for Ph.D. and sixteen candidates for M. Phil. He has also written some books for under graduate students.

Sonia Goel is a Research Scholar at Maharishi Markandeshwar (Deemed to be University) Mullana Ambala, INDIA. She has completed her master degree from Kurukshetra University Kurukshetra.